



# ***ARRAY PROCESSING II (2012)***

-Course Notes at:

<http://www.cttc.es> “training” “Graduate and undergraduate lectures.

Spanish: ARRAY PROCESSING II (Chapters 0-11)

English: MIMO PROCESSING (English Version)

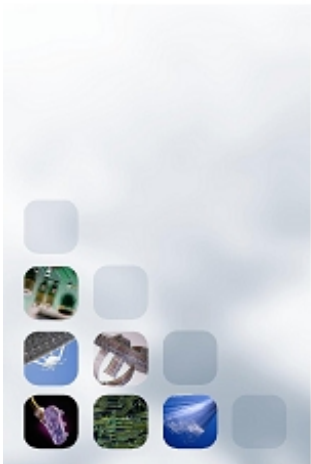
Recommended: ARRAY PROCESSING I (Spanish)

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[m.a.lagunas@cttc.es](mailto:m.a.lagunas@cttc.es) <http://www.cttc.es> Final Exam.



## *ARRAY PROCESSING II (2012)*

- March 8 NO
- Feb-March: M.A.Lagunas
- April 23: Ana Perez-Neira
- Final exam



innovating communications

## The Centre Tecnològic de Telecomunicacions de Catalunya

*A gateway to advanced communication technologies*

# MIMO:INTRODUCTION

Miguel Ángel Lagunas

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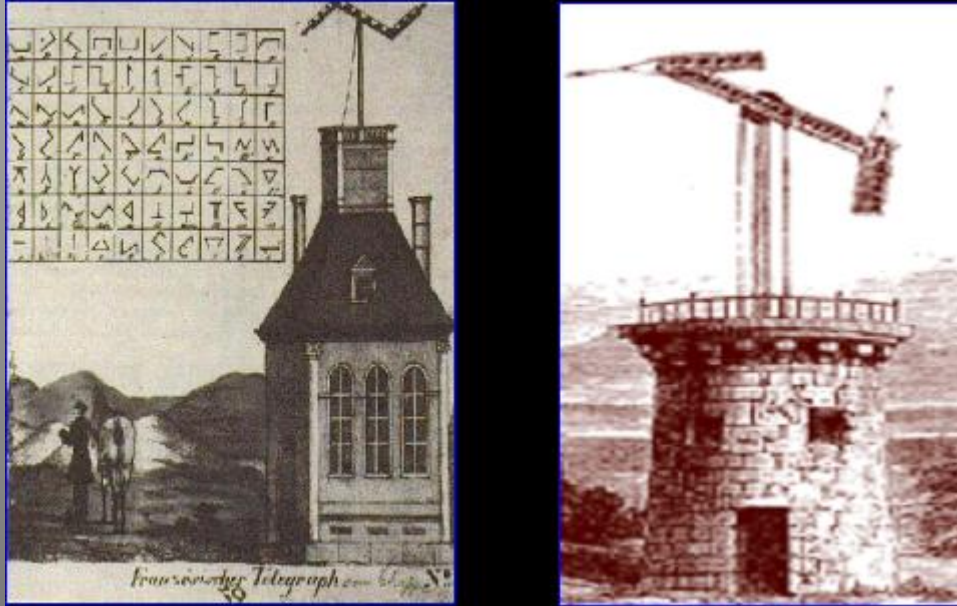
# The roots of communications



$$B \cdot B^H = H \cdot s1(n)$$

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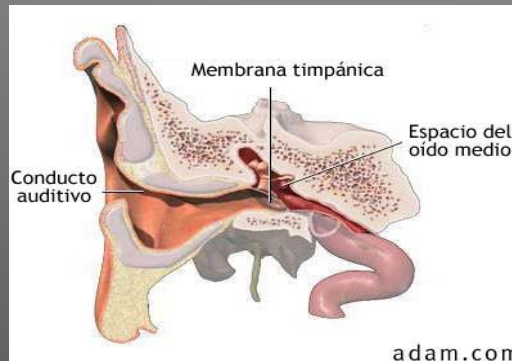


0.33 bits/sec

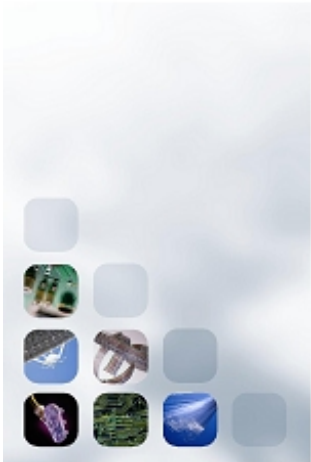
Chapé / Betancourt

100 Kbps.

15 Mbps.







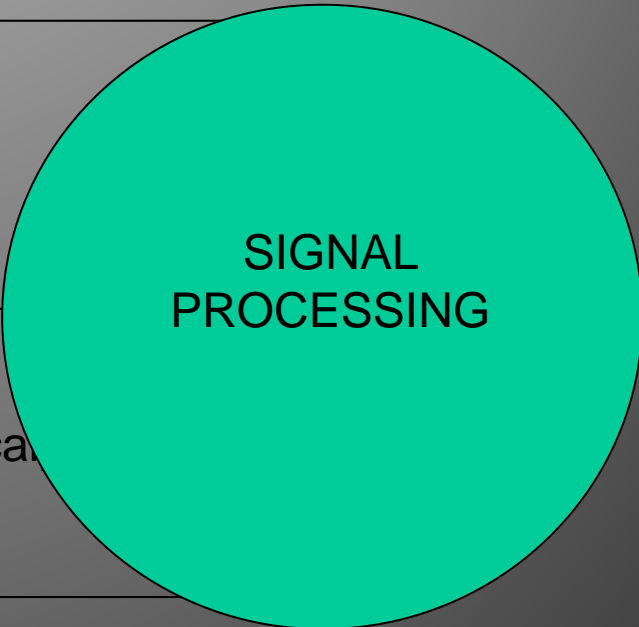
# Comm. Layers

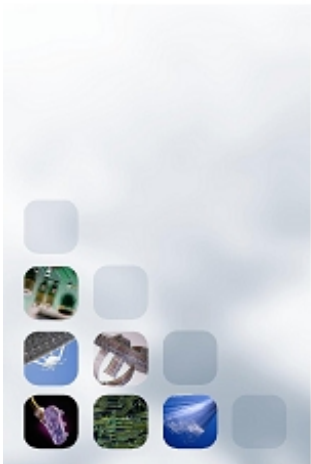
Session

⋮

Access Layer

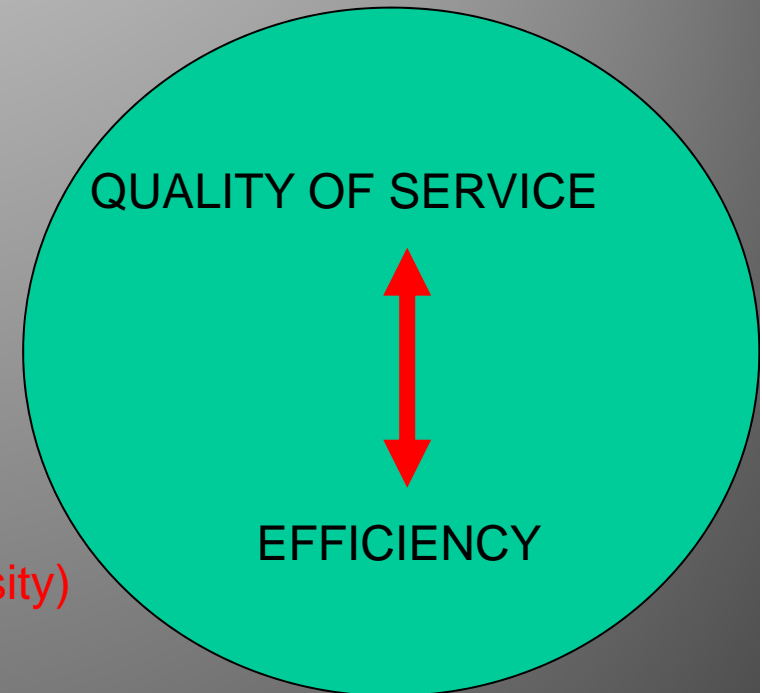
Physical Layer (wired, wireless, optical)





## Wireless Systems

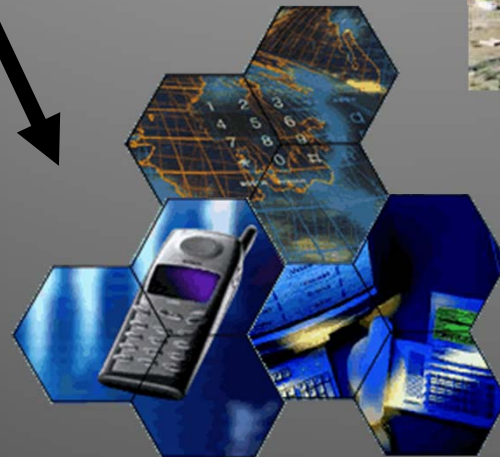
- Challenges
  - Higher data rate
  - Wider coverage
  - Energy efficient
  - Affordable
  - Reliable
- Possible solution
  - **Multiple antenna systems:**  
**ANTENNA ARRAYS (Spatial diversity)**



# Combination of Wired Communications / Spatial Diversity contributions



DSL Comm.



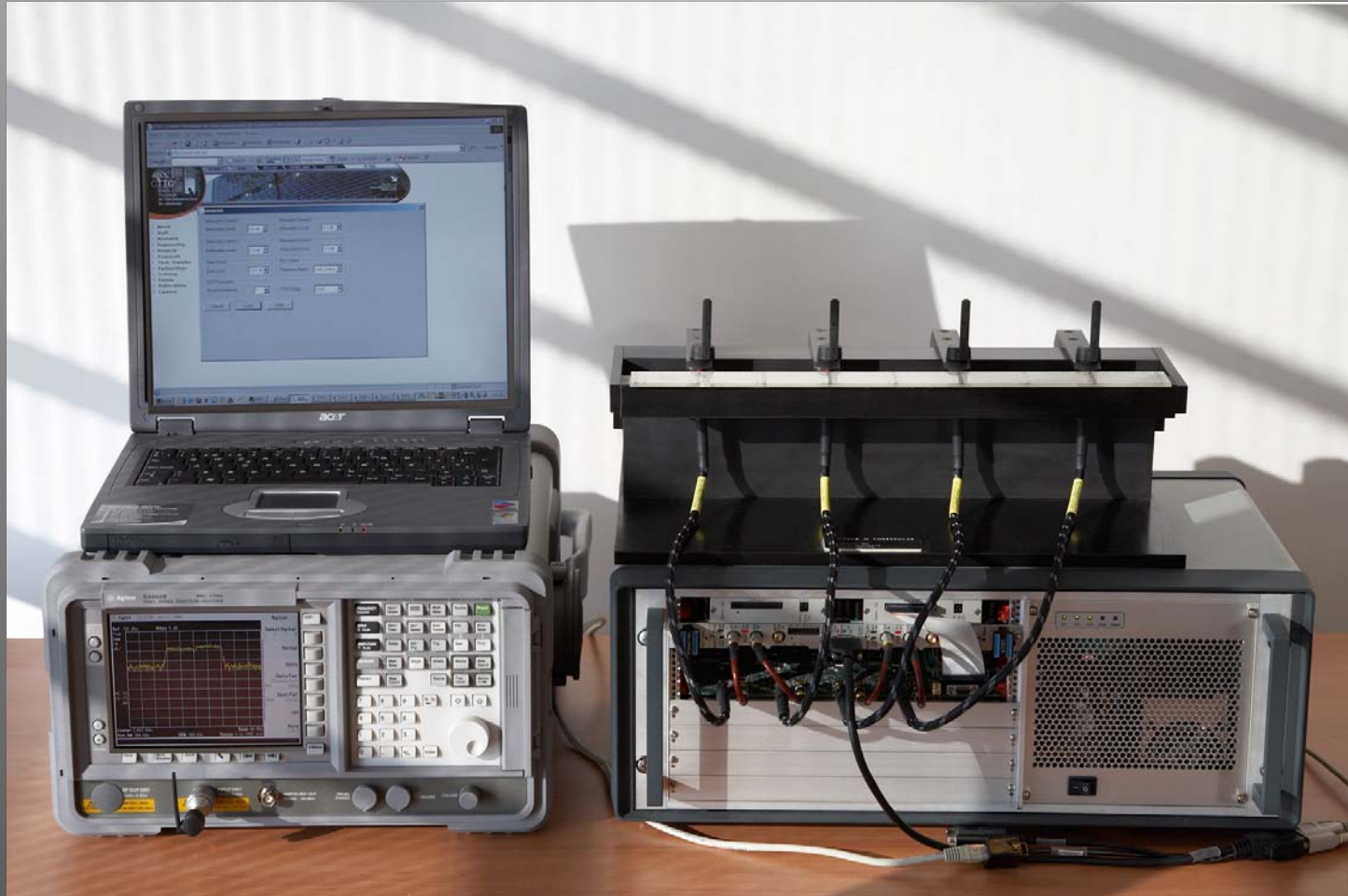
Radar/Sonar

Wireless Communications





# Acoustics/Sonar/Radar/Civil Eng./Wireless







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## PROYECTORES DE SONIDO. El Cine en Casa desde un sólo elemento



Los sistemas YSP de Yamaha incluyen en un sólo componente gran número de altavoces de reducidas dimensiones los cuales y mediante un avanzado método de aplicación de tiempo de retardo entre ellos permite proyectar "haces" sonoras que pueden ser orientados de manera precisa para conseguir un efecto sonoro envolvente óptimo.

Los haces direccionados producen ondas sonoras directas y otras reflejadas creando un verdadero sonido envolvente multicanal así como sonido estereo de alta calidad o en 3 canales para el máximo realismo en conciertos musicales. La ruptura tecnológica de los sistemas YSP abren una nueva era en el cine en casa facilitando su instalación y adaptándose a cualquier decoración. En cine en Casa sin cables.

¡ SOLICITE UNA DEMOSTRACION !



YSP-4000

- Dimensiones : 1030 (An) x 198 (al) x 144 (F) mm
- 42 altavoces con "TruBass"
- Potencia total: 120W
- Incorpora sintonizador FM con RDS

- Salida HDMI (repeater) compatible 1080p con conversión ascendente de video (a 1080i)
- Sistema de calibración automática "IntelliBeam" con comienzo directo

- Entrada de minijack frontal para lectores de MP3 externos con sistema de mejora de música comprimida
- Nuevos modos: 5ch stereo y My Surround
- Disponible plata y negro

PARA 42" Y SUPERIORES



Disponible en plata

**PVP: 1.319€**



YSP-3000

- Dimensiones: 800 (An) x 155 (al) x 152 (F) mm
- 23 altavoces con "TruBass"
- Potencia total: 82W

- Incorpora sintonizador FM con RDS
- Salida HDMI (switcher) compatible 1080p
- Sistema de calibración automática "IntelliBeam" con comienzo directo

- Nuevos modos: 5ch stereo y My Surround
- Disponible plata y negro

PARA 32" Y SUPERIORES

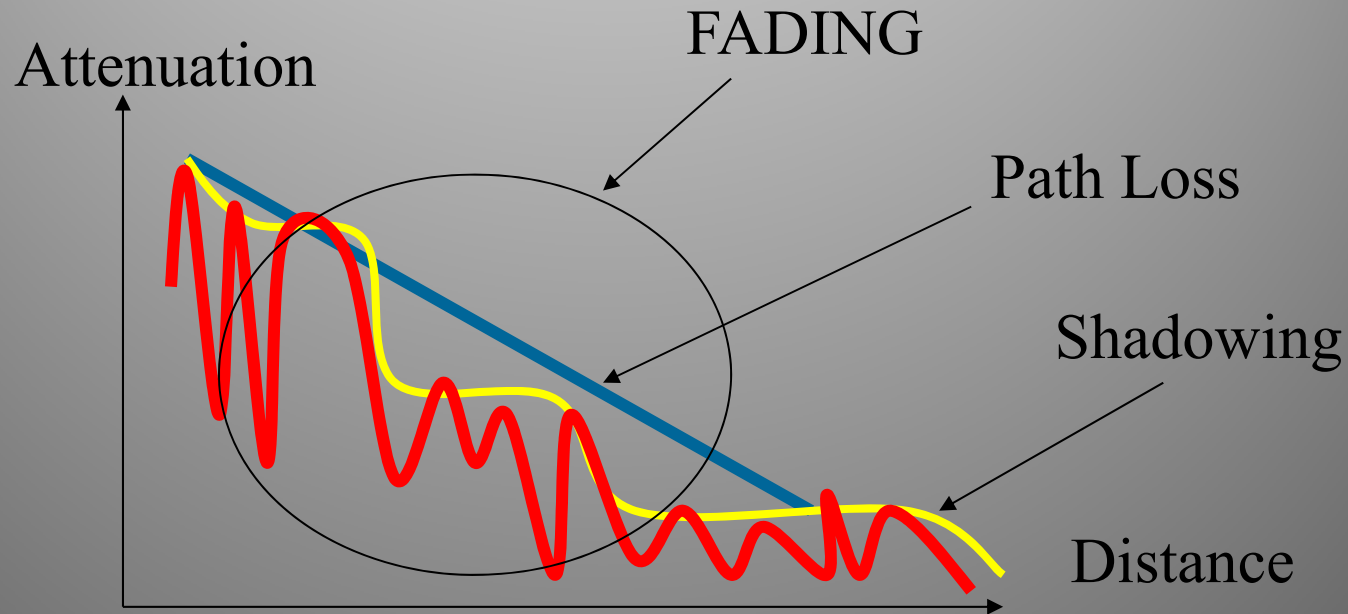


Disponible en negro

**PVP: 849€**



# The Wireless Channel



Frequency Selective

$$h(t) = \alpha \cdot g(t)$$

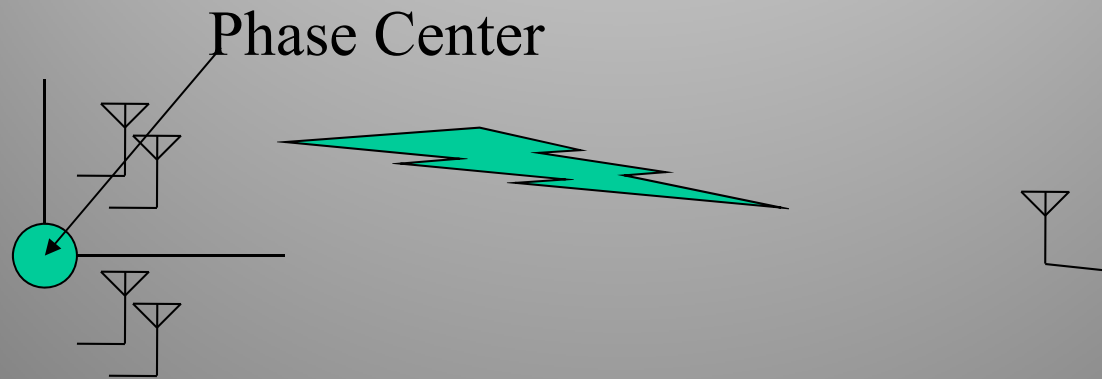
Flat Fading

$$h(t) = \alpha$$



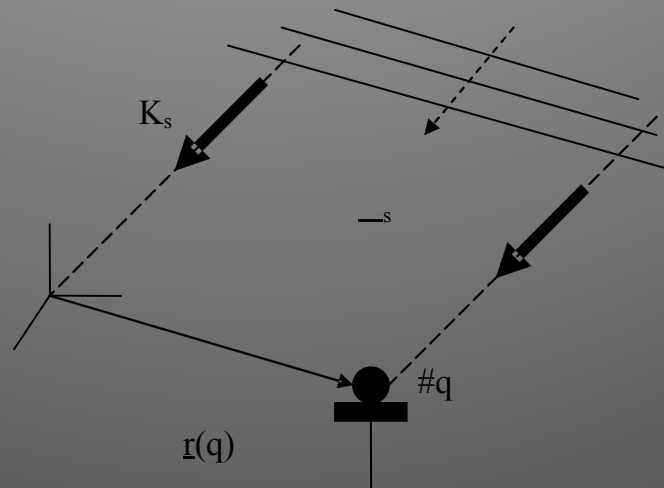
Multicarrier Systems -> OFDM, Poly-Phase Filter Bank

# THE SPATIAL CHANNEL (LOS)



LOS: Line of Sight

$$e^{j\omega_0 t} \longrightarrow e^{j\omega_0 t} \cdot e^{j \cdot K_s \cdot r_q}$$







$$\underline{K}_s = \frac{2\pi}{\lambda} (\text{sen}(\theta_s) \cdot \cos(\varphi_s), \text{sen}(\theta_s) \cdot \text{sen}(\varphi_s), \cos(\theta_s))$$

$$\underline{r}_q = d_q (\cos(\varphi_q), \text{sen}(\varphi_q), 0)$$

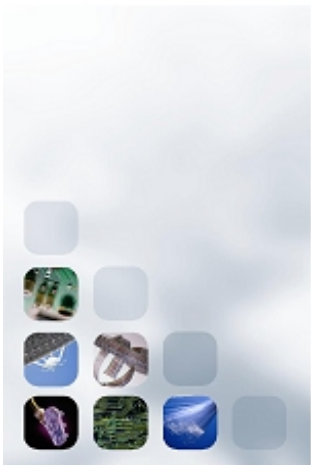
Wideband-Narrowband  $\rightarrow$  Small group delay

$$x_q(t) = x_s(t - \tau_{qs}) \cong a_s(t) \cdot \exp(-j2\pi f_o t) \cdot \exp(j\zeta_{qs});$$

$$\zeta_{qs} = 2\pi f_o \cdot \tau_{qs} = \frac{2\pi f_o}{c} \cdot d_q \cdot \text{sen}(\theta_s) \cdot \cos(\varphi_s - \varphi_q)$$

$$\underline{h} = \alpha \cdot \underline{S}_s \quad \text{Flat-Fading, LOS STEERING VECTOR}$$

Source Signal  $x_r(t) = s(t) \cdot \alpha \cdot \underline{S}_s + \underline{w}(t)$



The path loss is almost deterministic in the LOS scenario for frames of continuous per channel uses of the link.

The receiving aperture captures all the symbol information of the source in a single snapshot

$$\underline{Y}_n = s(n) \cdot \alpha \underline{S}_s + \underline{w}_n$$

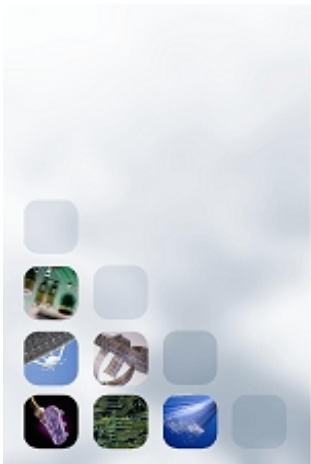
Snapshot

Steering  
vector of the  
source

Noise and  
interference

This requires “small” size  
apertures

$$\frac{D}{\lambda_c} < \frac{f_c}{B}$$



When multipath do not produces ISI the model is still valid but the steering vector multiplied by the path loss pass to be the “Spatial signature” of the source.

$$\underline{Y}_n = s(n) \cdot \underline{b}_s + \underline{w}_n$$

where

$$\underline{b}_s = \sum_1^{\#ofpaths} \alpha_s \underline{S}_s$$

Note that the received vector due to the source still preserves the rank one structure. When the group delay in the multiple paths cannot be neglected, the received vector increases its rank

$$\underline{Y}_t = \sum_1^{\#ofpaths} s(t - \tau_s) \alpha_s \underline{S}_s + \underline{w}_t$$





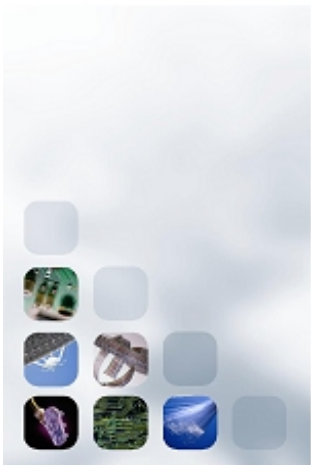
When the group delay is not small across the aperture (i.e. the spatial dimension of the aperture is high), the receiver suffers ISI.

The receiver, in order to capture the information of a given transmitted symbol needs to redefine a new snapshot including time-diversity.

$$\underline{Y}_{=n} = \left[ \underline{Y}_n \quad \underline{Y}_{n-1} \quad \underline{Y}_{n-2} \quad \dots \quad \underline{Y}_{n-L} \right]$$

Where  $L$  (LT seconds) is the time that is between the signal arrives to the first antenna and fully appears in the last antenna due to the propagation across the aperture.





## The NLOS contribution

Most of the wireless systems work under only NLOS channel conditions

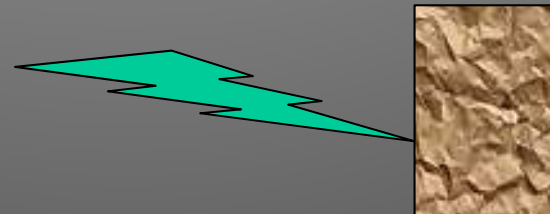
$$\underline{h} = \underline{\bar{h}} + \underline{h}_{NLOS}$$

Channel  
Mean LOS

Rayleigh component  
NLOS

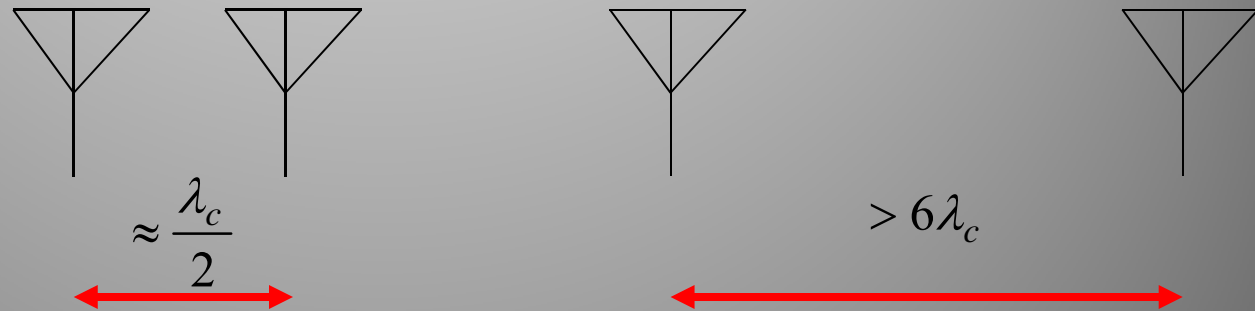
$$\underline{h} = G(\underline{\bar{h}}, \underline{\underline{\Sigma}})$$

- Steering and/or Scattering cluster





## Remember



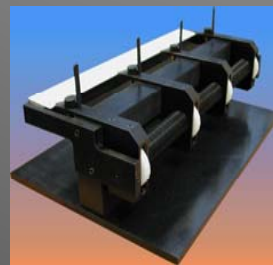
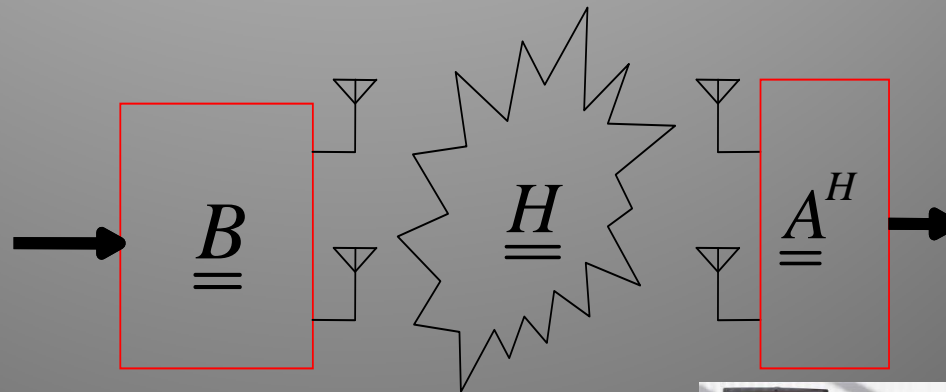
- Full coherent receiver aperture
- $h_{\text{NLOS/LOS}}$  almost the same for all antennas
- Aiming high QoS
- Low BER and high SNR
- Channels uncorrelated, i.e. covariance of the channel matrix diagonal
- Aiming Capacity
- MULTIPLEXING capabilities



## Multi-Antenna Tx/Rx (MIMO)

- INCREASE SPECTRAL EFFICIENCY -> RATE
- 4x4 -> 16 Channels (???) RELIABILITY

QUALITY/RATE trade-off



The radiated power differs substantially from the used power ***and RF effects***

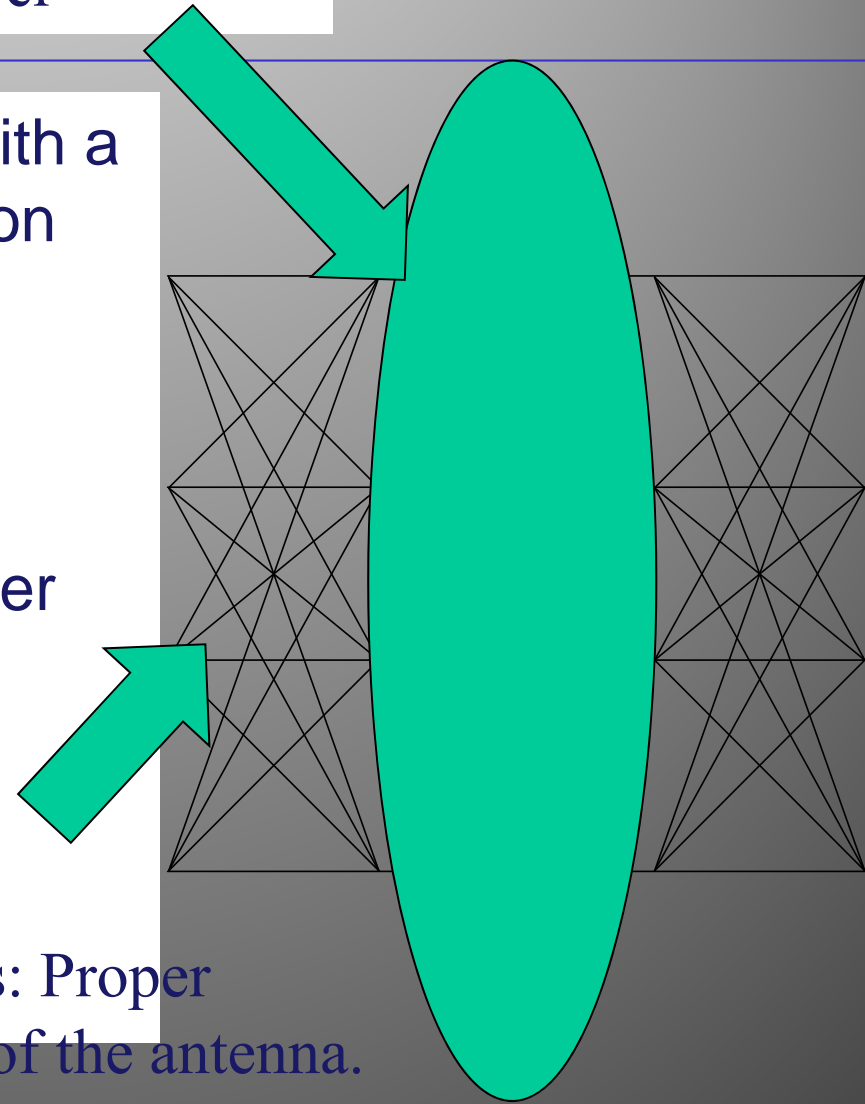
- $\mathbf{H}$  is modeled statistically with a general Gaussian distribution

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{R}_R^{1/2} \mathbf{G} \mathbf{R}_T^{T/2}$$

- Also known as the Kronecker model model

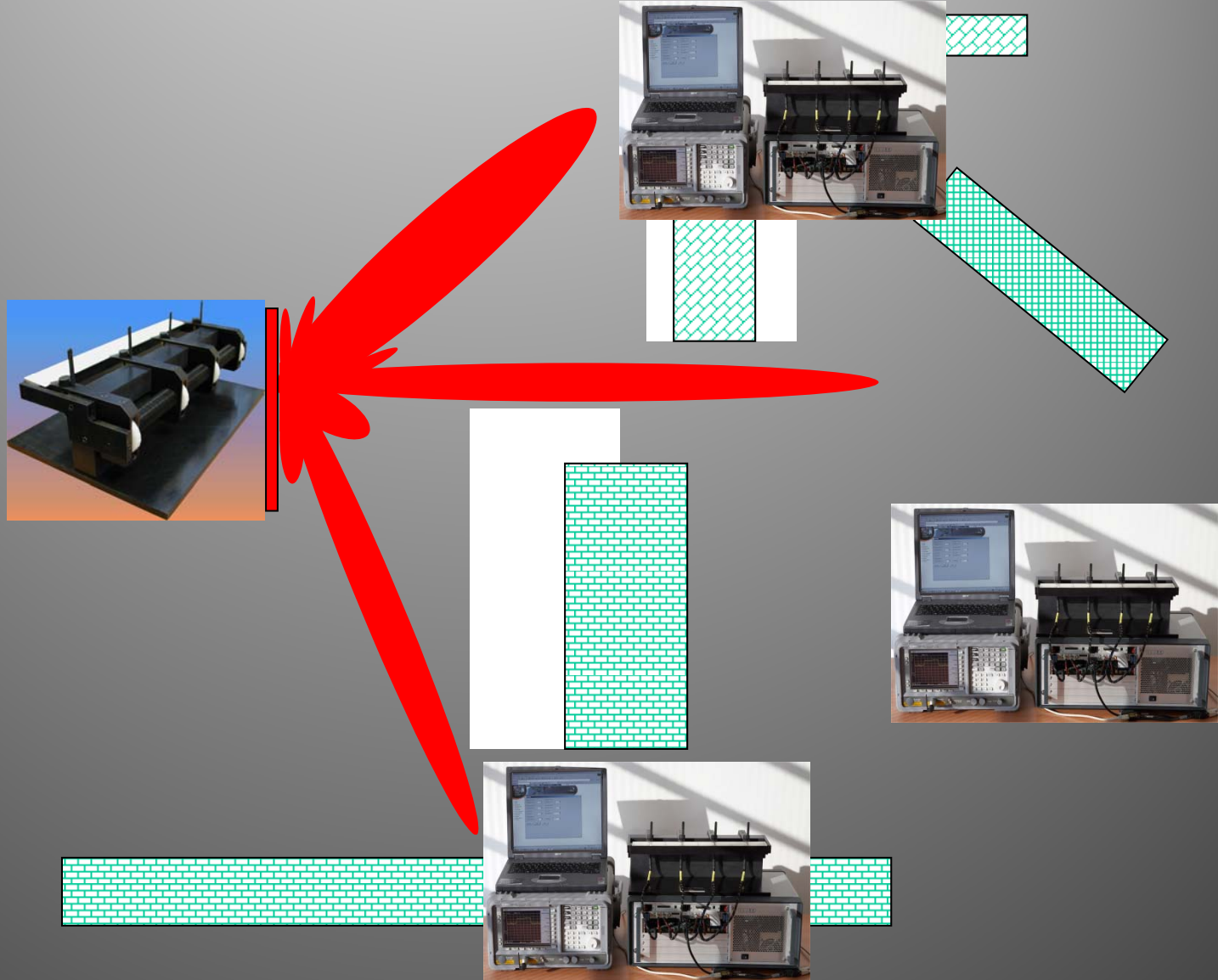
$$\mathbf{H} \sim \mathcal{CN}(\bar{\mathbf{H}}, \mathbf{R}_T \otimes \mathbf{R}_R)$$

RF Effects: Proper matching of the antenna. Mutual coupling, etc.





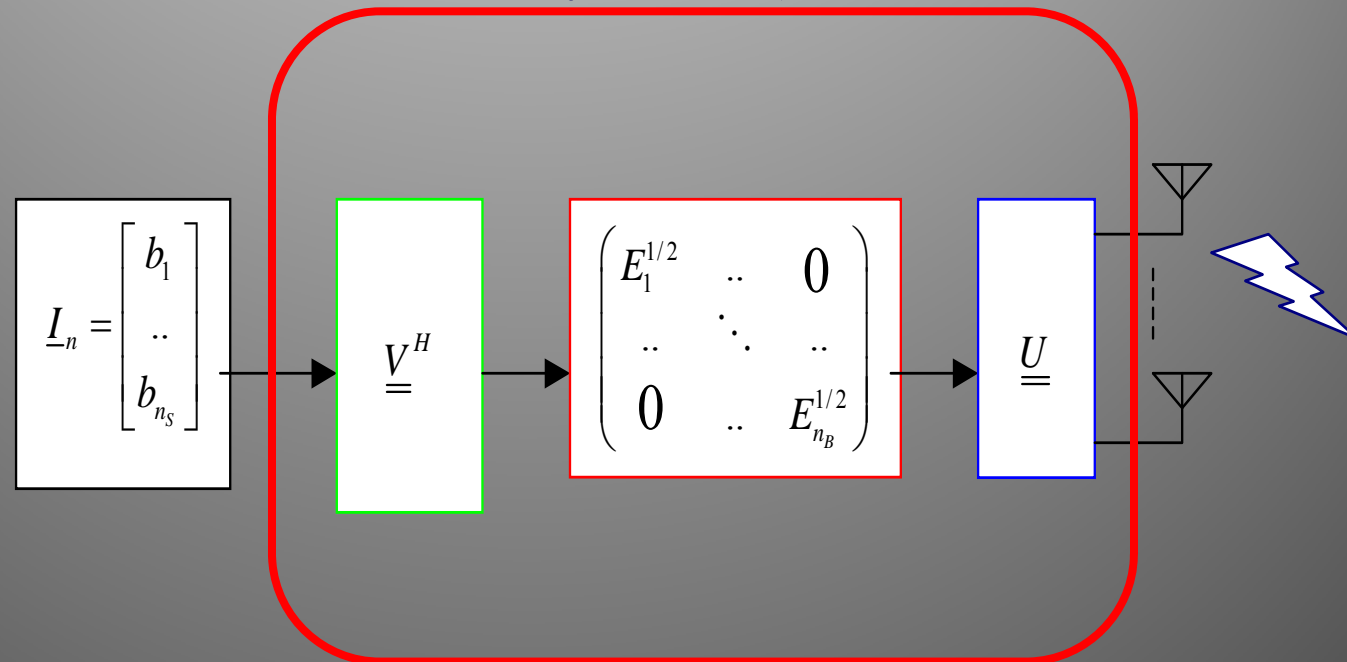
# Spatial Diversity





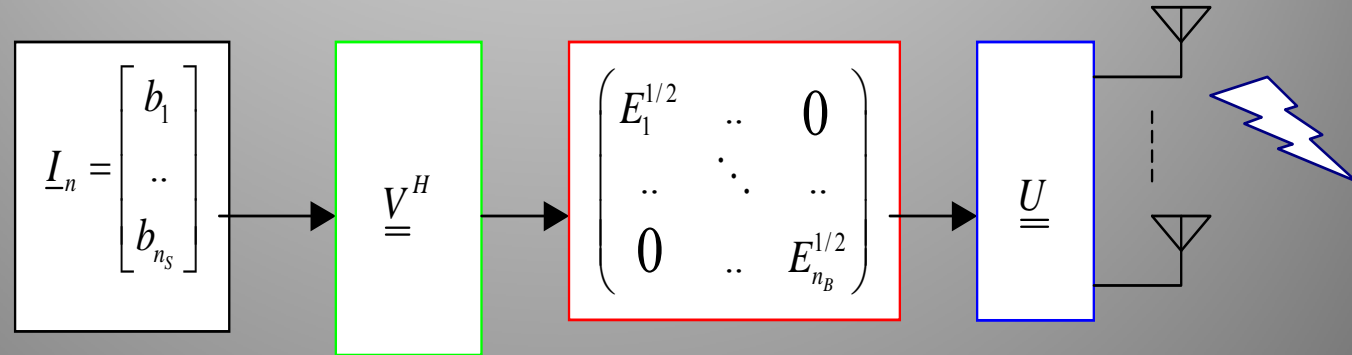
## Tx Architecture

The Tx pre-coder matrix  $B$  is always formed by 3 blocks (mathematically speaking is the SVD on any matrix)





## Tx Architecture: Packing in Channel Symbols (1/3)



The first matrix packs bits/symbols in channel symbols

$$\underline{I}_n = [b(1) \quad b(2) \quad \dots \quad b(n_s)] \quad \text{con} \quad b(i) = \{+1, -1\} \quad \forall i = 1, n_s$$

$$\underline{S}_n = \underline{V}^H \cdot \underline{I}_n$$

Linear/Non-Linear

Code - Constellation

$$\underline{V}^H = \begin{bmatrix} 1 & j & 0 & 0 \\ 0 & 0 & 1 & j \end{bmatrix}$$

$$\underline{V}^H = [1 \quad 3 \quad j \quad 3j]$$





We will use this matrix to pack a given set of symbols in order to accommodate the number of symbols transmitted to the number of channels available.

Other examples:

DFT transform

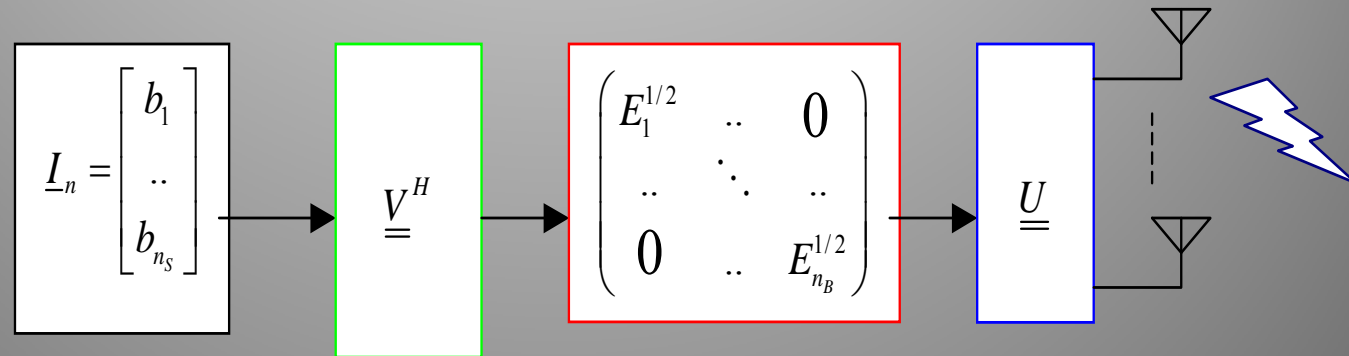
$$V^H = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}$$

Real entries

$$V^H = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$



## ***Tx Architecture: Power allocation to every channel symbol (2/3)***



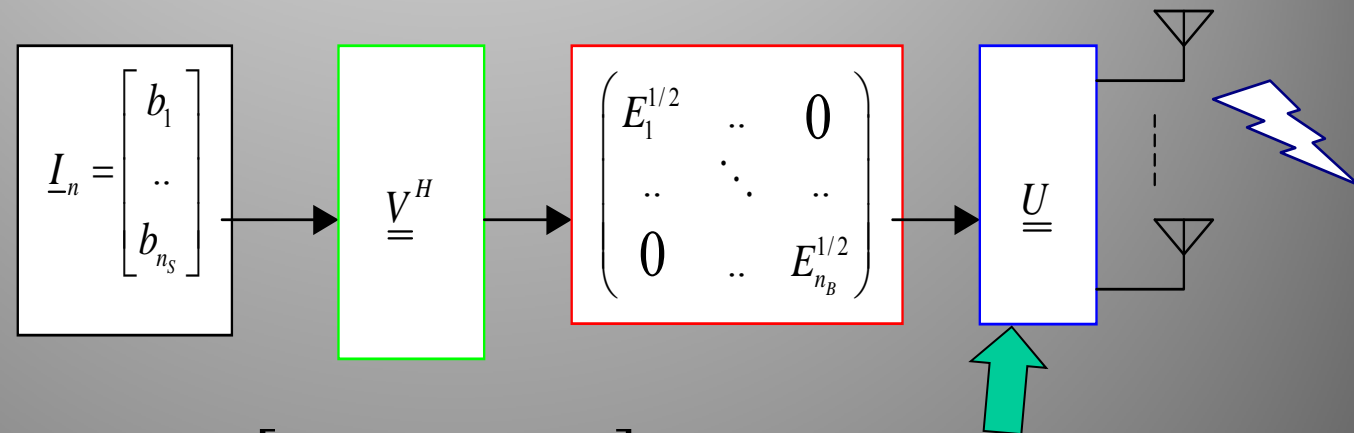
$$\underline{I}_n = [b(1) \quad b(2) \quad \dots \quad b(n_s)] \quad \text{con} \quad b(i) = \{+1, -1\} \quad \forall i = 1, n_s$$

Power allocation to each symbol ( $n_B$  symbols)

$$\underline{\theta}_n = \text{diag} \left[ E_1^{1/2}, \dots, E_{n_B}^{1/2} \right] \cdot \underline{V}^H \cdot \underline{I}_n = \underline{\underline{P}} \cdot \underline{V}^H \cdot \underline{I}_n$$



## Tx architecture: Beamforming 3/3



$\underline{U} = [b_1 \quad \dots \quad b_L]$  L beamformers for multiplexing transmitted energy across space

Beamforming

$$\underline{X}_{T,n} = \underline{U} \cdot \underline{P} \cdot \underline{\theta}_n = \underbrace{\underline{U} \cdot \underline{P} \cdot \underline{V}^H}_{\underline{B}} \cdot \underline{I}_n$$







## Tx Architecture: Power at Tx

$$\underline{\underline{Q}} = E\left(\underline{\underline{X}}_{T,n} \cdot \underline{\underline{X}}_{T,n}^H\right) = \underline{\underline{U}} \cdot \underline{\underline{P}} \cdot \underline{\underline{V}}^H E\left(\underline{\underline{I}}_n \cdot \underline{\underline{I}}_n^T\right) \cdot \underline{\underline{V}} \cdot \underline{\underline{P}} \cdot \underline{\underline{U}}^H = \underline{\underline{U}} \cdot \underline{\underline{P}} \cdot \underline{\underline{V}}^H \cdot \underline{\underline{V}} \cdot \underline{\underline{P}} \cdot \underline{\underline{U}}^H$$

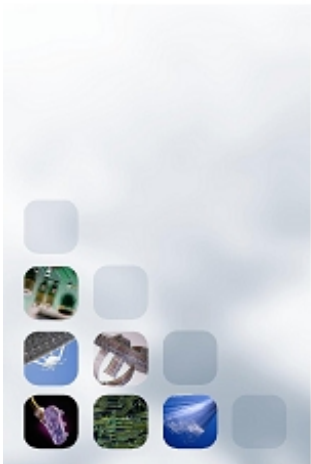
With  $\underline{\underline{U}}^H \cdot \underline{\underline{U}} = \underline{\underline{I}}_{ns}$        $\underline{\underline{V}}^H \cdot \underline{\underline{V}} = \underline{\underline{I}}_{ns}$

$$\underline{\underline{Q}} = \underline{\underline{U}} \cdot \underline{\underline{P}}^2 \cdot \underline{\underline{U}}^H = \underline{\underline{U}} \cdot \underline{\underline{Z}} \cdot \underline{\underline{U}}^H \quad E_T = \text{Trace}\left(\underline{\underline{Q}}\right) = \text{Trace}\left(\underline{\underline{Z}}\right)$$

Power budget or total energy available at Tx

Alternatives more practical and complex to manage

- Peak power per RF chain
- Radiated power (regulation)
- .....



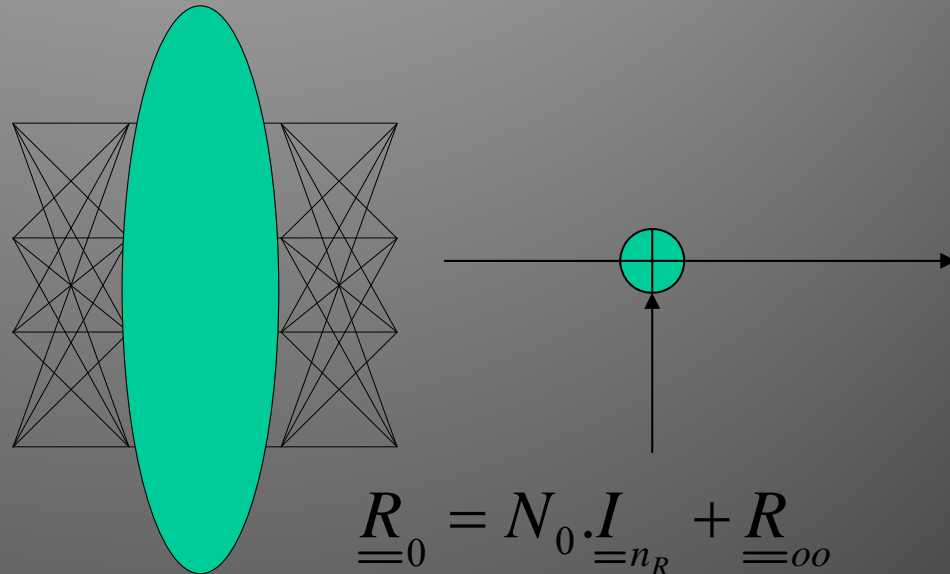
## ***MIMO Scenarios depending on Coordination (Joint processing)***

<b>T<sub>x</sub></b>	<b>R<sub>x</sub></b>	<b>Description</b>
YES	YES	MIMO PTP Point to Point
YES	NO	Broadcast MIMO
NO	YES	Access MIMO
NO	NO	Interference channel

## The channel architecture

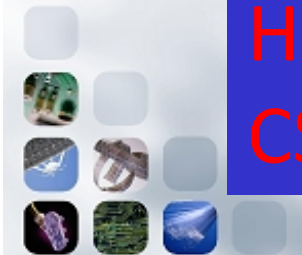
$$\underline{\underline{H}} = \underline{\underline{V}}_{=h} \cdot \underline{\underline{\Gamma}}_{=h} \cdot \underline{\underline{U}}_{=h}^H \quad \text{con} \quad \underline{\underline{\Gamma}}_{=h} = \text{diag} \left[ \gamma_{H1} \quad \dots \quad \gamma_{H \min(n_T, n_R)} \right]$$

$$\underline{\underline{H}}^H \cdot \underline{\underline{H}} = \sum_{q=1}^{n_R} \underline{\underline{h}}_q \cdot \underline{\underline{h}}_q^H = \underline{\underline{U}}_{=h} \cdot \underline{\underline{\Sigma}}_{=h} \cdot \underline{\underline{U}}_{=h}^H \quad \text{con} \quad \underline{\underline{\Sigma}}_{=h} = \left[ \lambda_{H1} \quad \lambda_{H2} \quad \dots \quad \lambda_{H \min(n_T, n_R)} \right]$$

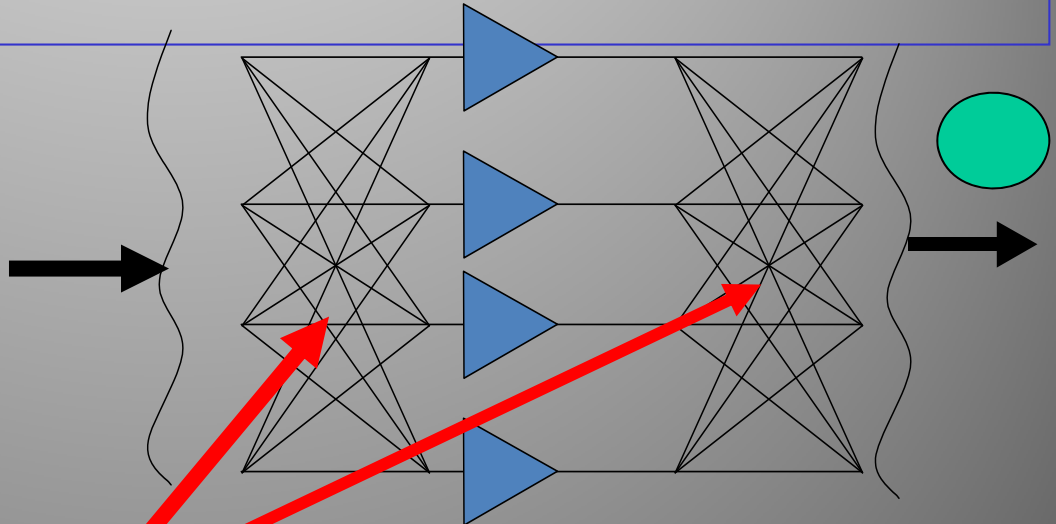


$$\underline{\underline{R}}_{=0} = N_0 \cdot \underline{\underline{I}}_{=n_R} + \underline{\underline{R}}_{=oo}$$



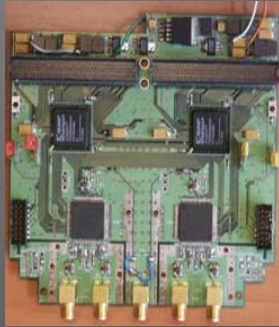
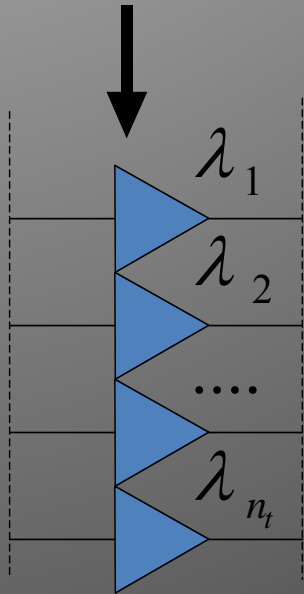


# CSI Channel State Information

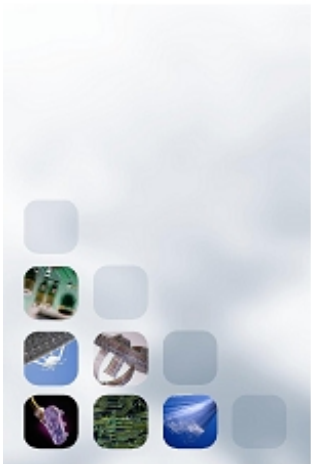


CSI: Channel State Information

Hard to afford perfect CSI, especially CSIT







## ***MIMO Scenarios by CSI***

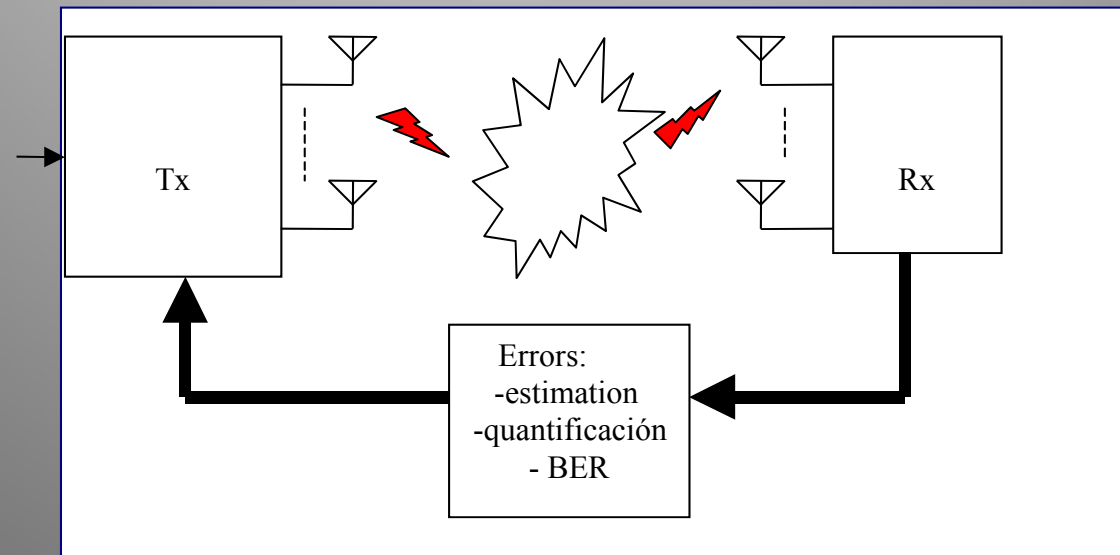
<b>Tx</b>	<b>Rx</b>	<b>Description</b>
YES	YES	Full CSI (CSIT and CSIR)
NO	YES	Partial CSI (CSIR only)
NO	NO	No CSI

CSIR only requires space time codes

No CSI requires differential space time codes



## *CSIT (Complexity)*

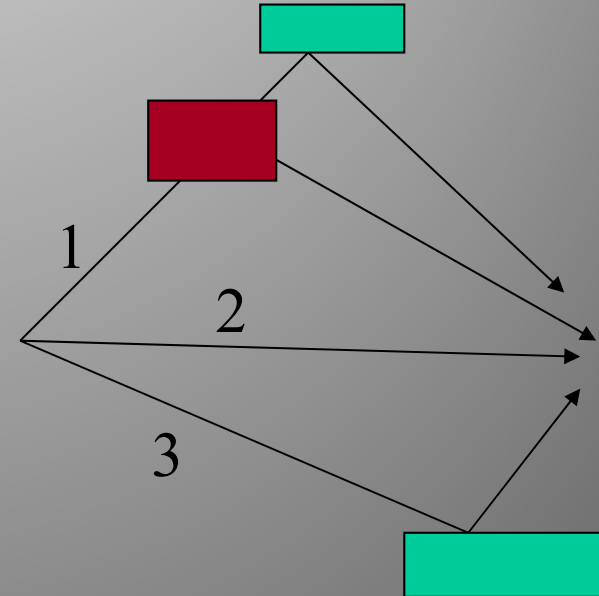


- Capacity of the feedback channel
- Not accurate
- User Mobility, FDMA, TDMA

## Remember: The LOS MIMO Channel

$$\underline{\underline{H}} = \begin{bmatrix} \underline{S}_{T1} & \underline{S}_{T2} & \underline{S}_{T3} \end{bmatrix}$$

$$\underline{\underline{H}}^H = \begin{bmatrix} \underline{S}_{R1} & \underline{S}_{R2} & \underline{S}_{R3} \end{bmatrix}$$



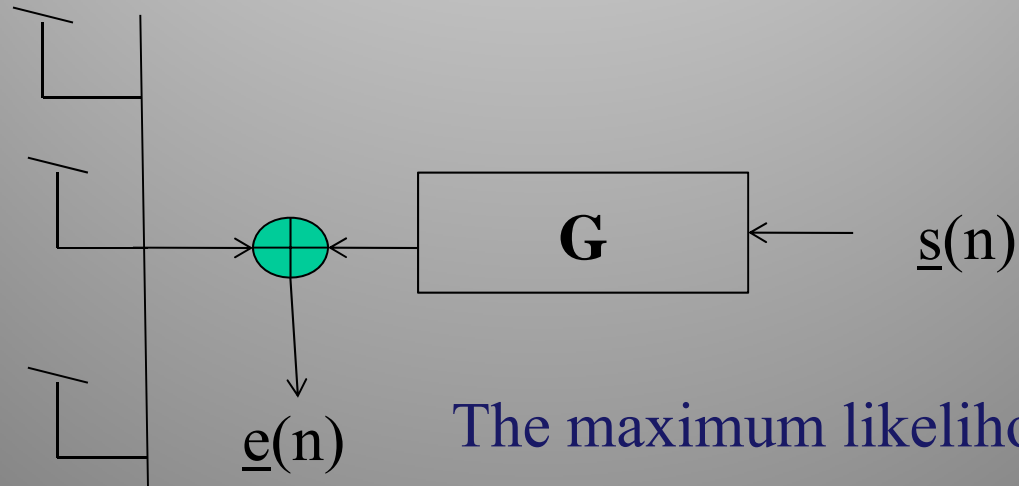
Use the best channel (#2) or all the available paths?

Comment on the Eigenvectors and the steering vectors relationship





## The Optimum Receiver



The maximum likelihood receiver:

$$\hat{\underline{s}}_n = \min_{\underline{s}_n \in \text{Constellation}} \text{trace} \left\{ \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n \right) \underline{R}^{-1} \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n \right)^H \right\}$$

- $\underline{G} = E(\underline{X}_n \cdot \underline{s}(n)^H)$  is the so-called DIR of the receiver
- For white noise  $\sigma_{e(n)}^2$  minimum over  $\underline{s}(n) \in \text{Tx Alphabet}$
- Scenario free of interference

COMPLEXITY/INTERFERENCE



## Suboptimum Receiver: Interference as noise

When interference is present, the ML forces to detect also the interference.

$$\hat{\underline{s}}_n = \min_{\underline{s}_n \in \text{Constellation}} \text{trace} \left\{ \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n - \underline{F} \underline{j}_n \right) \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n - \underline{F} \underline{j}_n \right)^H \right\}$$

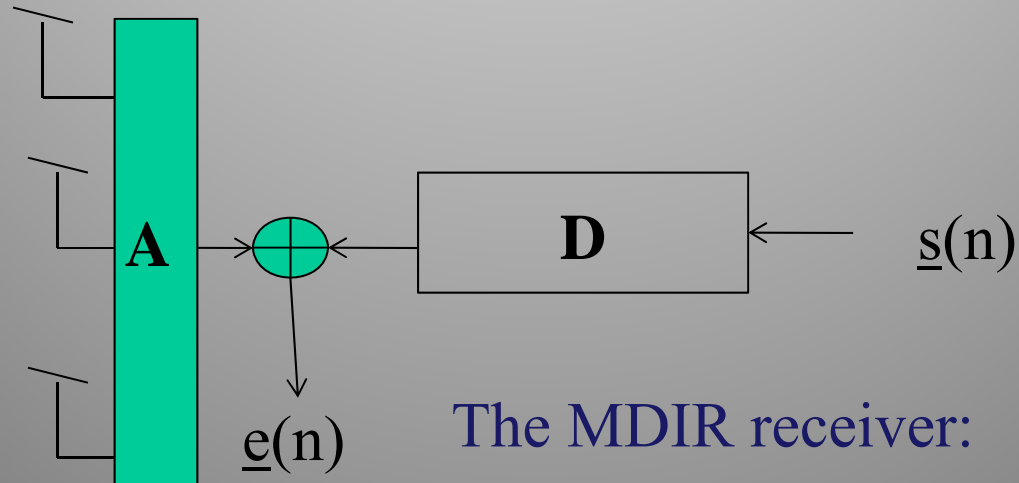
The advantage is that the front-end noise is white, but imposes to detect the interference and its channel to the receiver.

The suboptimum approach is to include the interference (not Gaussian in general in the noise matrix)

$$\hat{\underline{s}}_n = \min_{\underline{s}_n \in \text{Constellation}} \text{trace} \left\{ \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n \right) \underline{R}_{=0}^{-1} \left( \underline{X}_n - \underline{G} \cdot \underline{s}_n \right)^H \right\}$$



## Suboptimum receiver: Include beamforming to remove interference, the MDIR receiver

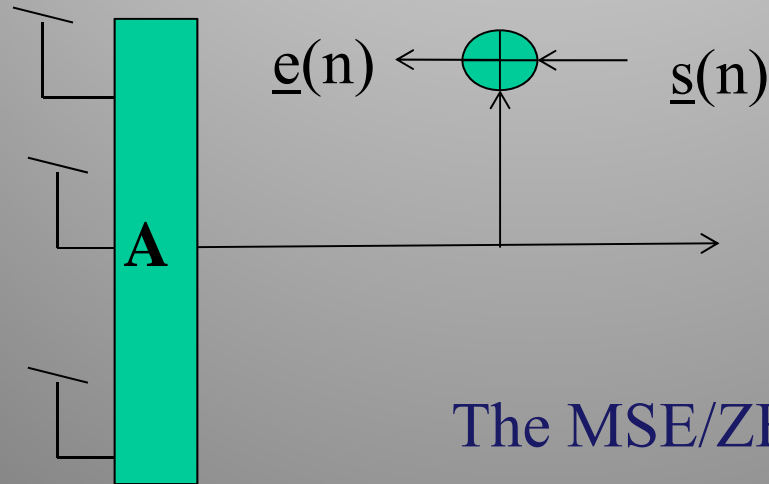


$$\hat{\underline{s}}_n = \min_{\underline{s}_n \in Constellation} \text{trace} \left\{ \left( \underline{A}^H \underline{X}_n - \underline{D} \cdot \underline{s}_n \right) \left( \underline{A}^H \underline{X}_n - \underline{D} \cdot \underline{s}_n \right)^H \right\}$$

- Forward and backward processing architecture
- Noise matrix depends on the forward (beamforming stage)
- Not instantaneous detection of the stream's vector.



## Suboptimum receiver: Instantaneous detection. MSE/ZF receivers



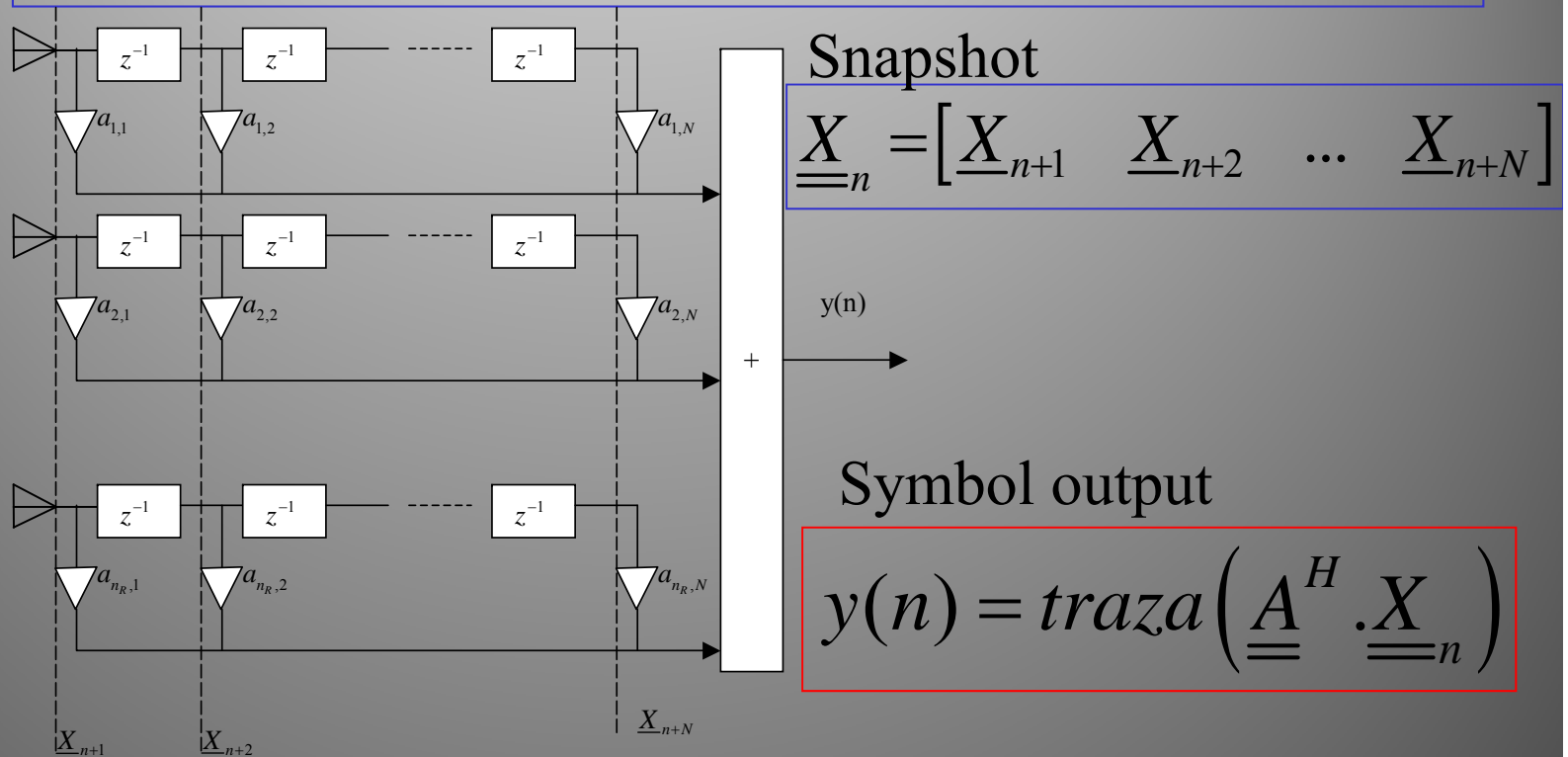
The MSE/ZF:

$$\hat{\underline{s}}_n = \underline{\underline{A}}^H \underline{\underline{X}}_n$$

- Multichannel processing for multiple streams
- Not easy to define a function of the vector error
- Single symbol much more easier that the multiple symbols problem



# Single symbol beamforming processing



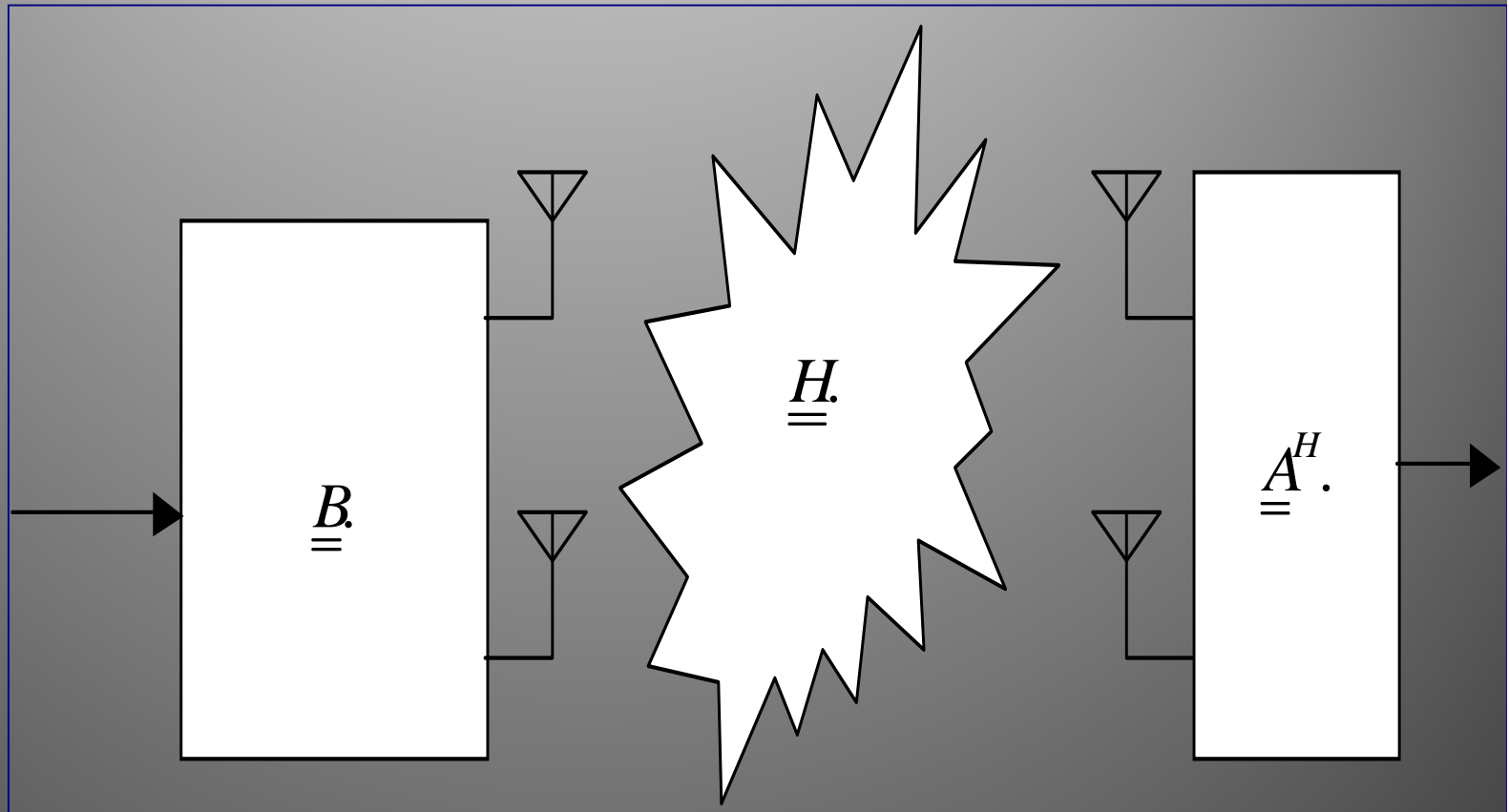
$$\underline{\underline{A}} = \begin{bmatrix} \underline{a}_1 & \underline{a}_2 & \dots & \underline{a}_N \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,N} \\ a_{2,1} & a_{2,2} & \dots & a_{2,N} \\ \dots & \dots & \dots & \dots \\ a_{n_R,1} & a_{n_R,2} & \dots & a_{n_R,N} \end{bmatrix}$$

Receiver  
Matrix





## SUMMARY



Flat Fading, CSIT and CSIR, Single Symbol



innovating communications

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*A gateway to advanced communication technologies*



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