MIMO3: MSE/ZF Receivers with CSIT

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ML, MSE and the MDIR Receiver

-Tx-Rx optimization can be done over the ML receiver.

- Complexity can be very high for ns (# of streams) greater than 4.

- Existing alternatives is to consider MSE/ZF designs for the receiver, less demanding in terms of memory and computational load (but sub-optimum).

$$\Lambda(\underline{I}_0) = -Trace\left[\left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)^H . \underline{\underline{R}}_0^{-1} . \left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)\right] = \left| \begin{matrix} \text{assume white} \\ \text{noise scenario} \end{matrix} \right| =$$

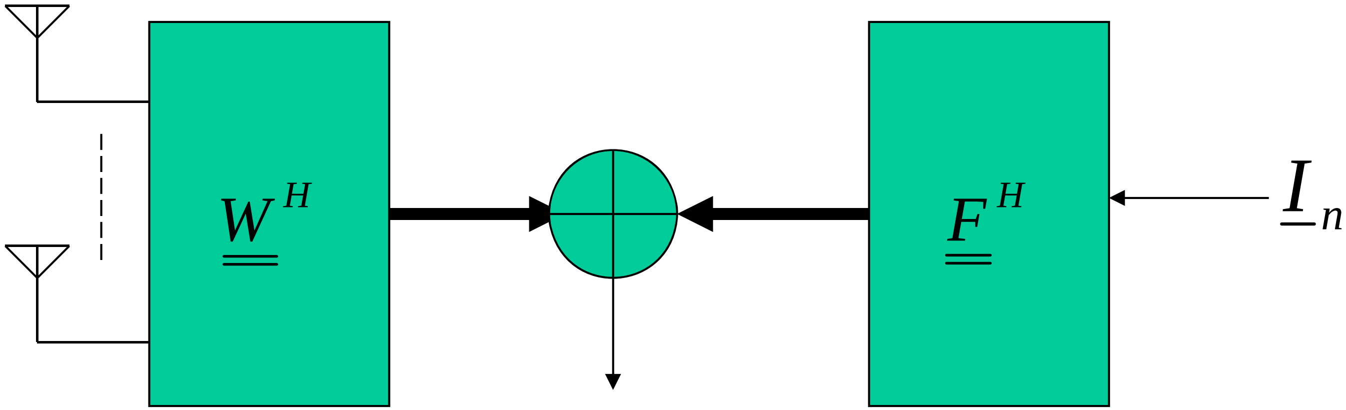
$$\propto -Trace\left[\left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)\left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)^H\right] = -Trace\left[\underline{\underline{E}}\right]$$

The likelihood is minus the trace of the MSE matrix

Assuming a square root decomposition of the noise matrix we arrive to the MDIR detector.

$$\Lambda(\underline{I}_0) = -Trace\left[\left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)^H.\underline{\underline{R}}_0^{-1}.\left(\underline{X}_n - \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n\right)\right] = \left| \begin{array}{l} assume \\ \underline{\underline{R}}_0^{-1} = \underline{\underline{W}}.\underline{\underline{W}}^H \end{array} \right| =$$

$$\propto -Trace\left[\left\| \underline{\underline{W}}^H \underline{X}_n - \underline{\underline{W}}^H \underline{\underline{H}}.\underline{\underline{B}}.\underline{I}_n \right\|^2\right] = -Trace\left[\left\| \underline{\underline{W}}^H \underline{X}_n - \underline{\underline{F}}.\underline{I}_n \right\|^2\right]$$



MDIR to reduce the ML with the presence of interferers to white noise (easier metric computation than in the original. Matched DIR on the stream processing. Forward equalizer concept.

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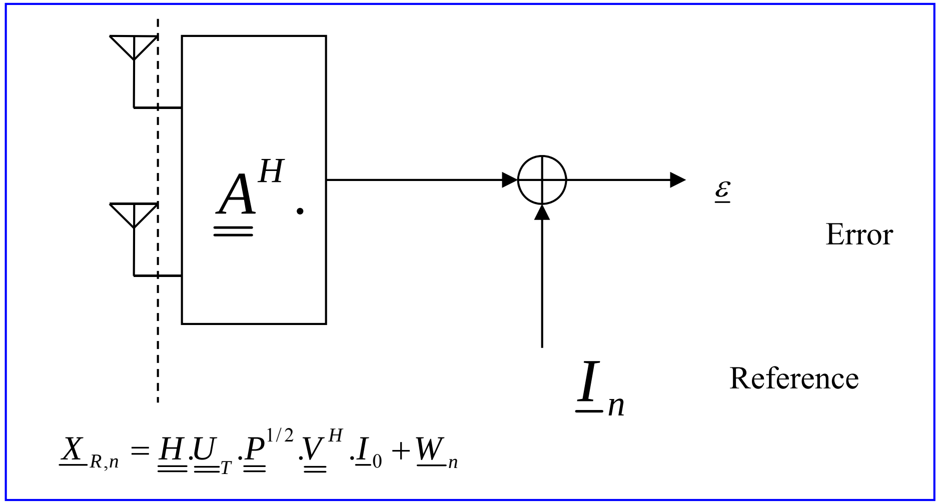
The MSE/ZF Receiver



The error is: $\underline{\varepsilon} = \left(\underline{\underline{A}}^H.\underline{\underline{H}}.\underline{\underline{B}} - \underline{\underline{I}}\right)\underline{I}_n + \underline{\underline{A}}^H.\underline{w}_n$

and, the MSE matrix

$$\underline{\underline{E}} = E\left(\underline{\varepsilon}_n.\underline{\varepsilon}_n^H\right) = \left(\underline{\underline{A}}^H.\underline{\underline{H}}.\underline{\underline{B}} - \underline{\underline{I}}\right)\left(\underline{\underline{A}}^H.\underline{\underline{H}}.\underline{\underline{B}} - \underline{\underline{I}}\right)^H + \underline{\underline{A}}^H.\underline{\underline{R}}_0.\underline{\underline{A}}$$

Note that bit-streams are assumed uncorrelated which holds also when using block (repetition) or convolutional codes

$$\underline{\underline{E}} = E\left(\underline{\varepsilon}_n . \underline{\varepsilon}_n^H\right) = \left(\underline{\underline{A}}^H . \underline{\underline{H}} . \underline{\underline{B}} - \underline{\underline{I}}\right)\left(\underline{\underline{A}}^H . \underline{\underline{H}} . \underline{\underline{B}} - \underline{\underline{I}}\right)^H + \underline{\underline{A}}^H . \underline{\underline{R}}_0 . \underline{\underline{A}}$$

perfect square

$$\underline{\underline{E}} = \left[\underline{\underline{A}} - \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)^{-1} \underline{\underline{H}}\,\underline{\underline{B}}\right]^H \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)$$
$$\left[\underline{\underline{A}} - \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)^{-1} \underline{\underline{H}}\,\underline{\underline{B}}\right] + \left[\underline{\underline{I}} - \underline{\underline{H}}^H \underline{\underline{B}}^H \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)^{-1} \underline{\underline{H}}\,\underline{\underline{B}}\right]$$

The matrix of the minimum error and the receiver are:

$$\underline{\underline{E}}_{\min} = \left[\underline{\underline{I}} - \underline{\underline{B}}^H \underline{\underline{H}}^H \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)^{-1} \underline{\underline{H}}\,\underline{\underline{B}}\right]$$

$$\underline{\underline{A}} = \left(\underline{\underline{R}}_0 + \underline{\underline{H}}^H \underline{\underline{B}}^H \underline{\underline{BH}}\right)^{-1} \underline{\underline{H}}\,\underline{\underline{B}}$$

Matrix inverse lemma

$$\left(\underline{\underline{A}} + \underline{\underline{B}}.\underline{\underline{C}}.\underline{\underline{D}}\right)^{-1} = \underline{\underline{A}}^{-1} - \underline{\underline{A}}^{-1}.\underline{\underline{B}}.\left(\underline{\underline{D}}.\underline{\underline{A}}^{-1}\underline{\underline{B}} + \underline{\underline{C}}^{-1}\right)^{-1}\underline{\underline{D}}.\underline{\underline{A}}^{-1}$$

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The MSE Receiver and MSE Matrix

$$\underline{\underline{E}} = \left[\underline{\underline{I}}_{ns} - \underline{\underline{B}}^H \underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{\underline{B}} \right] = \left[\underline{\underline{I}}_{ns} + \underline{\underline{B}}^H \underline{\underline{R}}_H . \underline{\underline{B}} \right]^{-1}$$

$$\underline{\underline{A}} = \left(\underline{\underline{H}} . \underline{\underline{B}} . \underline{\underline{B}}^H . \underline{\underline{H}}^H + \underline{\underline{R}}_0 \right)^{-1} . \underline{\underline{H}} . \underline{\underline{B}} = \underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{\underline{B}} . \underline{\underline{E}}$$

In case of ZF remove from the MSE the Identity matrix

Before facing the Tx design, some preliminars.....

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QUALITY equivalent to MSE or SNR or BER ??

For single channel (SISO) it is easy to conect these 3 quality measures:

$$snr = \frac{1}{mse} - 1 \quad BER < \exp(-k_0.snr)$$

For low SNR regime the BER is concave for high SNR (BER<0.1) is convex

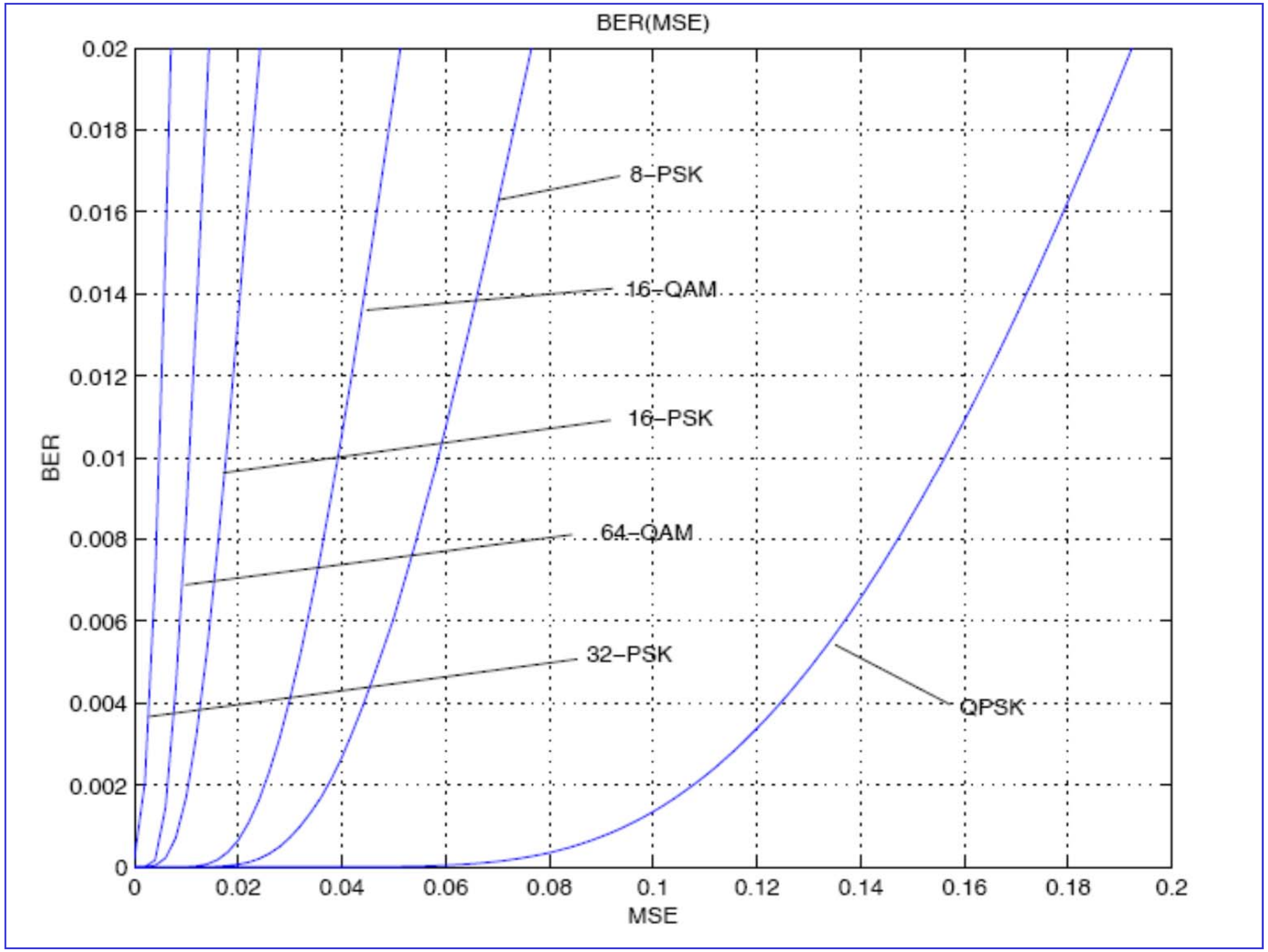
$$\underline{\varepsilon} = \underline{I}_n - \underline{\hat{I}}_n \rightarrow \underline{I}_n = \underline{\hat{I}}_n + \underline{\varepsilon}$$

For MSE receiver the error is independent of data or filter input vector

$$\underline{\underline{I}} = \underline{\underline{S}} + \underline{\underline{E}}$$

$$\underline{\underline{SNR}} \triangleright \underline{\underline{S}}.\underline{\underline{E}}^{-1} = \underline{\underline{E}}^{-1} - \underline{\underline{I}}$$

UNLESS the MSE matrix is diagonal the SNR per stream is not properly defined



Convexity of BER(MSE) for BER<0.02

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Majorization Theory

Majorization makes precise the notion that the components of a vector are "less spread" than the components of other vector.

Let us assume two vectors with components arranged in decreasing order

$$\underline{x} \quad x(1) \geq x(2) \geq \ldots \geq x(N)$$

$$\underline{y} \quad y(1) \geq y(2) \geq \ldots \geq y(N)$$

Vector $\underline{x}$ is majorized by $\underline{y}$ $\quad \underline{x} \prec \underline{y}$

$$\sum_{i=1}^{n} x(i) \leq \sum_{i=1}^{n} y(i) \quad 1 \leq n \leq N-1 \quad and \quad \sum_{i=1}^{N} x(i) = \sum_{i=1}^{N} y(i)$$

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When the sum up N is also smaller for x than for y we denote this as "y weakly majorizes x"

$$\underline{x} \prec^{w} \underline{y}$$

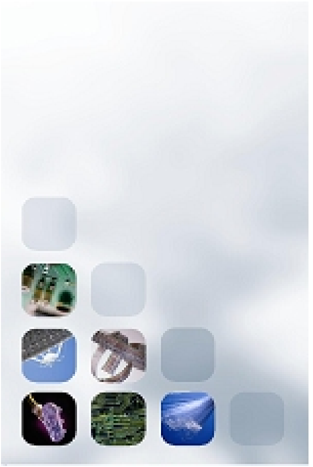
Also, using logarithms we may extend the concept of majorization to the product of the vector components instead to their sum.

Some examples:

$$\underline{1} \prec \underline{x} \quad \text{with} \quad \underline{1} = \sum_{1}^{N} \frac{x(i)}{N}.ones(1,N)$$

For any $\underline{x} \prec \underline{y}$ *there exists a sequence of transforms* $\underline{\underline{T}}^{(1)},\underline{\underline{T}}^{(2)},\ldots,\underline{\underline{T}}^{(K)}$ *with* $K \le N$ *such that* $\underline{x} = \underline{\underline{T}}^{(K)},\underline{\underline{T}}^{(K-1)},\ldots,\underline{\underline{T}}^{(1)}.\underline{y}$

For any Hermitian matrix the eigenvalues majorize the diagonal entries $\underline{\lambda} \succ \underline{d} \quad \left(\det\left(\underline{\underline{A}}\right) \le \prod a_{ii}\right)$



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Schur-Concave and Schur-Convex

A real valued function $\Phi(\underline{x})$ is said to be Schur-convex if

$$\underline{x} \prec \underline{y} \quad \Rightarrow \quad \Phi(\underline{x}) \leq \Phi(\underline{y})$$

It is said Schur-concave if

$$\underline{x} \prec \underline{y} \quad \Rightarrow \quad \Phi(\underline{x}) \geq \Phi(\underline{y})$$

Sc-cx and Sc-ce do not cover the complete set of real valued function defined over x

Function $\Phi(\underline{x}) = \sum_{1}^{N} x(i)$ is Sc-ce and Sc-cx

$$\Phi(\underline{x}) = x(1) + 2.x(2) + x(3) \quad \textit{is neither } Sc - cx \textit{ not } Sc - ce$$

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MSE Defined Designs

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SC-ce and SC-cx functions of MSE and MIMO Tx Designs

The objective for the Tx design will be of the form

$$\min \ f\left(diag\left\lfloor \underline{\underline{MSE}} \right\rfloor\right)$$

Since, diagonal is majorized by the eigenvalues of the matrix, for Schur-Concave functions the optimal solution will be to diagonalize the MSE

As a consequence, the number of available channels will be equal to the rank of the channel meaning that if we try to send more symbols than the minimum number of antennas (Tx-Rx) the excess symbols will be systematically on error.

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For Schur-Concave functions of the MSE DO NOT send more symbol streams than the minimum number of antennas at Tx-Rx

In summary, the Tx matrix will contain only the power allocation matrix and the spatial processor

$$\underline{\underline{B}} = \underline{\underline{U}}.diag(\underline{z}^{1/2})$$

The eigenvectors of

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}} = \underline{\underline{U}}.diag(\underline{\lambda}).\underline{\underline{U}}^H$$

Power Allocation

$$\underline{\underline{Q}} = \underline{\underline{U}}.diag(\underline{z}).\underline{\underline{U}}^H$$

The solution diagonalices the MSE

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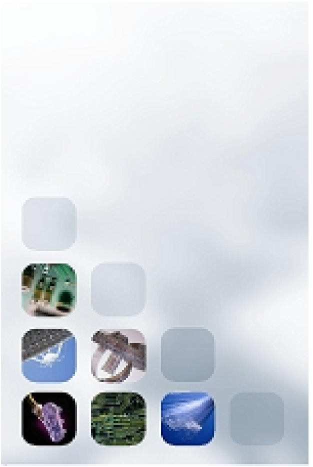

$$\min f\left(diag\left\lfloor \underline{\underline{MSE}} \right\rfloor\right)$$

When f(.) is Schur-Convex since the uniform MSE (the same value for all diagonal entries) is majorized by any other choice, this implies that the solution has to set all the diagonal elements of the MSE matrix equal.

At the same time, when there are more symbols than eigen-channels an unitary matrix (rotation) is needed to spread the energy of all the streams on the available channels.

$$\underline{\underline{B}} = \underline{\underline{U}}.diag\left(\underline{z}^{1/2}\right).\underline{\underline{V}}^{H} \qquad \underline{\underline{Q}} = \underline{\underline{U}}.diag\left(\underline{z}\right).\underline{\underline{U}}^{H}$$

In this case the optimum Tx-Rx does not diagonalices the MSE matrix

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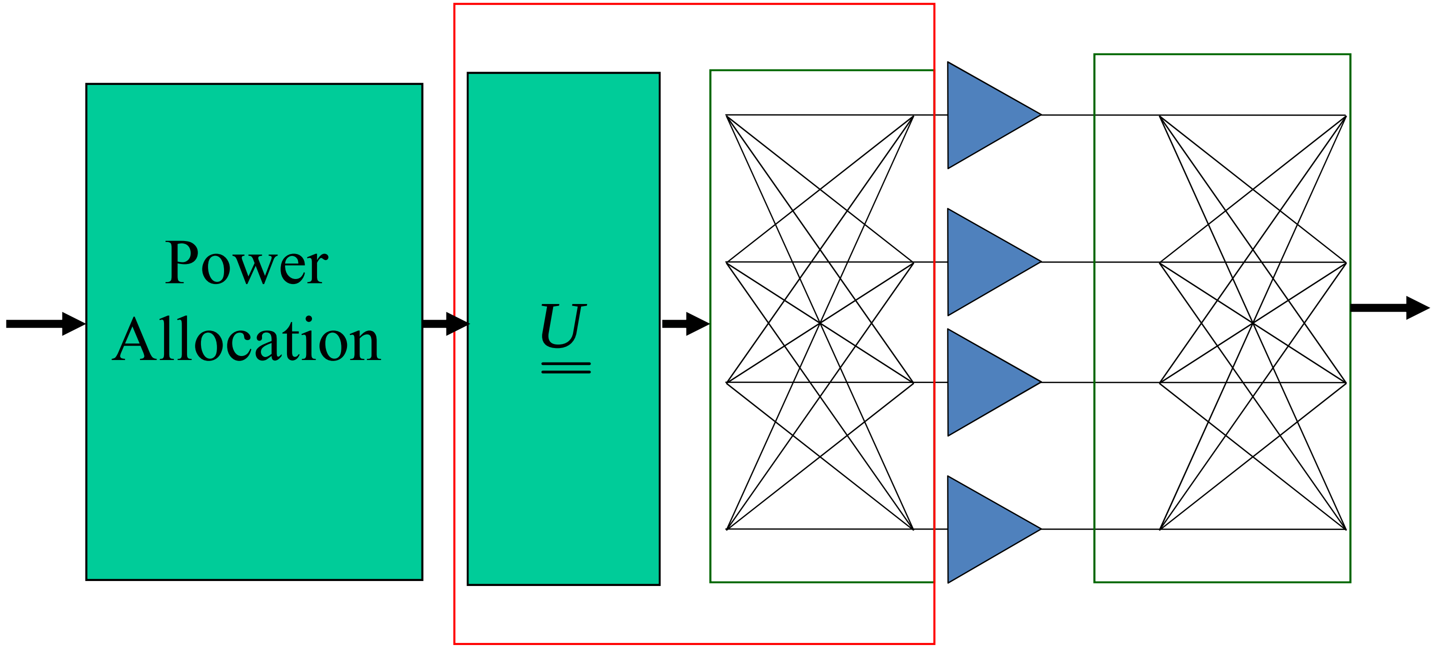
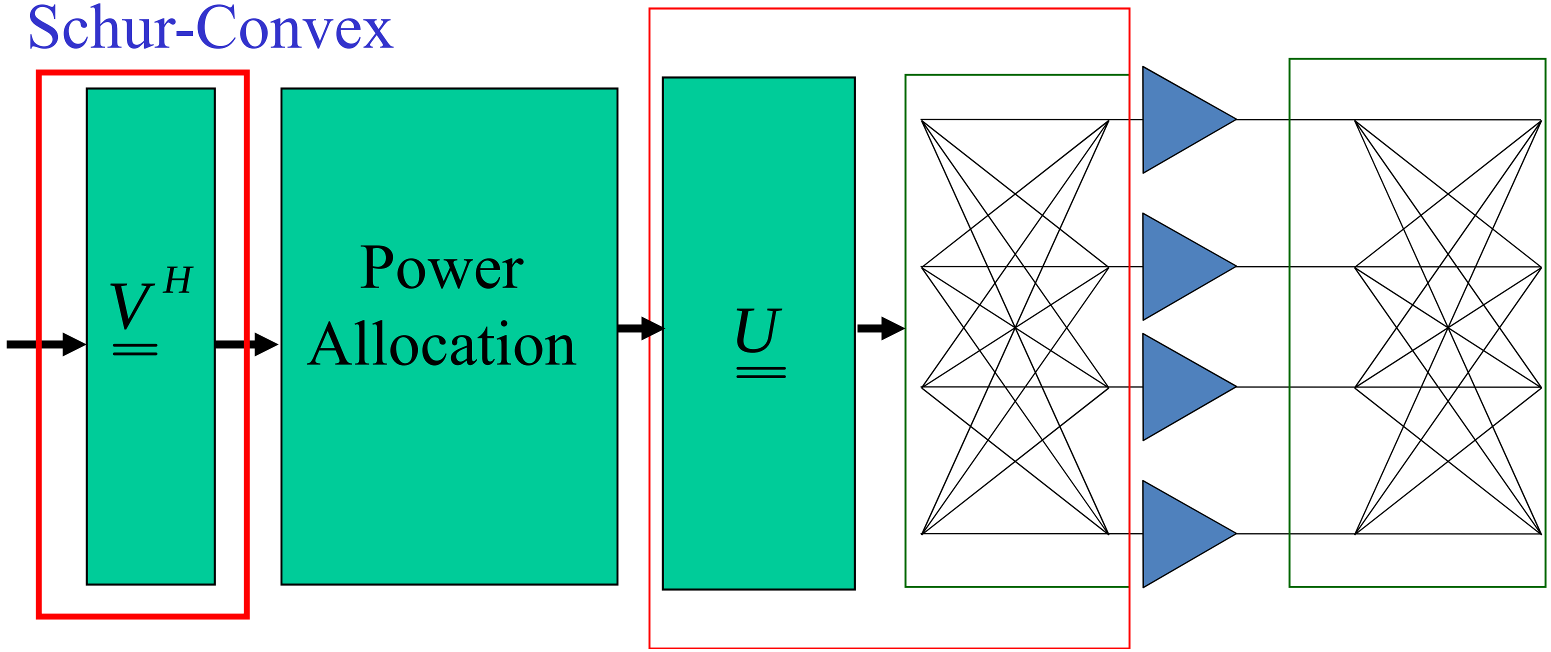
Schur-Concave



Schur-Convex



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Arithmetic Mean of the MSE diagonals

The problem is:

$$\min_{\underline{\underline{B}}}\left[trace\left(\underline{\underline{MSE}}\right)\right]=\min_{\underline{\underline{B}}}\left[trace\left(\left(\underline{\underline{I}}+\underline{\underline{B}}^{H}.\underline{\underline{R}}_{H}.\underline{\underline{B}}\right)^{-1}\right)\right]$$

$$s.t.\quad trace\left(\underline{\underline{B}}.\underline{\underline{B}}^{H}\right)=E_{T}$$

The trace is a Schur-Concave function of the diagonal entries, in consequence:

$$\underline{\lambda}\succ\underline{d}\Rightarrow sum\left(\underline{\lambda}\right)\leq sum\left(\underline{d}\right)$$

The solution is that B diagonalizes R_H

$$\underline{\underline{R}}_{H}=\underline{\underline{U}}.diag\left(\underline{\lambda}\right).\underline{\underline{U}}^{H} \qquad \underline{\underline{B}}=\underline{\underline{U}}.diag\left(\underline{z}^{1/2}\right)$$

The optimum Tx diagonalizes the channel and also the MSE (for ns, #of streams, equal to no, rank of the MIMO channel)

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New problem

Power allocation:
$$\min_{\underline{z}} \sum_{q=1}^{no} \frac{w(q)}{1+\lambda(q).z(q)}$$

$$s.t. \quad \sum_{1}^{no} z(q) \leq E_T \qquad z(q) \geq 0, \forall q$$

Assume w(q)=1 q=1,n0

The Lagrangian is:

$$L = \sum_{q=1}^{n0} \frac{1}{1+\lambda(q).z(q)} - \mu \left(\sum_{q=1}^{n0} z(q) - E_T \right) + \sum_{q=1}^{n0} \upsilon_q . z(q)$$

KKT.- Energy with equality, Lagrange factor μ greater than cero, and ν_i cero

$$\partial_{z(q)} L = 0 \quad \Rightarrow \quad \frac{\lambda(q)}{\left(1+\lambda(q).z(q)\right)^2} - \mu = 0 \quad \Rightarrow \quad z(q) = \left\lfloor \mu_0 . \lambda(q)^{-0.5} - \lambda(q)^{-1} \right\rfloor^{+}$$

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- Not all modes are activated
- More power to high eigenmodes (fairness!)
- Waterfilling algorithm
- When all the modes are activated:

$$trace\left(\underline{\underline{MSE}}/n_0\right)=\frac{1}{n_0}\frac{\left(\sum\lambda(q)^{1/2}\right)^2}{E_T+\sum\lambda(q)^{-1}}$$

$$\frac{1}{n_0}\frac{\left(\sum\lambda(q)^{1/2}\right)^2}{E_T+\sum\lambda(q)^{-1}}\le\frac{\sum\lambda(q)^{-1}}{E_T+\sum\lambda(q)^{-1}}=\sum\frac{\lambda(q)^{-1}}{E_T+\sum\lambda(q)^{-1}}=$$

$$\le\sum\frac{1}{n_0+E_T\lambda(q)}=\frac{1}{n_0}\sum\frac{1}{1+\left(\frac{E_T}{n_0}\right)\lambda(q)}$$

Full CSIT Solution

UPA No CSIT solution

The Geometric mean of the MSE

The problem is:

$$\min_{\underline{\underline{B}}}\left[\det\left(\underline{\underline{MSE}}\right)\right]=\min_{\underline{\underline{B}}}\left[\det\left(\left(\underline{\underline{I}}+\underline{\underline{B}}^{H}.\underline{\underline{R}}_{H}.\underline{\underline{B}}\right)^{-1}\right)\right]$$

$$s.t.\quad trace\left(\underline{\underline{B}}.\underline{\underline{B}}^{H}\right)=E_{T}$$

The determinat or geometric mean is a Schur-Concave function (use log(.) at the objective) of the diagonal entries, in consequence:

The solution is that B diagonalizes R_H

$$\underline{\underline{R}}_{H}=\underline{\underline{U}}.diag\left(\underline{\lambda}\right).\underline{\underline{U}}^{H}$$

$$\underline{\underline{B}}=\underline{\underline{U}}.diag\left(\underline{z}^{1/2}\right)$$

The optimum Tx diagonalizes the channel and coincides with the capacity solution

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The MIN-MAX MSE

The problem is:

$$\min_{\underline{\underline{B}}}\left[\max\left(\underline{\underline{MSE}}\right)\right]=\min_{\underline{\underline{B}}}\left[\max\left(\left(\underline{\underline{I}}+\underline{\underline{B}}^{H}.\underline{\underline{R}}_{H}.\underline{\underline{B}}\right)^{-1}\right)\right]$$

$$s.t.\quad trace\left(\underline{\underline{B}}.\underline{\underline{B}}^{H}\right)=E_{T}$$

The trace is a Schur-Convex function of the diagonal entries, in consequence:

$$\underline{1}\prec\underline{d}\Rightarrow all\ MSE\ entries\ equal$$

Assuming that the number of symbol streams is L and the rank of the MIMO channel is L_0<L. Implies that the solution for the Tx will be in its general form:

$$\underline{\underline{B}}=\underline{\underline{U}}.diag\left(\underline{z}^{1/2}\right)\underline{\underline{V}}^{H}$$

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The MSE matrix will be:

$$\underline{\underline{MSE}} = \left(\underline{\underline{I}} + \underline{\underline{B}}^H . \underline{\underline{R}}_H . \underline{\underline{B}}\right)^{-1} = \underline{\underline{V}}.\left(\underline{\underline{I}} + \underline{\underline{D}}_z . \underline{\underline{D}}_{Rh}\right)^{-1} . \underline{\underline{V}}^H = \underline{\underline{V}}.\underline{\underline{MSE0}}.\underline{\underline{V}}^H$$

Since $D_{Rh}=\text{diag}(\lambda_1,\lambda_2,.., \lambda_{L0},0,..,0)$ it is non-sense to allocate power to the null eigenvalues; thus $L-L_0$ cahnnels will be with an MSE equal 1 (full error) and its trace would be:

$$trace\left(\underline{\underline{MSE0}}\right) = (L - L_0) + \sum_{q=1}^{L_0} \frac{1}{1 + z(q).\lambda(q)}$$

Since the unitary matrix V does not modify the trace, it means that the trace of the MSE is the same that the trace of MSE0. In consequence the mission of V is to transform MSE0 in order that all the diagonal entries of MSE are equal to:

$$MSE_{ii} = \left(1 - \frac{L_0}{L}\right) + \left(\frac{1}{L}\right) \sum_{q=1}^{L_0} \frac{1}{1 + z(q).\lambda(q)}$$

Now we can formulate the following convex problem:

$$\min \quad t_i \quad s.t.$$

$$t_i \leq \left(1 - \frac{L_0}{L}\right) + \left(\frac{1}{L}\right) \sum_{q=1}^{L_0} \frac{1}{1 + z(q).\lambda(q)}$$

$$\sum_{1}^{L_0} z(q) \leq E_T \quad z(q) \geq 0$$

The solution for the power loading is identical to the trace of the MSE. The Tx differs in this case in the necessity of matrix V at Tx. The MSE matrix is not longer diagonaliced

There is a sub-optimal solution which is to force by the power loading that all the entries of the MSE0 matrix are equal.

$$z(q) = E_T . \lambda(q)^{-1} . \frac{1}{\sum_{1}^{L_0} \lambda(q)^{-1}}$$

And the MSE0 entries are:

$$MSE0_{ii} = \left(1 - \frac{L_0}{L}\right) + \frac{\sum \lambda(q)^{-1}}{E_T + \sum \lambda(q)^{-1}}$$

The second term is larger (see slide 26) than the obtained from the optimal solution

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Arithmetic mean of the SNR

REMARK: Only properly defined if the MSE matrix is diagonalized.

The problem is:

$$\min_{\underline{\underline{B}}}\left[trace\left(\underline{\underline{MSE}}^{-1}-\underline{\underline{I}}\right)\right]=\min_{\underline{\underline{B}}}\left[trace\left(\left(\underline{\underline{B}}^{H}.\underline{\underline{R}}_{H}.\underline{\underline{B}}\right)^{-1}\right)\right]$$

$$s.t.\quad trace\left(\underline{\underline{B}}.\underline{\underline{B}}^{H}\right)=E_{T}$$

The trace is a Schur-Concave function of the diagonal entries, in consequence:

$$\underline{\lambda}\succ\underline{d}\Rightarrow sum\left(\underline{\lambda}\right)\leq sum\left(\underline{d}\right)$$

$$\boxed{\underline{\underline{B}}=\underline{\underline{U}}.diag\left(\underline{z}^{1/2}\right)}$$

New problem

Power allocation:

$$\max_{\underline{z}} \sum_{q=1}^{no} \lambda(q).z(q)$$

$$s.t. \quad \sum_{1}^{no} z(q) \le E_T \qquad z(q) \ge 0, \forall q$$

The solution for this problem is just to provide all the available energy to the best eigen-channel. BEAMFORMING and pack all the streams in a single stream

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The Determinant of the SNR (Geometric mean)

The solution is UPA and no CSIT is necessary.

The MAX-MIN of the SNR and BER

The solution, as in the Min-max of the MSE, is the same since both functions are convex.

Arithmetic mean of the BER

Since for small MSE the BER is convex ($Pe<10^{-1}$) the solution reduces also to the mean MSE plus a rotation to get the same value at all the diagonal entries.

Geometric mean of the BER

Beamforming

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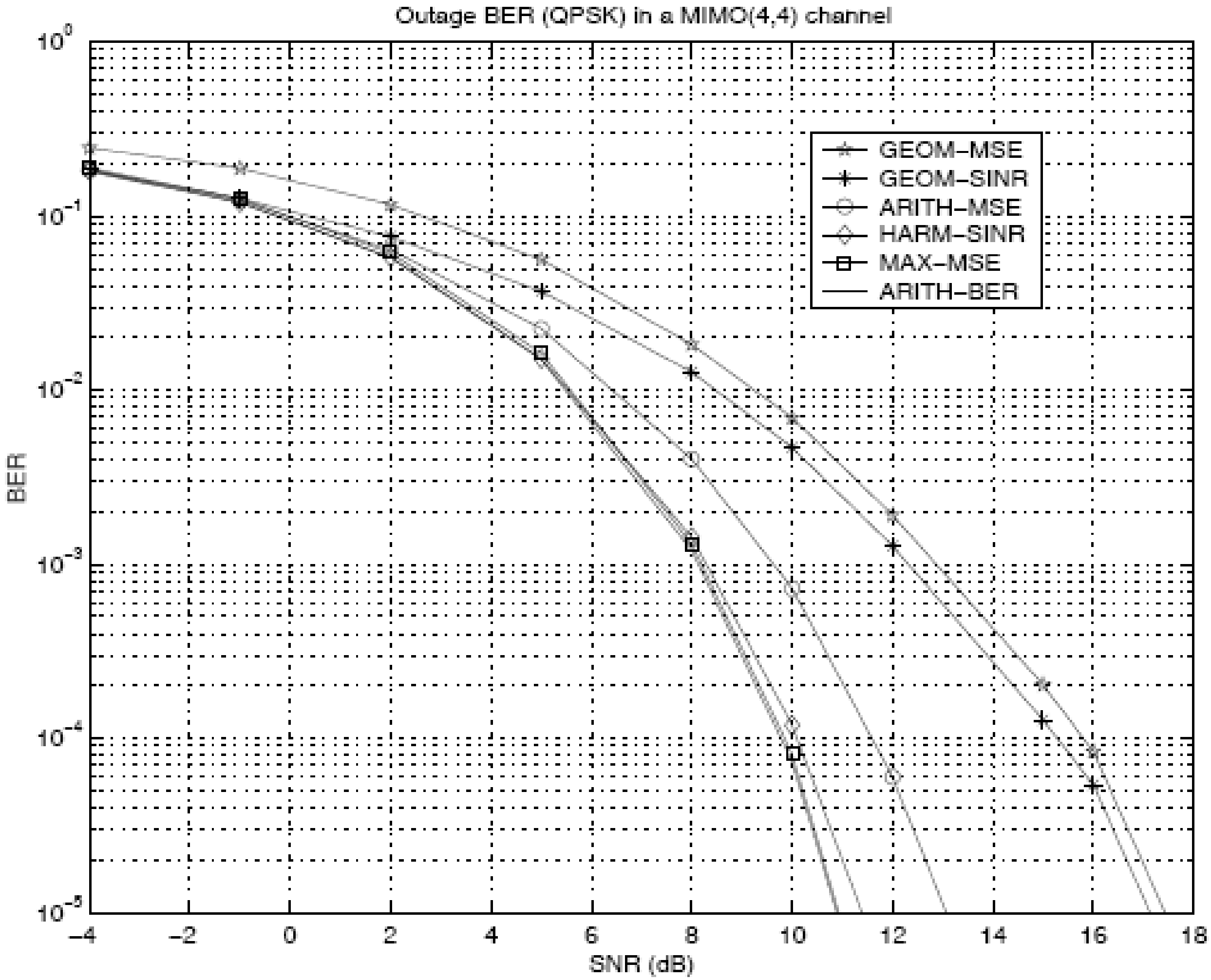
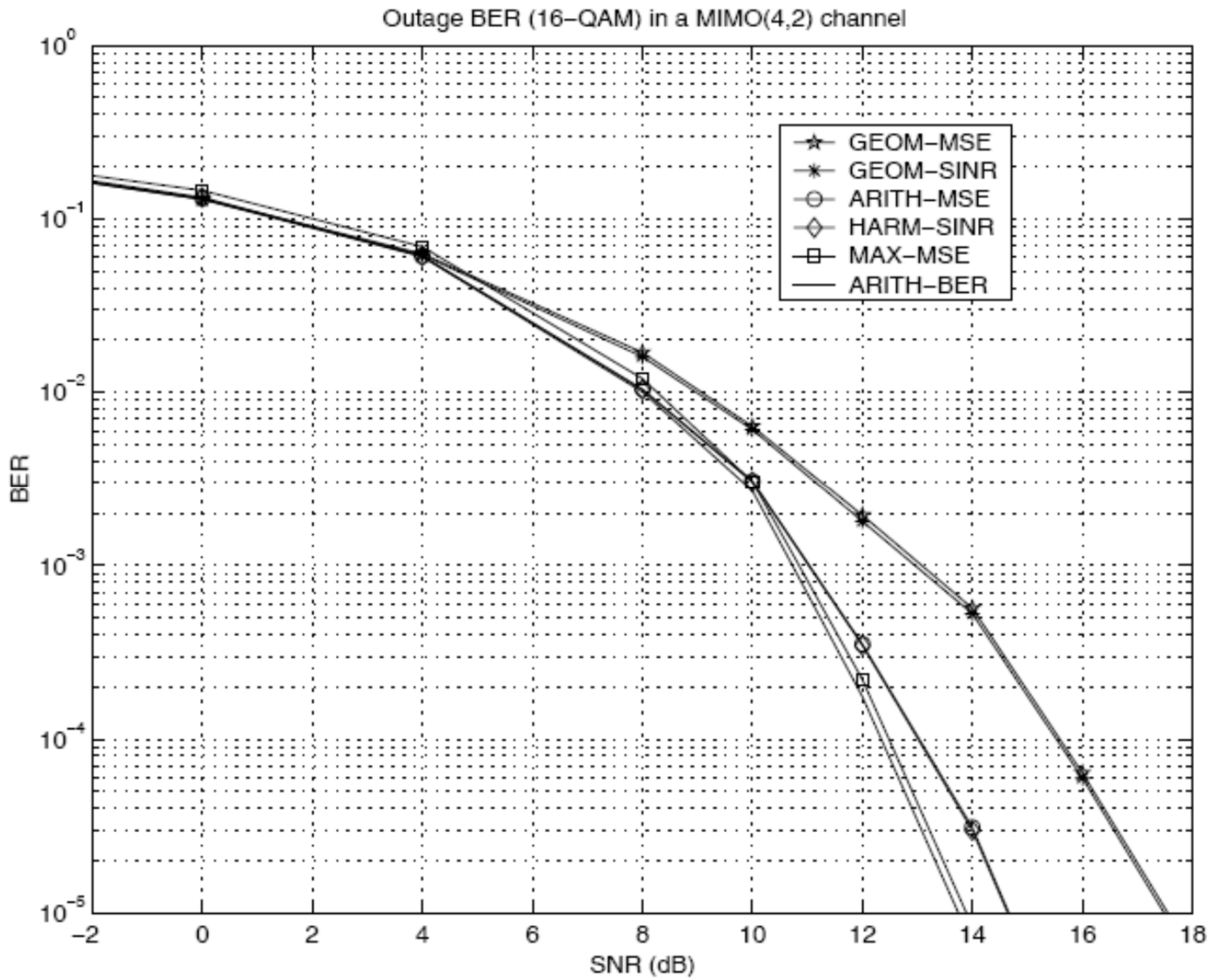
SIMULATIONS





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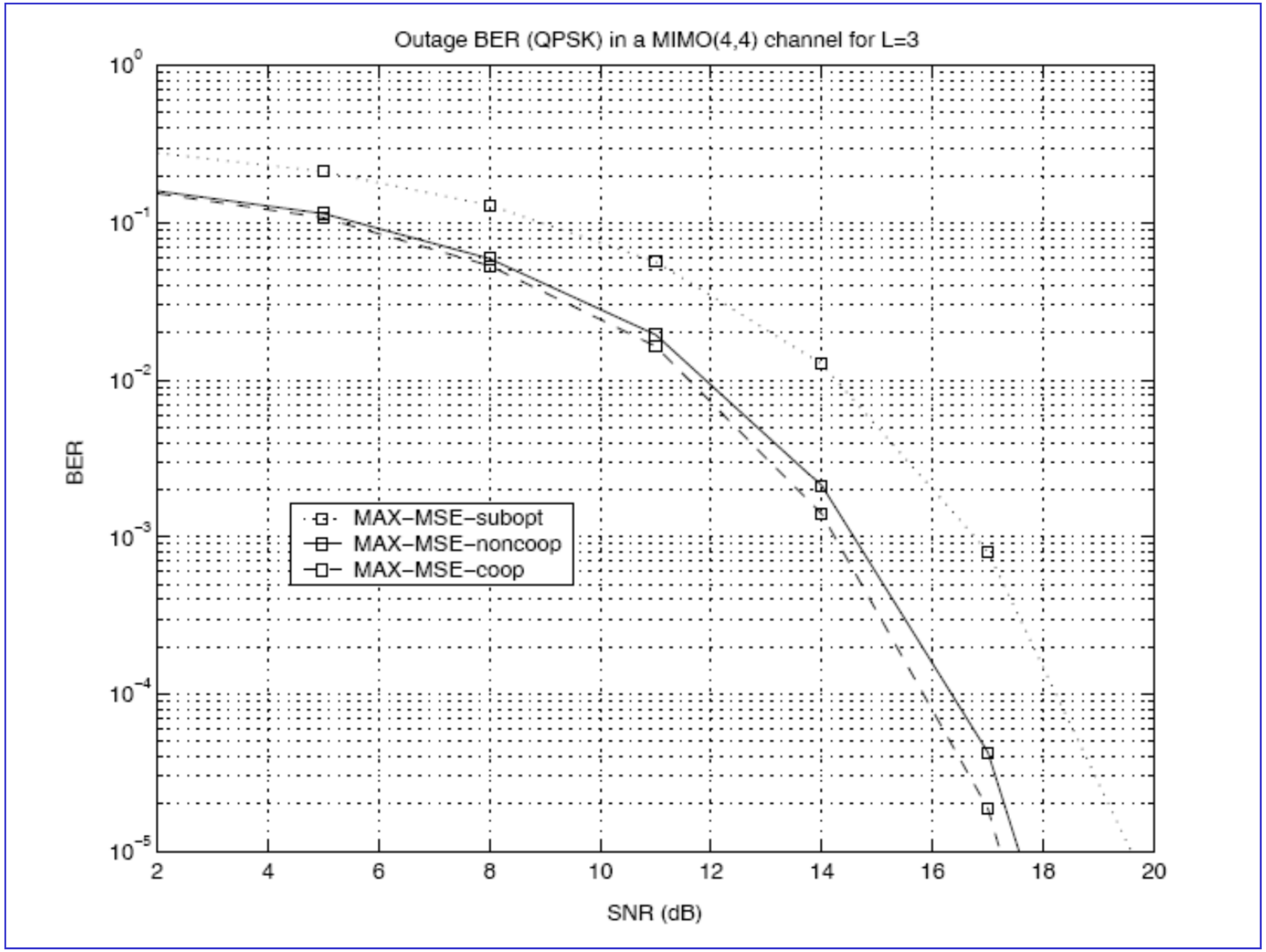
Outage BER (QPSK) in a MIMO(4,4) channel for L=3
BER
SNR (dB)
MAX–MSE–subopt
MAX–MSE–noncoop
MAX–MSE–coop
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2
4
6
8
10
12
14
16
18
20

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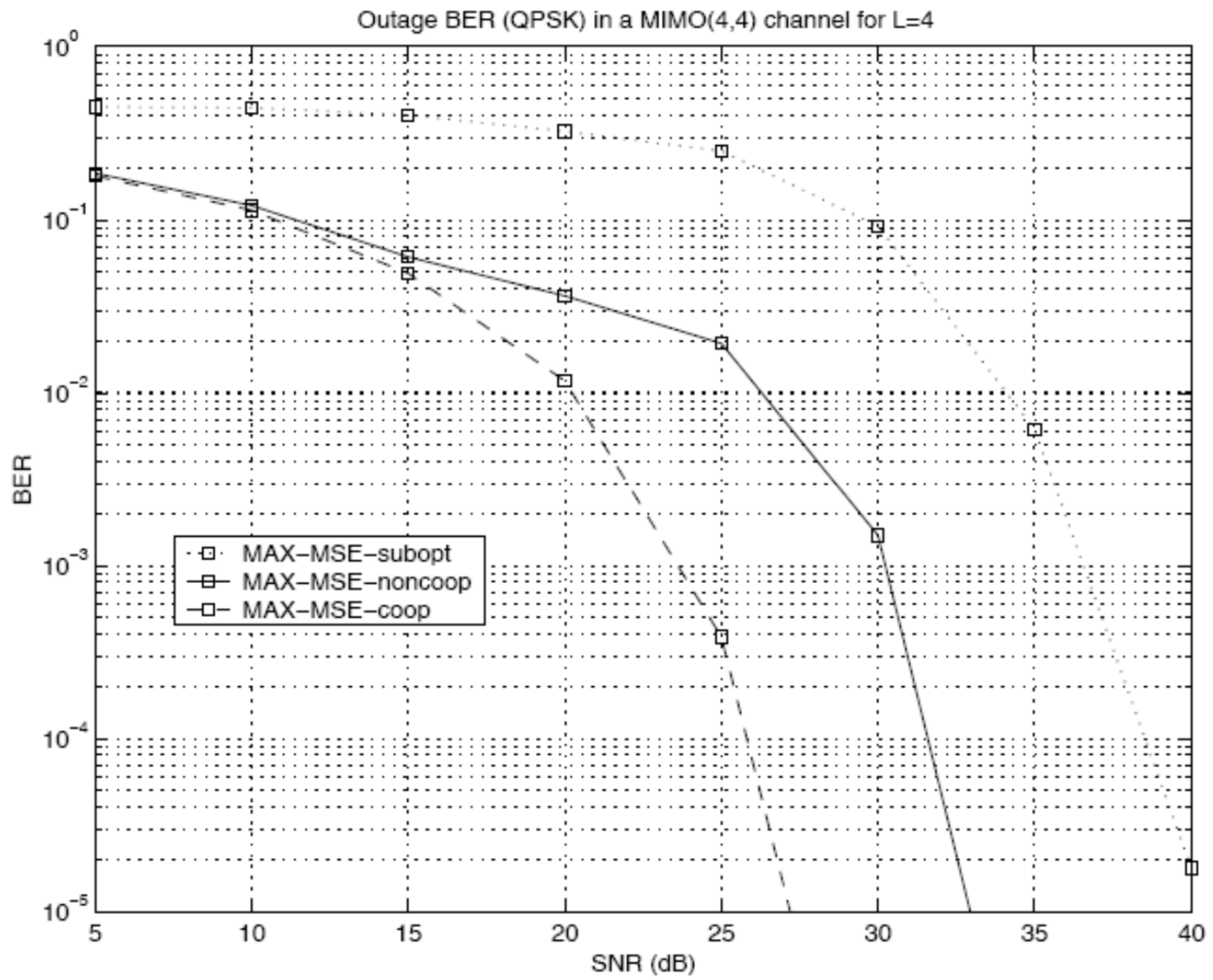

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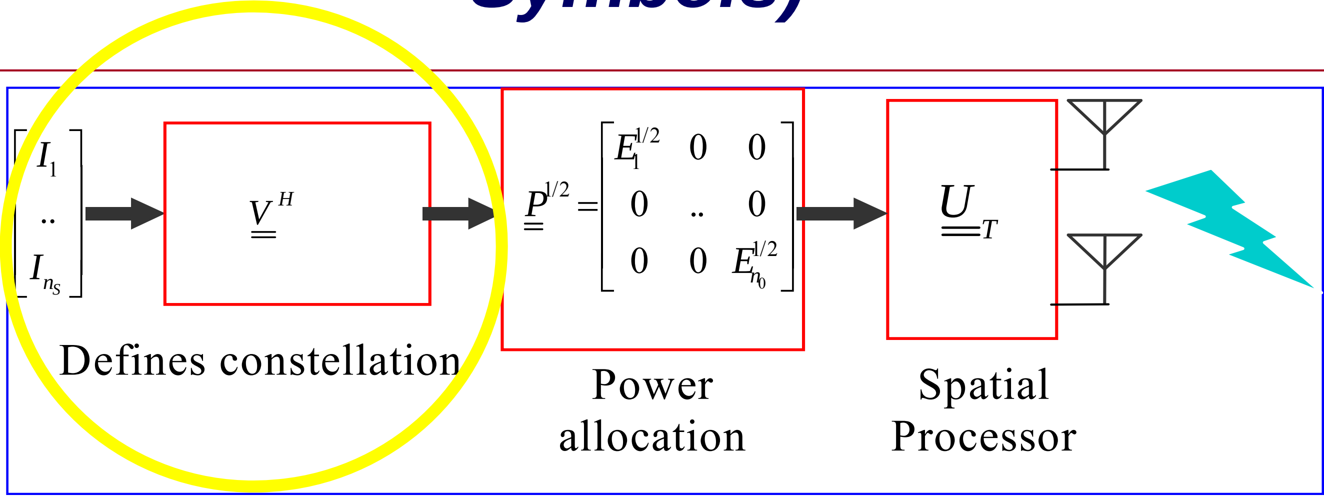
The TX Architecture (Single/Multiple Symbols)



For single symbol at Tx the ML detector focused on the symbol not in the bit-streams was:

$$Traza\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^2\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Traza\left(\underline{\underline{W}}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right]$$

The ML detector for optimum bit decoding will be:

$$Traza\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\underline{\tilde{I}}_0.\underline{\tilde{I}}_0^H\right|^2\right] > 2.\mathrm{Re}\left[Traza\left(\underline{W}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}.\underline{\tilde{I}}_0\right)\right]$$

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Assuming only an error in a single component of the streams vector and with:

$$t(q) = (q,q)\ element\ of\ \underline{\underline{B}}^H\, \underline{\underline{R}}_H\, \underline{\underline{B}}$$

The probability of error per stream will be:

$$P_e(q) = Q\left(\sqrt{\frac{d_q^2 . t(q)}{2.N_0}}\right)$$

Since d_q is selected for unitary energy per symbol for each #q constellation, the design would be to minimice some objective (minimax or average) contrained to the maximum energy to be used by Tx.

Note that it is assumed that there is no room for bit allocation which is assumed done (no sense?)

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Note that the objective of max average SNR will end up in to diagonalice the channel. This implies that the manner that streams will pass the channel will be traveling along the eigenvalues of it.

Proper design: TX under maximum capacity and RX with ML detection

Point To Point MIMO: MIMO PTP

-Served as single user

- Served as contrained rate streams

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The ML Receiver (Baseline)

Taking into account the Tx architecture,

$$\underline{\underline{B}} = \underline{\underline{U}}_T . \underline{\underline{P}}^{1/2} . \underline{\underline{V}}^H$$

The detector formulation in order to compute the pair-wise error probability reduces to:

$$Traza\left[\left(\underline{\underline{U}}_T^H . \underline{\underline{H}}^H . \underline{\underline{H}} . \underline{\underline{U}}_T\right) . \left| \underline{\underline{P}}^{1/2} . \underline{\underline{V}}^H . \underline{\tilde{I}}_0 . \underline{\tilde{I}}_0^H . \underline{\underline{V}} . \underline{\underline{P}}^{1/2} \right|^2 \right] > 2.\mathrm{Re}\left[Traza\left(\underline{W}_n^H . \underline{\underline{H}} . \underline{\underline{B}} . \underline{\tilde{I}}_0\right)\right]$$

where $\underline{\phi} = \underline{\underline{V}}^H . \underline{I}_0 \quad y \quad \underline{\tilde{\phi}} = \underline{\underline{V}}^H . \underline{\tilde{I}}_0$

Note that the spatial processing, the power allocation and the constellation are clearly separated in this formula

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Duality Theory and KKT Conditions

Convex problems

$$\min_{\underline{x}} \; f_0(\underline{x})$$
$$s.t. \quad f_i(\underline{x}) \leq 0 \quad i = 1, m$$
$$h_i(\underline{x}) = 0 \quad i = 1, p$$

Feasible solutions and optimal value f^*

The Lagrangian is:

$$L(\underline{x}, \underline{\lambda}, \underline{\mu}) = f_0(\underline{x}) + \sum \lambda_i . f_i(\underline{x}) + \sum \mu_i(\underline{x})$$

$\underline{\lambda}, \underline{\mu}$ Dual variables

$\underline{x}$ Primal variables

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$$L(\underline{x},\underline{\lambda},\underline{\mu}) = f_0(\underline{x}) + \sum \lambda_i . f_i(\underline{x}) + \sum \mu_i(\underline{x})$$

The dual function (always concave regardless the original could be not)

$$g(\underline{\lambda},\underline{\mu}) = \inf_{\underline{x}\ feasible} L(\underline{x},\underline{\lambda},\underline{\mu})$$

since

$$f_0(\underline{x}) \geq f_0(\underline{x}) + \sum \lambda_i . f_i(\underline{x}) + \sum \mu_i . h_i(\underline{x}) =$$

$$\geq \inf_{\underline{x}} L(\underline{x},\underline{\lambda},\underline{\mu}) = g(\underline{\lambda},\underline{\mu})$$

then

$$\min_{\underline{x}} f_0(\underline{x}) \geq \max_{\underline{\lambda},\underline{\mu}} g(\underline{\lambda},\underline{\mu})$$

$$(\underline{\lambda},\underline{\mu}) \quad dual\ \ feasible \quad if\ \underline{\lambda} > \underline{0}$$

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$$L(\underline{x}, \underline{\lambda}, \underline{\mu}) = f_0(\underline{x}) + \sum \lambda_i . f_i(\underline{x}) + \sum \mu_i(\underline{x})$$

$$g(\underline{\lambda}, \underline{\mu}) = \inf_{\underline{x}\ feasible} L(\underline{x}, \underline{\lambda}, \underline{\mu})$$

DUALITY GAP

$$\min_{\underline{x}} f_0(\underline{x}) - \max_{\underline{\lambda}, \underline{\mu}} g(\underline{\lambda}, \underline{\mu}) \begin{cases} 0 & strong\ duality \\ > 0 & weak\ duality \end{cases}$$

In summary, solve the new problem:

$$\max_{\underline{\lambda}, \underline{\mu}} g(\underline{\lambda}, \underline{\mu})$$

$$s.t. \quad \underline{\lambda} > \underline{0}$$

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KKT (Karush,Kuhn,Tucker) and duality gap

Cero duality gap implies equality in the previous relationship between primal, dual and min-dual functions

$$g(\underline{\lambda},\underline{\mu}) = L(\underline{x},\underline{\lambda},\underline{\mu}) = f_0(\underline{x})$$

implies

$$\nabla_{\underline{x}} L = \underline{0}$$

$$\lambda_i . f_i(\underline{x}) = 0$$

Complementary slackness condition

In summary:

$$h_i(\underline{x}^*) = 0 \qquad f_i(\underline{x}^*) \leq 0$$

$$\lambda_i^* \geq 0 \qquad \nabla_{\underline{x}} L = \underline{0}$$

$$\lambda_i^* . f_i(\underline{x}^*) = 0$$

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Sensitivity Analysis

$$\min_{\underline{x}} f(\underline{x}) \quad s.t. \quad f_i(\underline{x}) \le u_i \quad h_i(\underline{x}) \le v_i$$

$f^*(\underline{u}, \underline{v})$ differentiable at u=v=0

strong duality holds $\quad \lambda_i^* = -\dfrac{\partial f^*(\underline{0},\underline{0})}{\partial u_i} \quad \mu_i^* = -\dfrac{\partial f^*(\underline{0},\underline{0})}{\partial v_i}$

for u_i small and positive $\quad f^* \;\; \textit{increases as} \;\; -\lambda_i^* . u_i$

λ_i^* tell us how active is the constraint at the optimum