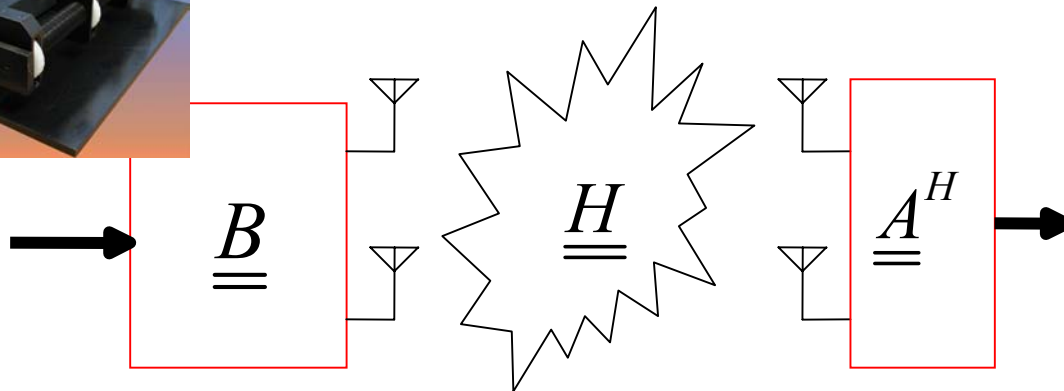


innovating communications

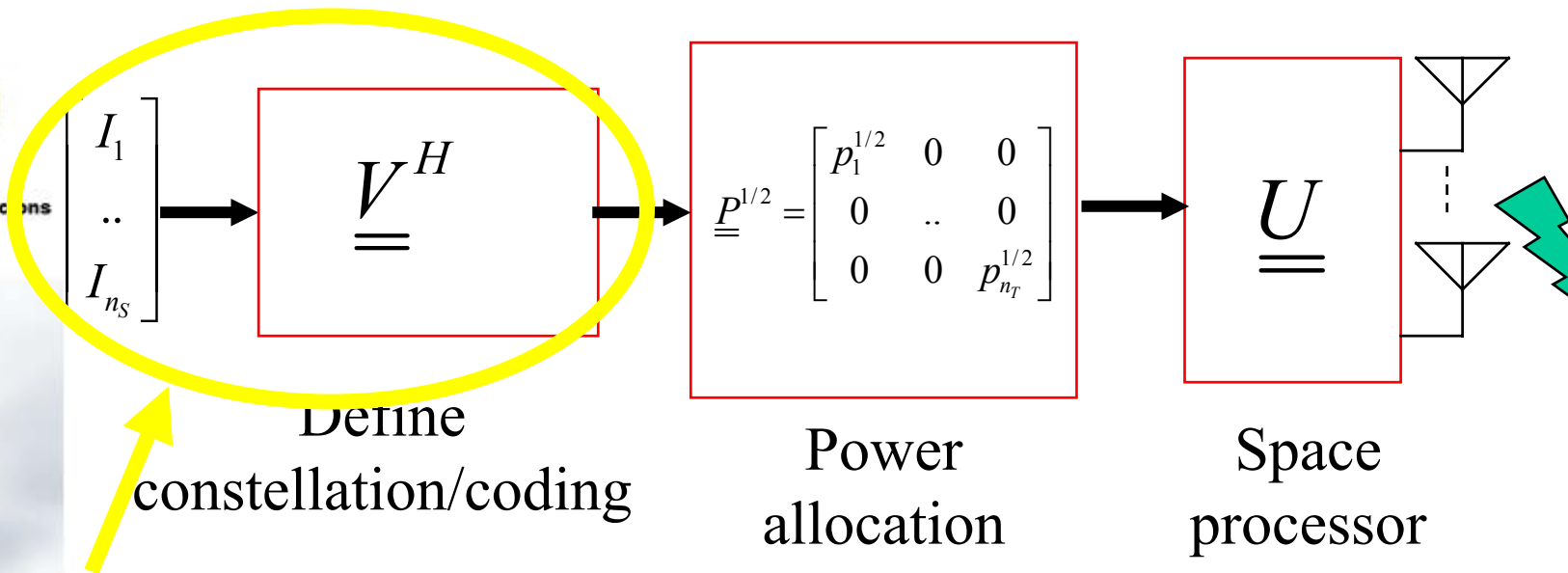
**The Centre Tecnològic de
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A gateway to advanced communication technologies

MIMO5: ML RECEIVERS FOR PTP, MAC and BC

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A detailed view of the space-time processor at Tx is:



REPLACE BY PROPER BIT-ALLOCATION

THE ADEQUATE DESIGN

- TX under maximum capacity
- Bit-Allocation under quality constraints
- RX as Maximum likelihood

BER and RATE

For M by M constellations, the BER can be bounded by:

$$BER \approx \left(\frac{1 + \frac{(2M-3)}{M^2}}{\log_2(M^2)} \right) Q \left(\sqrt{\frac{3}{2^{n_b} - 1} \cdot \left(\frac{E_R}{N} \right)} \right)$$

Where it is assumed that:

- The error is only produced to the nearest neighborhood
- n_b is the number of bits loaded
- E_R is the signal energy received
- N_0 is the gaussian power density of noise plus interferences all assumed white and gaussian distributed



Since, for full square constellations $M_1 \times M_2 = 2^{nb}$, the use of constellations loaded with 3 and 5 bits, where M_1 is different from M_2 , the factor inside the $Q(\cdot)$ function changes to $12/(M_1^2 + M_2^2 - 2)$.

Other fractional rates can be achieved by the use of coding (i.e. Repetition $N:1$ codes)

Also for BPSK the factor 3 in the numerator have to be set equal 2.

Aiming a global quality versus rate formula, the following is selected:

$$BER \approx 0.58 Q \left(\sqrt{\frac{3}{2^{nb} - 1} \cdot \left(\frac{E_R}{N} \right)} \right)$$



In summary, for a Tx energy, with respect the noise level, equal to $z(i)$ and channel gain $\lambda(i)$, the resulting BER can be approximated by:

$$BER(i) \approx 0,58.Q\left(\sqrt{\frac{3}{2^{nb(i)} - 1} \cdot z(i) \cdot \lambda(i)}\right)$$

For a full-diagonalized MIMO, the sum-rate will be:

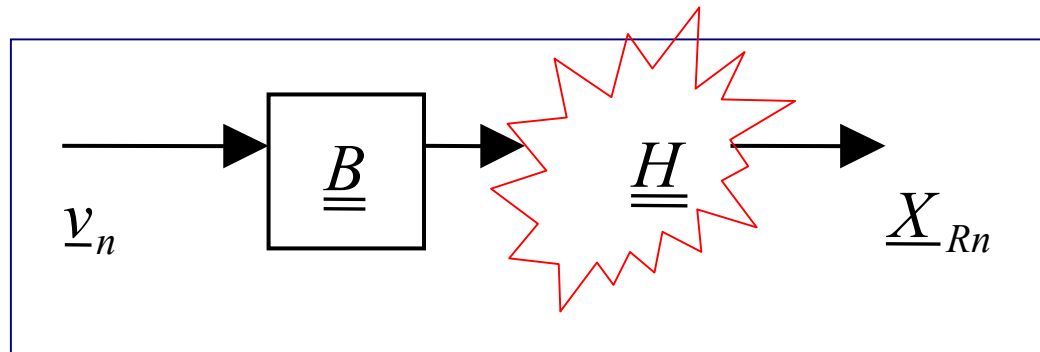
$$R = \sum_{i=1}^{no} nb(i) = \sum_{i=1}^{no} \log_2 \left(1 + \frac{z(i) \cdot \lambda(i)}{\left[Q^{-1}(1,7 \cdot BER)\right]^2 / 3} \right)$$

Thus, for a target-BER and capacity achieving power loading the corresponding fractional rate can be obtained. Reducing fractional rate to integers matching constellations will require, whenever it is possible, to change the power allocation strategy.



MIMO PTP: The ML Detector

$$\Gamma = \left(\underline{X}_{Rn} - \underline{\underline{H}} \cdot \underline{\underline{B}} \cdot \underline{v}_n \right)^H \cdot \underline{\underline{R}}_0^{-1} \cdot \left(\underline{X}_{Rn} - \underline{\underline{H}} \cdot \underline{\underline{B}} \cdot \underline{v}_n \right)$$



Using the square-root, generalized, for the noise covariance:

$$\underline{\underline{R}}_0^{-1} = \underline{\underline{A}} \cdot \underline{\underline{A}}^H$$

$$\underline{\underline{A}} = \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{\Phi}} \quad \text{con} \quad \underline{\underline{\Phi}} \cdot \underline{\underline{\Phi}}^H = \underline{\underline{I}}$$



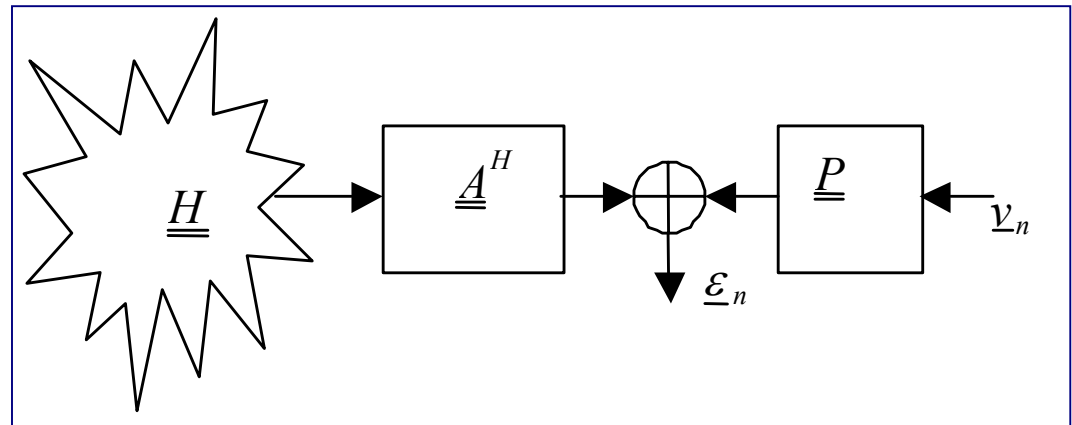
The likelihood is represented in terms of a forward equalizer plus the DIR of the ML detector

$$\Gamma = \left| \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{P}} \cdot \underline{\underline{v}}_n \right|^2$$

where $\underline{\underline{P}} = \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}}$

Note that the forward equalizer will be diagonal for the interference free scenario

The ML receiver is:



Note that in this optimum receiver the error is always diagonal, in consequence, we may relate directly MSE, SNR and BER without the problems associated with classical MSE and ZF receivers.

$$\underline{\underline{E}} = E\left(\underline{\underline{\varepsilon}} \cdot \underline{\underline{\varepsilon}}^H\right) = \underline{\underline{I}}$$

MAXIMUM CAPACITY DESIGN AT TX:

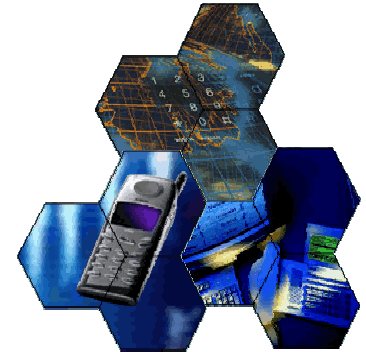
Tx and Rx have to diagonalize the channel

$$\underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} = \underline{\underline{V}}_H \cdot \underline{\underline{\Lambda}}_H^{1/2} \cdot \underline{\underline{U}}_H^H$$

Thus, the orthogonal matrix of the forward equalizer is set to the left eigenvector matrix of the channel above

Tx diagonalizes the channel and provides optimum power allocation

$$\underline{\underline{B}} = \underline{\underline{U}}^H \cdot \underline{\underline{\Lambda}}_B^{1/2}$$



Where power allocation water-fills the channel eigenmodes

$$\underline{\underline{\Lambda}}_B = WF(\underline{\underline{\Lambda}}_H)$$

$$q = 1, n_0 \quad \lambda_B(q) = \mu - \frac{1}{\lambda_H(q)}$$

$$\mu = \frac{E_T}{n_0} + \frac{1}{\lambda_{HAR}}$$

$$\lambda_{HAR}^{-1} = n_0^{-1} \cdot \sum_1^{n_0} \lambda_H^{-1}(q)$$

$$1 + \lambda_H(q) \cdot \lambda_B(q) = \mu \cdot \lambda_H(q)$$

$$q = n_0 + 1, N_t \quad \lambda_B(q) = 0$$

$$1 + \lambda_H(q) \cdot \lambda_B(q) = 1$$

$$C_{resul\ tan\ te} = n_0 \cdot \log_2 \left(\frac{E_T \cdot \lambda_{GEO}}{n_0} + \frac{\lambda_{GEO}}{\lambda_{HARM}} \right)$$

MIMO-PTP: Design summary

MIMO Point to Point FULL-CSI

$$0. - \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} = \underline{\underline{V}} \cdot \underline{\underline{\Lambda}}_H^{1/2} \cdot \underline{\underline{U}}^H$$

$$1. - \underline{\underline{\Lambda}}_B = WF(\underline{\underline{\Lambda}}_H)$$

2. - *BA according WF*

$$BA = \text{function}(BER_tar, SNR_disp = (\underline{\underline{\Lambda}}_B \cdot \underline{\underline{\Lambda}}_H),)$$

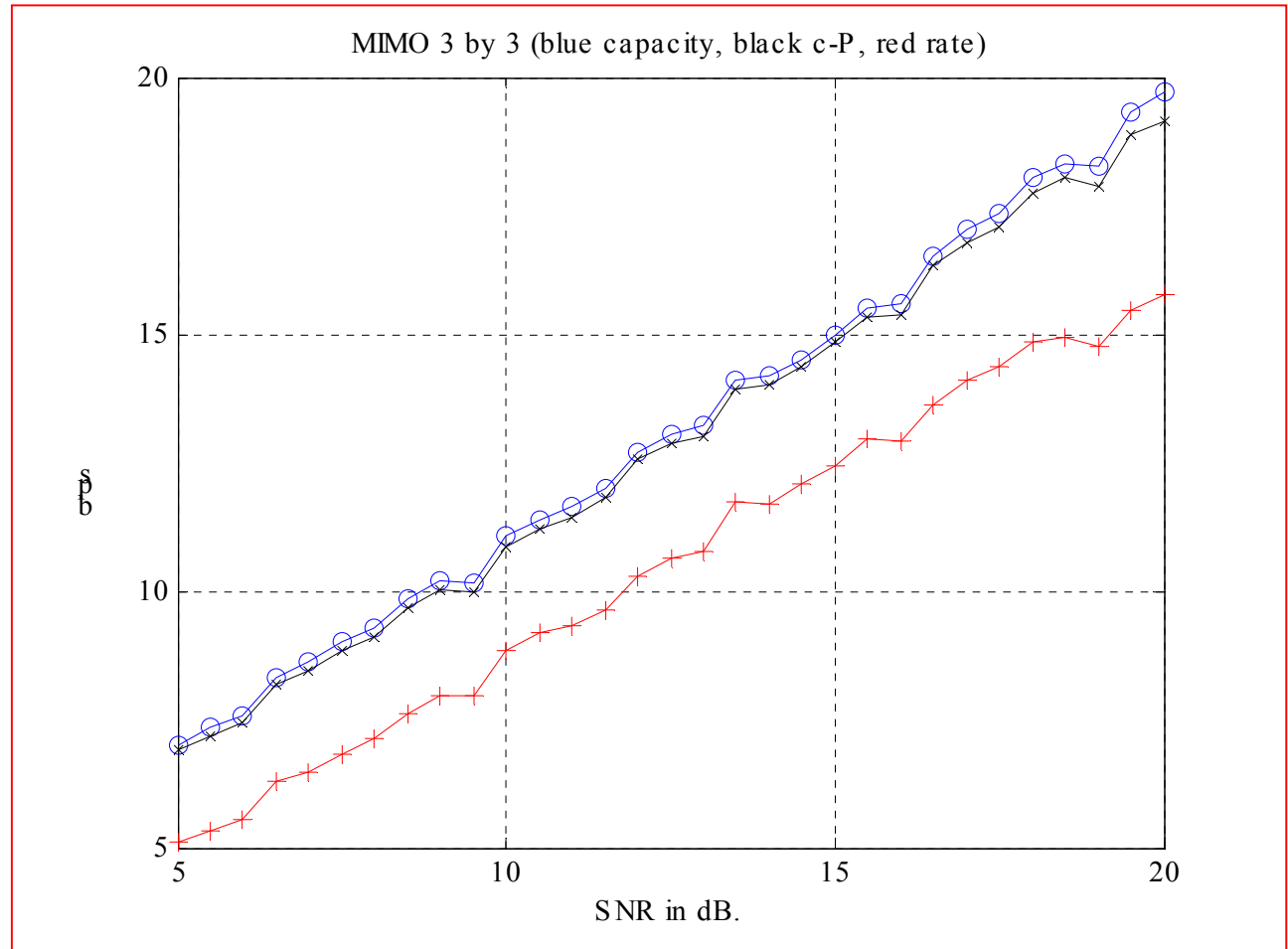
$$3. - \underline{\underline{A}} = \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{V}}$$

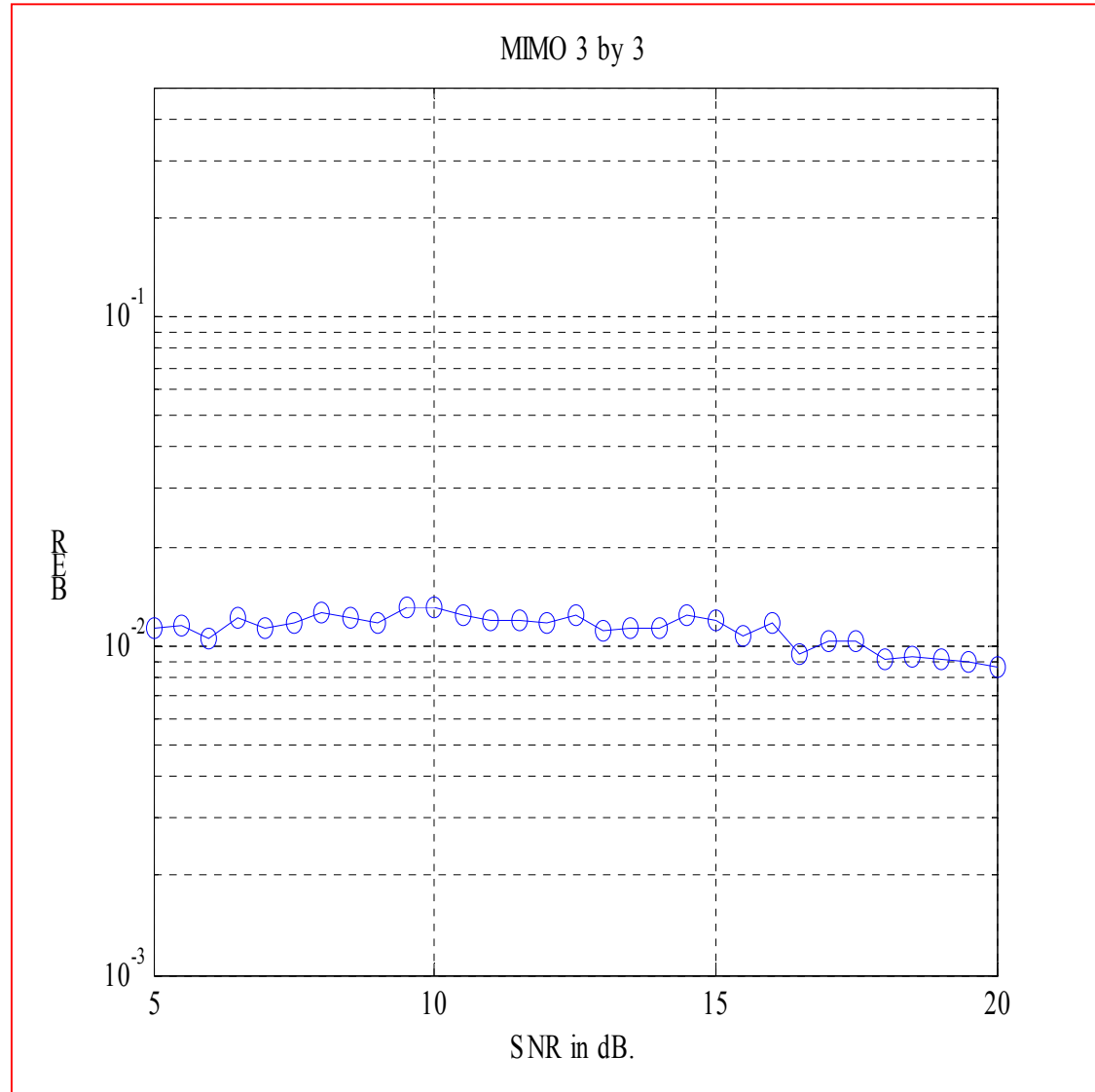
$$4. - \underline{\underline{E}} = \underline{\underline{I}}$$

$$5. - \underline{\underline{P}} = (\underline{\underline{\Lambda}}_H \cdot \underline{\underline{\Lambda}}_B)^{1/2} \quad (\text{direct - decission})$$

$$6. - \underline{\underline{B}} = \underline{\underline{U}} \cdot \underline{\underline{\Lambda}}_B^{1/2}$$

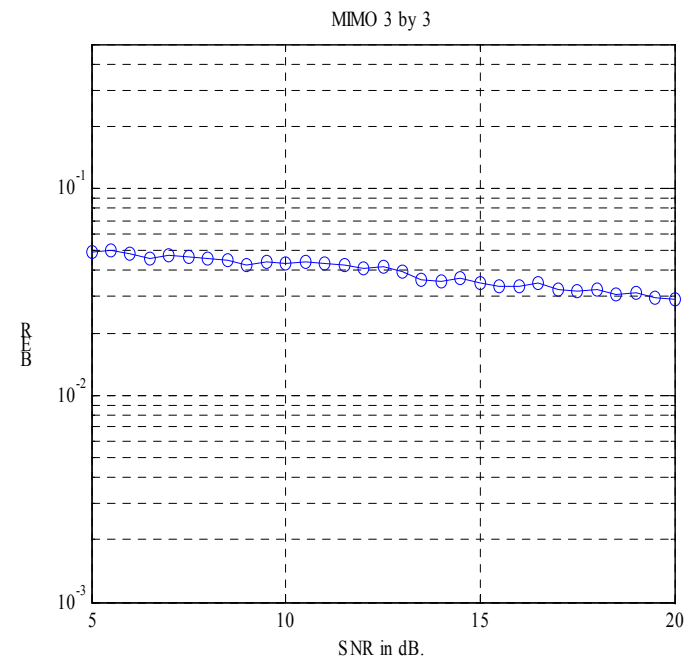
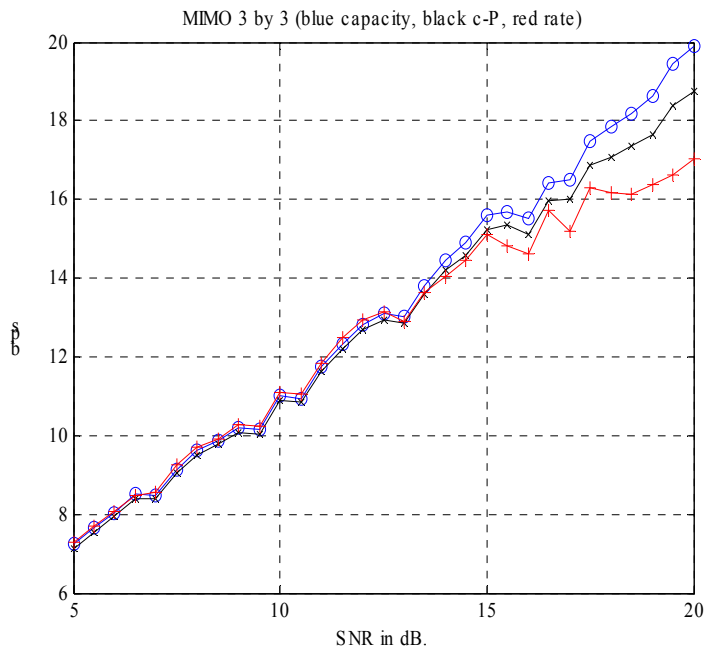
MIMO (3,3) Full CSIT, BER target 10^{-2} , constellations size bounded to 64-QAM, 50 averages on channel realizations.





No sense of maximum capacity designs without quality constraints

$$\text{BER} = 10^{-2}$$



The MIMO-MAC scenario

No cooperation at Tx $\underline{\underline{B}} = \text{diag}\left(Pot^{1/2}(i)\right) = \underline{\underline{\Lambda}}_B^{1/2}$

The DIR at the cooperative receiver is:

$$\underline{\underline{P}} = \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{\Lambda}}_B^{1/2} = \underline{\underline{V}}_H \underline{\underline{\Lambda}}_H^{1/2} \cdot \underline{\underline{U}}_H^H \cdot \underline{\underline{\Lambda}}_B^{1/2} \quad \text{with} \quad \underline{\underline{A}}^H = \underline{\underline{R}}_0^{-1/2}$$

The problem is to modify the Rx in order to implement direct decision instead the ML search.

With the QR decomposition of the Rx plus channel

$$qr\left(\underline{\underline{A}}^H \cdot \underline{\underline{H}} = \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}}\right) = \underline{\underline{Q}} \cdot \underline{\underline{R}}$$





The likelihood is:

$$\underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{s}}_n = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{Q}} \cdot \underline{\underline{R}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{s}}_n$$

Concerning the forward equalizer, it may start diagonalizing the channel by:

$$\underline{\underline{A}}^H = \underline{\underline{Q}}^H \cdot \underline{\underline{R}}_0^{-1/2}$$

Note that still we are in the ML framework.

$$\underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{s}}_n = \underline{\underline{z}}_n - \underline{\underline{R}} \cdot \underline{\underline{\Lambda}}_B \cdot \underline{\underline{s}}_n$$

At this stage, the only manner of implementing direct decision is to develop the DIR as a backward equalizer or DFE. To do so, we need that all the diagonal elements of R have to be equal to one.



Setting all the main diagonal elements of R to one implies that some power, from the available at the multiple Tx units have to be devoted to this.

$$\underline{\underline{\Lambda}}_B^{1/2} = \text{diag}(\text{diag}(\underline{\underline{R}}))^{-1} \underline{\underline{\Lambda}}_{BU}^{1/2}$$

Power used to diagonalize R (could be negative = channel gain)

Power available for BA.

also

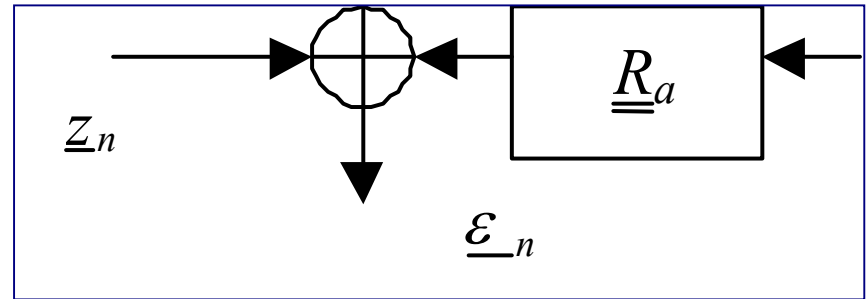
$$\underline{\underline{\Lambda}}_{BU} = \text{diag}(\text{diag}(\underline{\underline{R}}))^2 \cdot \underline{\underline{\Lambda}}_B$$



being

$$\underline{\underline{R}}_a = \left(\text{diag} \left[\text{diag}(\underline{\underline{R}}) \right] \right)^{-1} \cdot \underline{\underline{R}} = \underline{\underline{I}} + \underline{\underline{C}}$$

The DFE receiver is:



with

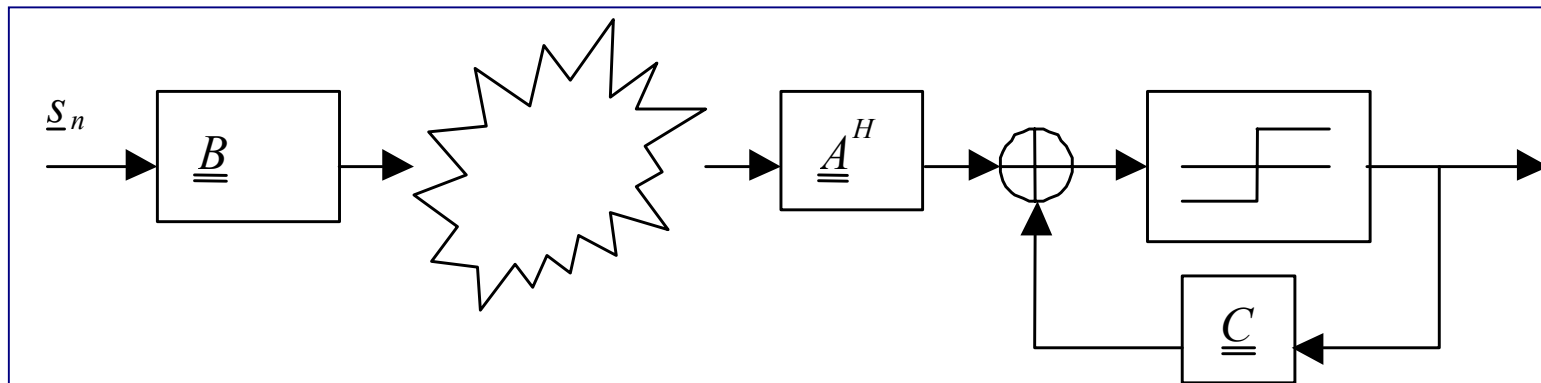
$$\underline{\underline{\varepsilon}}_n = \underline{\underline{R}}_a \cdot \underline{\underline{s}}_n - \underline{\underline{z}}_n \quad \rightarrow \quad \underline{\underline{s}}_n + \underline{\underline{C}} \cdot \underline{\underline{s}}_n - \underline{\underline{z}}_n = \underline{\underline{\varepsilon}}_n$$

$$\underline{\underline{s}}_n = \left(\underline{\underline{z}}_n - \underline{\underline{C}} \cdot \underline{\underline{s}}_n \right) + \underline{\underline{\varepsilon}}_n$$

The above equation changes as indicated below in order to remove the error term by making direct decision on every component of vector $\underline{\underline{s}}_n$

$$\hat{s}_n(q) = \text{dec} \left[z_n(q) - \sum_{i=1}^{NU-q} c(q, q+i) \cdot \hat{s}_n(q+i) \right]$$

MIMO-MAC Scheme and Design Summary



$$\underline{A}_0 = \underline{R}_0^{-1/2}$$

$$qr(\underline{A}_0^H \cdot \underline{H}) = \underline{Q} \cdot \underline{R}$$

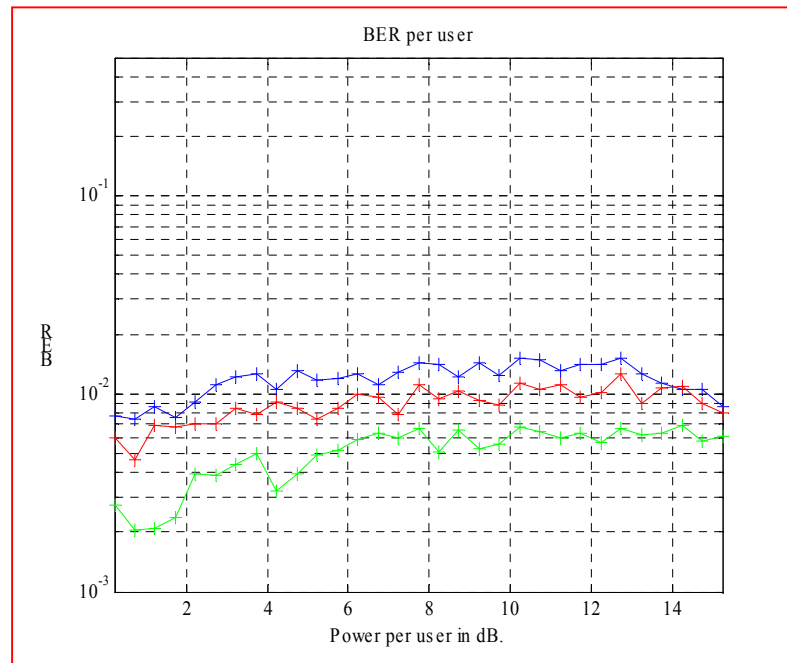
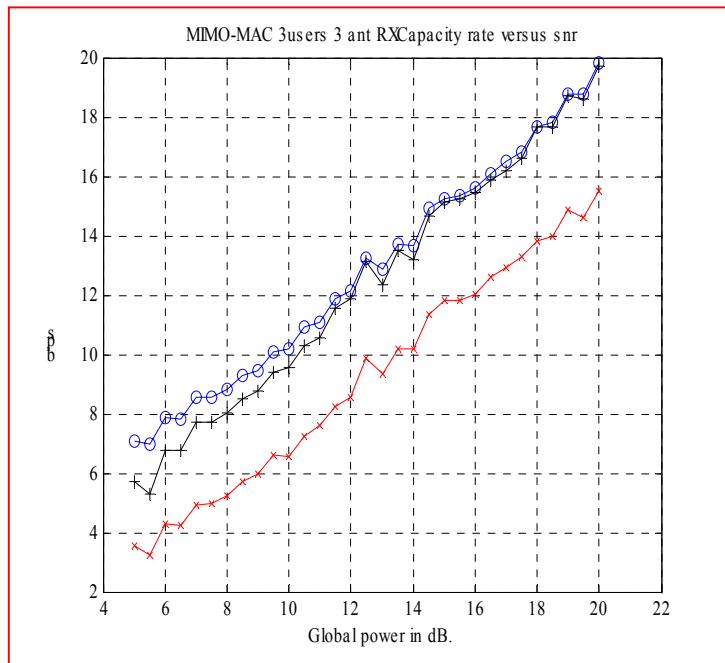
$$\underline{\Lambda}_{SNR_disp} = \underline{\Lambda}_B \cdot (\text{diag}(\text{diag}(\underline{R}))^2)$$

$$BA(\underline{\Lambda}_{SNR_disp}, BER_tar)$$

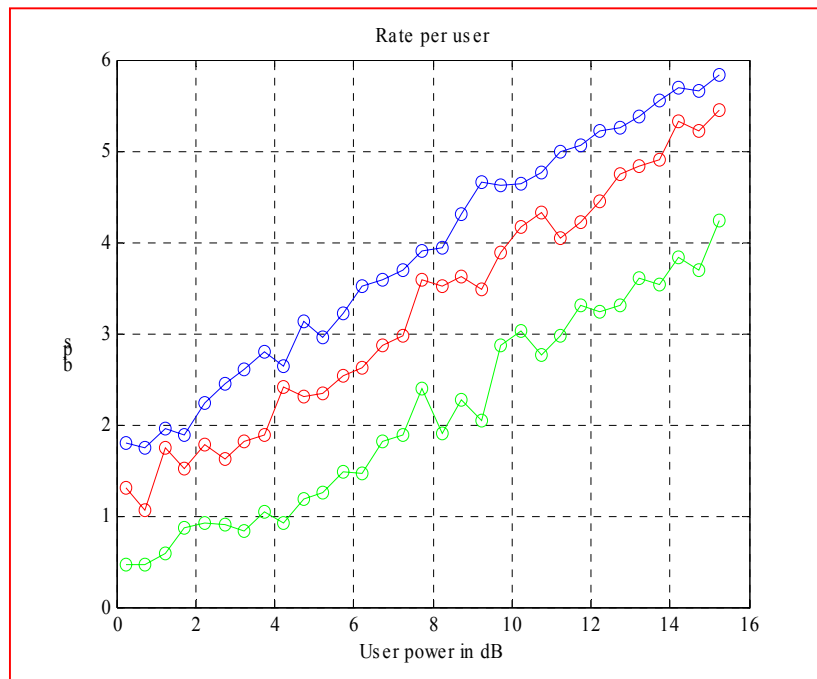
$$\underline{A}^H = \underline{Q}^H \cdot \underline{R}_0^{-1/2}$$

$$\underline{C} = \text{upper_triangular}(\underline{R})$$

MIMO-MAC: 3 users, 3 antennas at Rx, BER target 10^{-2} , 50 channel realizations.



Note the problem of users “labeling”. User #3 always favored in BER and lowered on average rate.....



To solve the labeling problem we need the generalized QR decomposition, where matrix T is a permutation that provides the diagonals of R in decreasing order

$$\underline{\underline{A}}^H \cdot \underline{\underline{H}} = \underline{\underline{Q}} \cdot \underline{\underline{R}} \cdot \underline{\underline{T}}^H$$

The new error is:

$$\underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{\Lambda}}_B^{1/2} = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \underline{\underline{Q}} \cdot \underline{\underline{R}}_a \cdot \text{diag}(\underline{\underline{R}}) \cdot \underline{\underline{T}}^H \cdot \underline{\underline{\Lambda}}_B^{1/2}$$

and

$$\begin{aligned} \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{Q}} \cdot \underline{\underline{R}}_a \cdot \text{diag}(\underline{\underline{R}}) \cdot \underline{\underline{T}}^H \cdot \underline{\underline{\Lambda}}_B^{1/2} &= \\ = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{Q}} \cdot \underline{\underline{R}}_a \cdot \text{diag}(\underline{\underline{R}}) \cdot \underline{\underline{T}}^H &\cdot \underline{\underline{\Lambda}}_{BO}^{1/2} \cdot \underline{\underline{T}} \cdot \underline{\underline{T}}^H \cdot \underline{\underline{\Lambda}}_{B_use} \end{aligned}$$

now to force one at the diagonal entries of R, we need

$$\text{diag}(\text{diag}(\underline{\underline{R}}))^{-1} = \underline{\underline{T}}^H \cdot \underline{\underline{\Lambda}}_{BO} \cdot \underline{\underline{T}}$$

$$\underline{\underline{\Lambda}}_{BO} = \underline{\underline{T}} \cdot \text{diag}(\text{diag}(\underline{\underline{R}}))^{-1} \cdot \underline{\underline{T}}^H$$

$$\underline{\underline{\Lambda}}_{B_use} = \underline{\underline{\Lambda}}_B \cdot \underline{\underline{\Lambda}}_{BO}^{-1}$$

Providing
the proper
available
power for
BA

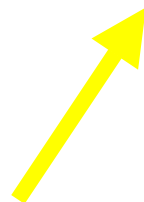
The new error is:

$$\underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{s}}_n = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}} \cdot \underline{\underline{Q}} \cdot \underline{\underline{R}}_a \cdot \underline{\underline{T}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{s}}_n$$

with

$$\underline{\underline{A}}^H = \underline{\underline{Q}}^H \cdot \underline{\underline{R}}_o^{1/2}$$

$$\underline{\underline{z}}_n - \underline{\underline{R}}_a \cdot \left(\underline{\underline{T}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \right) \underline{\underline{s}}_n$$



Users permuted
according T

Dynamic to be used
at the DFE decision
block:

$$\text{diag} \left(\underline{\underline{T}} \cdot \underline{\underline{\Lambda}}_B^{1/2} \cdot \underline{\underline{T}}^H \right)$$



MIMO-MAC Summary

$$\underline{\underline{A}}_0 = \underline{\underline{R}}_0^{-1/2}$$

$$qr(\underline{\underline{A}}_0^H \cdot \underline{\underline{H}}) = \underline{\underline{Q}} \cdot \underline{\underline{R}} \cdot \underline{\underline{T}}^H$$

$$\underline{\underline{\Lambda}}_{SNR_disp} = \underline{\underline{\Lambda}}_B \cdot \underline{\underline{T}}^H (\text{diag}(\text{diag}(\underline{\underline{R}})))^2 \cdot \underline{\underline{T}}$$

$$BA(\underline{\underline{\Lambda}}_{SNR_disp}, BER_tar)$$

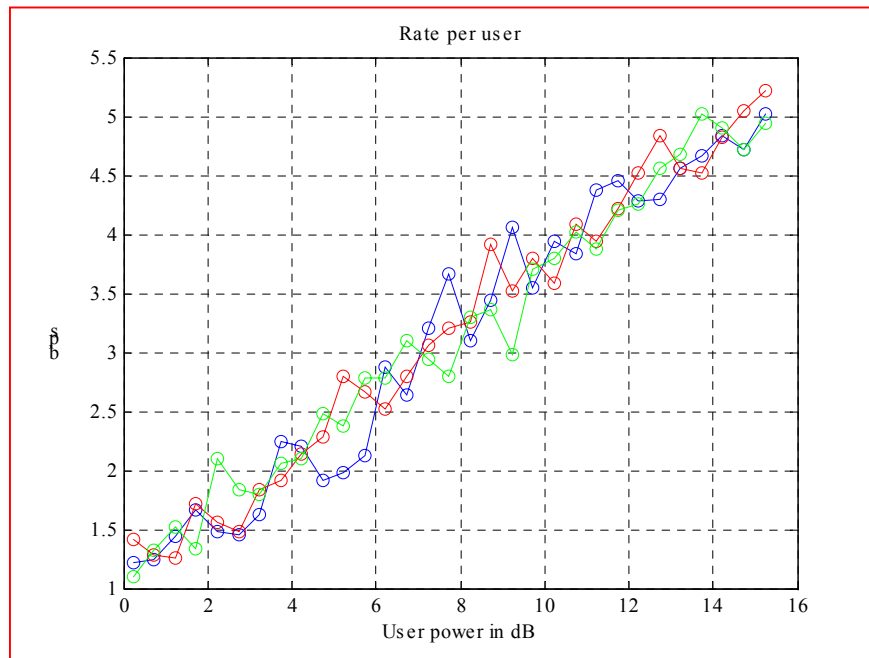
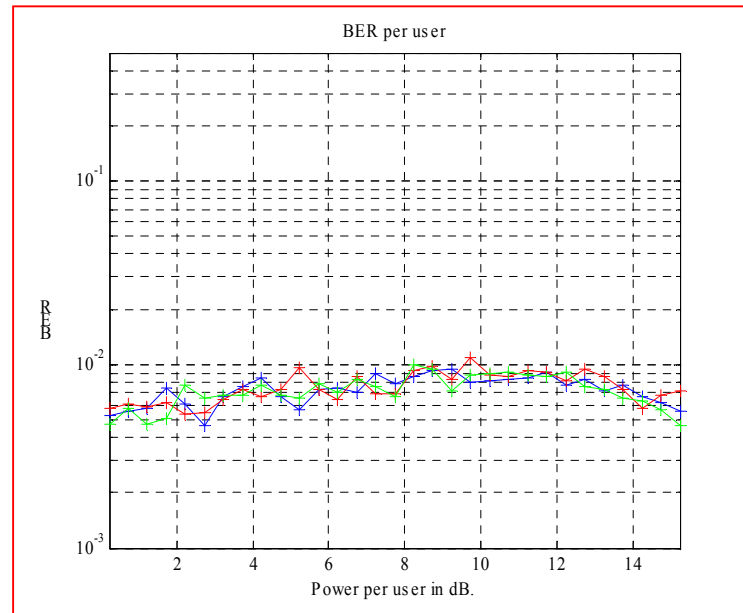
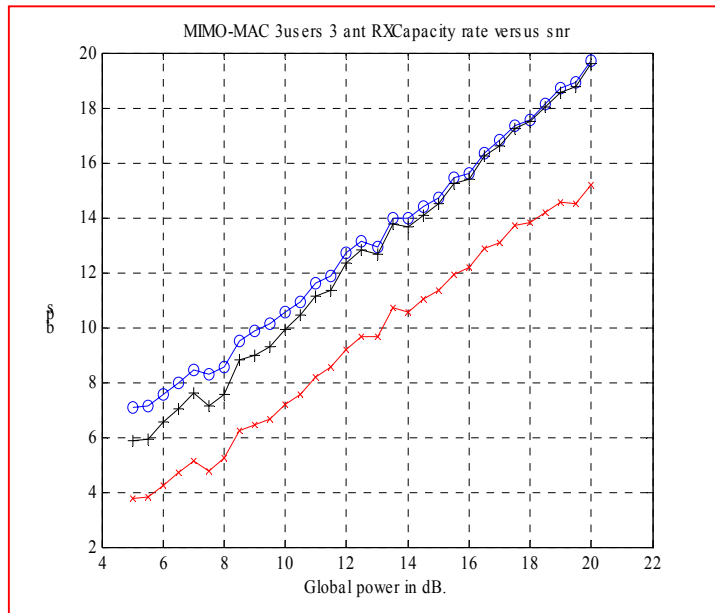
$$\underline{\underline{A}}^H = \underline{\underline{Q}} \cdot \underline{\underline{R}}_0^{-1/2}$$

$$\underline{\underline{C}} = \text{upper_triangular}(\underline{\underline{R}})$$

$$DFE \text{ sobre } \underline{\underline{T}} \cdot \underline{\underline{\Lambda}}_{SNR_disp}^{1/2}$$

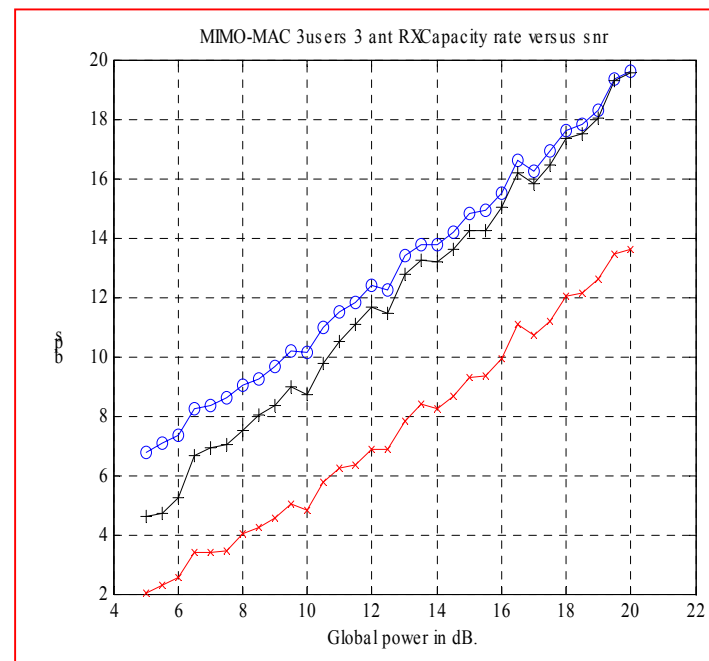
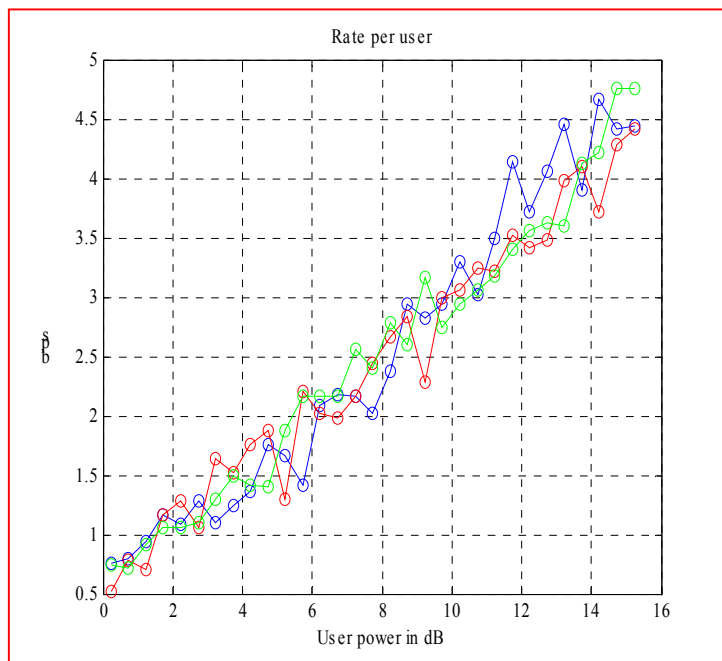


BER=10⁻²

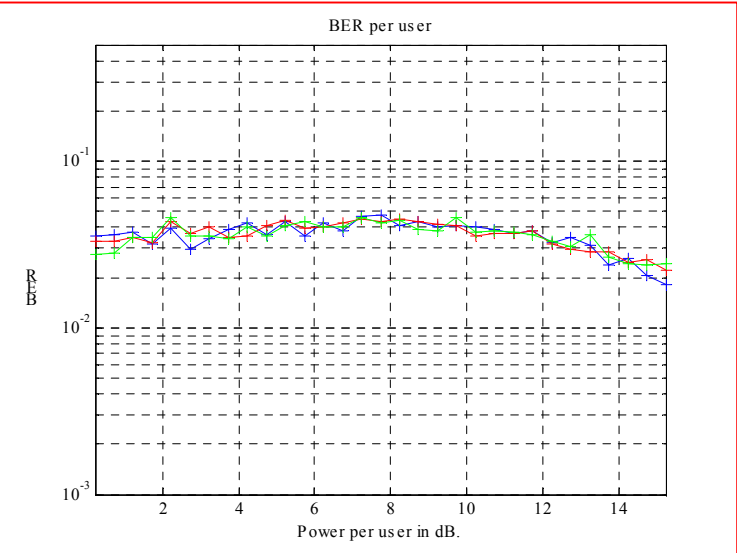
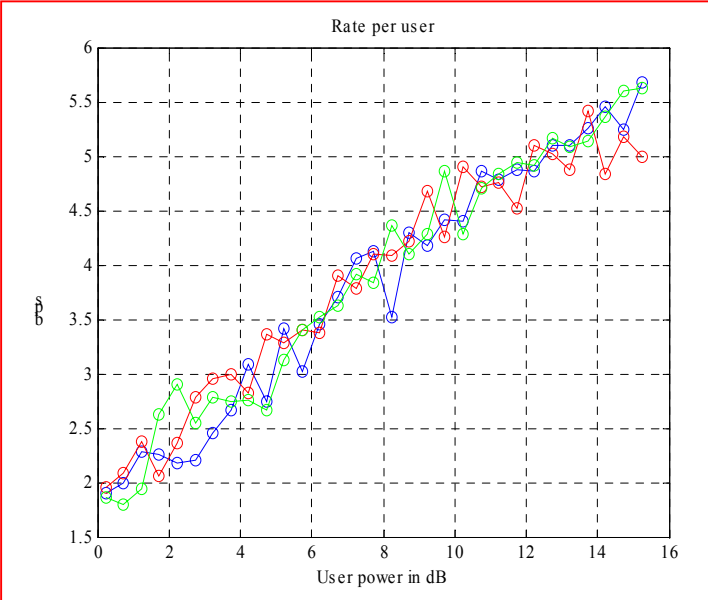
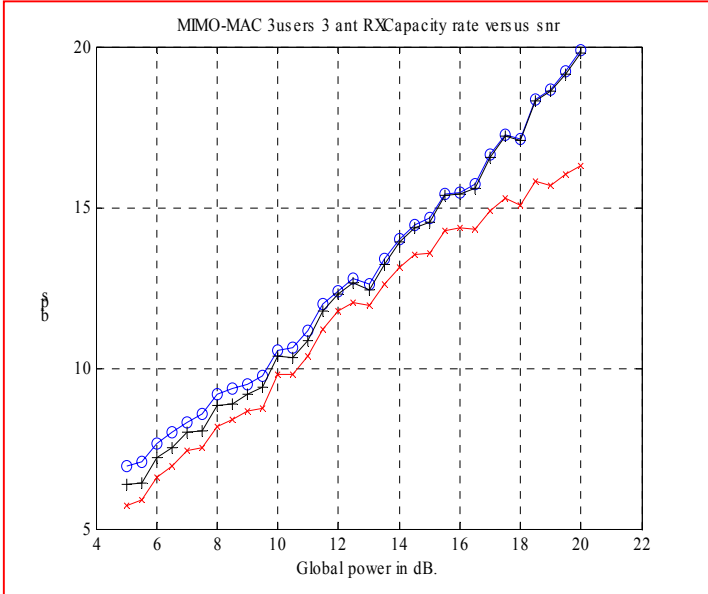




$$\text{BER}=10^{-3}$$



$BER=5 \cdot 10^{-2}$



THE MIMO-BC SCENARIO

Tx is cooperative so maximum capacity design is allowed. On the other hand, the receivers cannot cooperate.



This implies that the DIR have to pass to the Tx side where the signal processing is going to be concentrated

It is necessary a new Tx signal \underline{v} such that $\underline{v} = \underline{W}.s$ and $\underline{P}.\underline{W} = \underline{I}$ $\underline{W}.\underline{W}^H = \underline{I}$, this last constraint in order to preserve the max. Cap. design at Tx . In other words, the Tx implements this matrix preceding the power allocation and the beamforming matrix.....



The DIR to be passed to TX is: $\underline{\underline{P}} = \underline{\underline{A}}^H \cdot \underline{\underline{V}}_H \cdot \underline{\underline{\Lambda}}_H^{1/2} \cdot \underline{\underline{\Lambda}}_B^{1/2}$

Using the qr-decomposition (note that R^H is upper triangular)

$$\underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} \cdot \underline{\underline{U}} \cdot \underline{\underline{\Lambda}}_B \underline{\underline{Q}} = \underline{\underline{R}}^H$$

Note the difference
between MIMO PTP
and MIMO BC

$$\begin{array}{ll} \text{MIMO} - \text{BC} & \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} \cdot \underline{\underline{U}} \cdot \underline{\underline{Q}} = \underline{\underline{R}}^H \\ \text{MIMO} - \text{PP} & \underline{\underline{V}}^H \cdot \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} \cdot \underline{\underline{U}} = \underline{\underline{\Lambda}}_H \end{array}$$

Very important: Since the QR includes the power allocation matrix, the SNR available for proper BA under quality constraint is given by the diagonals of matrix R.



The receiver is:

$$\underline{\underline{A}}^H \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{A}}^H \underline{\underline{H}} \cdot \underline{\underline{B}} = \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{R}}^H \cdot \underline{\underline{Q}}^H \cdot \underline{\underline{v}}_n$$

Note that when the new vector of streams is introduced we start deviating from the optimum ML framework

$$\underline{\underline{s}}_n = \underline{\underline{R}}^H \cdot \underline{\underline{Q}}^H \cdot \underline{\underline{v}}_n \quad \text{or} \quad \underline{\underline{v}}_n = \underline{\underline{Q}} \cdot \underline{\underline{R}}^{-H} \cdot \underline{\underline{s}}_n$$

The problem is that the new Tx including matrixes Q, R, Power allocation and U, deviates from the optimum design. The optimum design is not preserved since the following property does not hold (exactly!!)

$$E(\underline{\underline{s}}_n \cdot \underline{\underline{s}}_n^H) = \underline{\underline{R}}^H \cdot \underline{\underline{R}} \stackrel{?}{=} \underline{\underline{I}}$$

We will go back to this problem hereafter

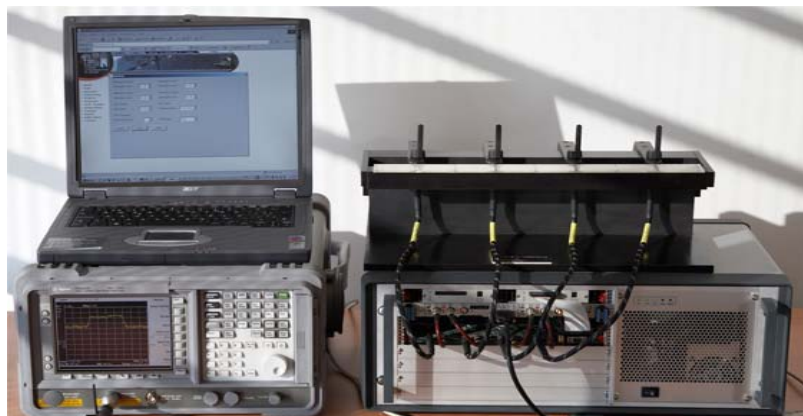




It is not longer needed to pass the complete R to Tx, furthermore, it is better for complexity issues to pass only its equal diagonal version (with ones in the main diagonal). To do it, a diagonal receiver matrix can be implemented containing the inverse of these diagonal entries. Thus the receiver is set as:

$$\underline{\underline{A}}^H = \text{diag}(\text{inv}(\text{diag}(\underline{\underline{R}}^H))).\underline{\underline{R}}_0^{-1/2}$$

Note also that the BA is performed in accordance to the BER target and the SNR available which is given by the diagonal entries of matrix R



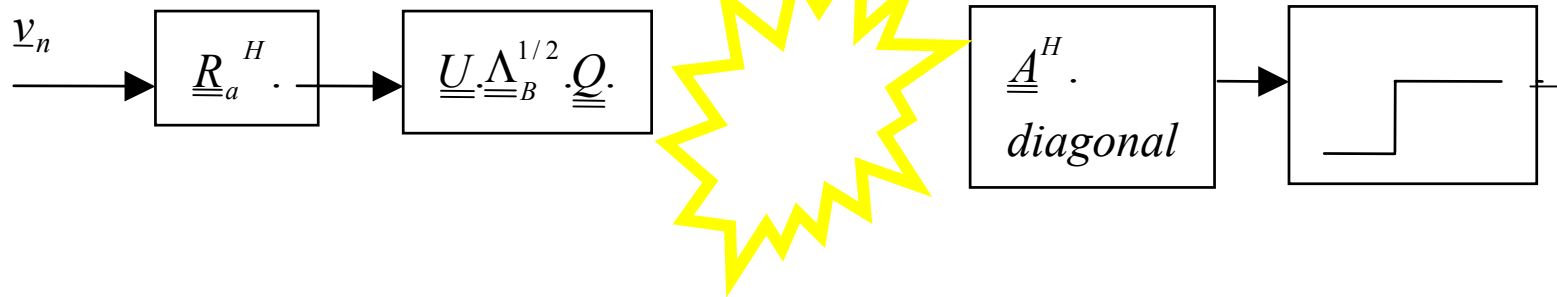
Thus, the new DIR is:

$$\underline{\underline{P}} = \underline{\underline{A}}^H \cdot \underline{\underline{R}}^H \cdot \underline{\underline{Q}}^H = \underline{\underline{R}}_a^H \cdot \underline{\underline{Q}}^H$$

The diagonal receiver (direct decision) is:

$$\underline{\underline{\varepsilon}} = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_R - \underline{\underline{R}}_o^H \cdot \underline{\underline{Q}}^H \cdot \underline{\underline{s}} = \underline{\underline{A}}^H \cdot \underline{\underline{X}}_R - \underline{\underline{v}}$$

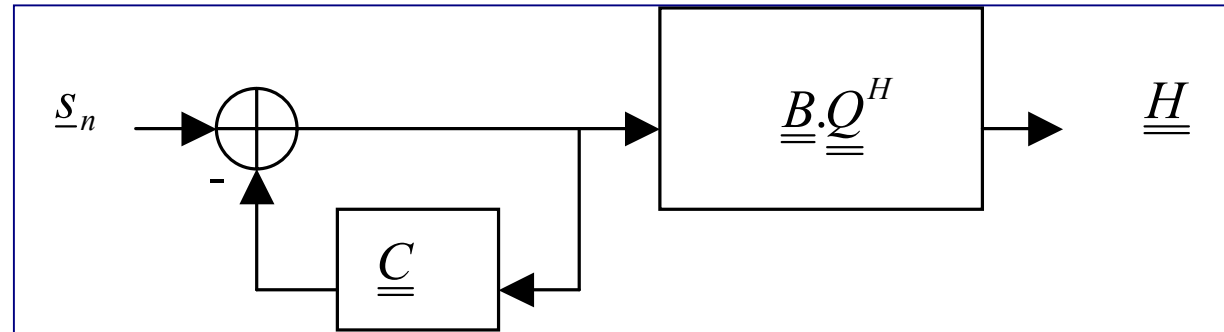
MIMO-BC



The power allocation for BA may produce modes unloaded. Here is the major difference with respect MIMO PTP since unloaded modes does not allow to redistribute the power loading. The reason is that the power loading used is also needed in canceling interference at the receivers.

To implement the inverse of R_a^H we use the recursive formula that follows:

$$\underline{\underline{R}}_a^H = \underline{\underline{I}} + \underline{\underline{C}}$$

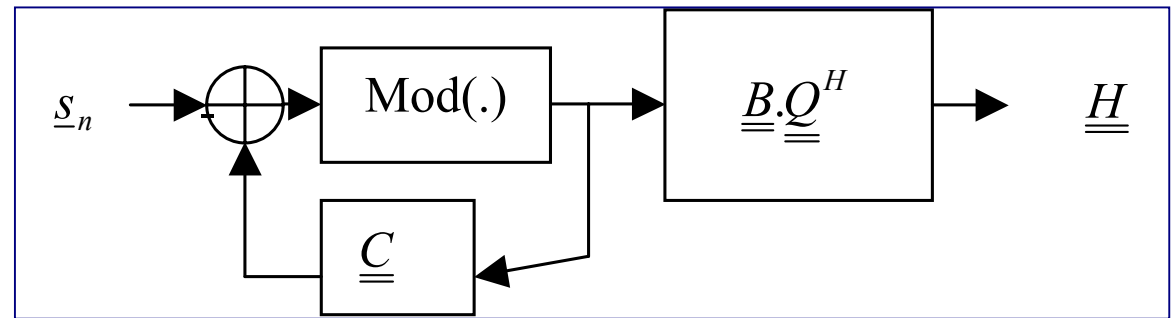


Note that matrix Q does not destroy the optimality of B , since it is orthonormal, but the inverse of $I+C$ does. In fact the recursive scheme, in addition to a non-diagonal character, introduces diagonal values which are greater than one.

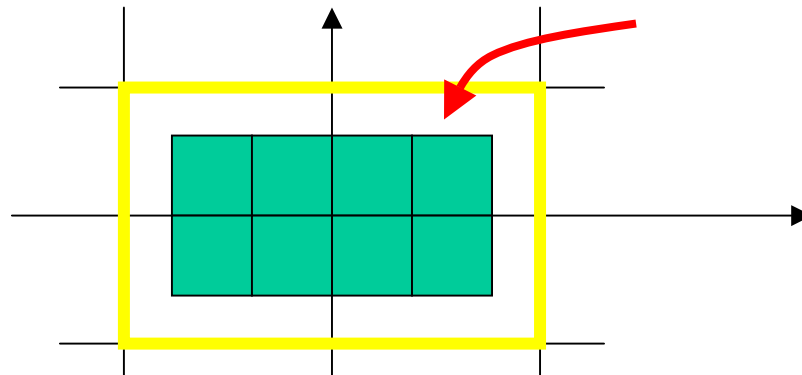
$$\underline{\underline{R}}_a^{-H} \cdot \underline{\underline{R}}_a^H \neq \underline{\underline{I}}$$



To solve the excess of power on the diagonals, the modulus operation is introduced in the recursive loop. This modulus operation have to be compensated also at each receiver.



The modulus operation have to be introduced independently For in-phase and quadrature components of the constellation



Thresholds for modulus operation on a 8QAM constellation



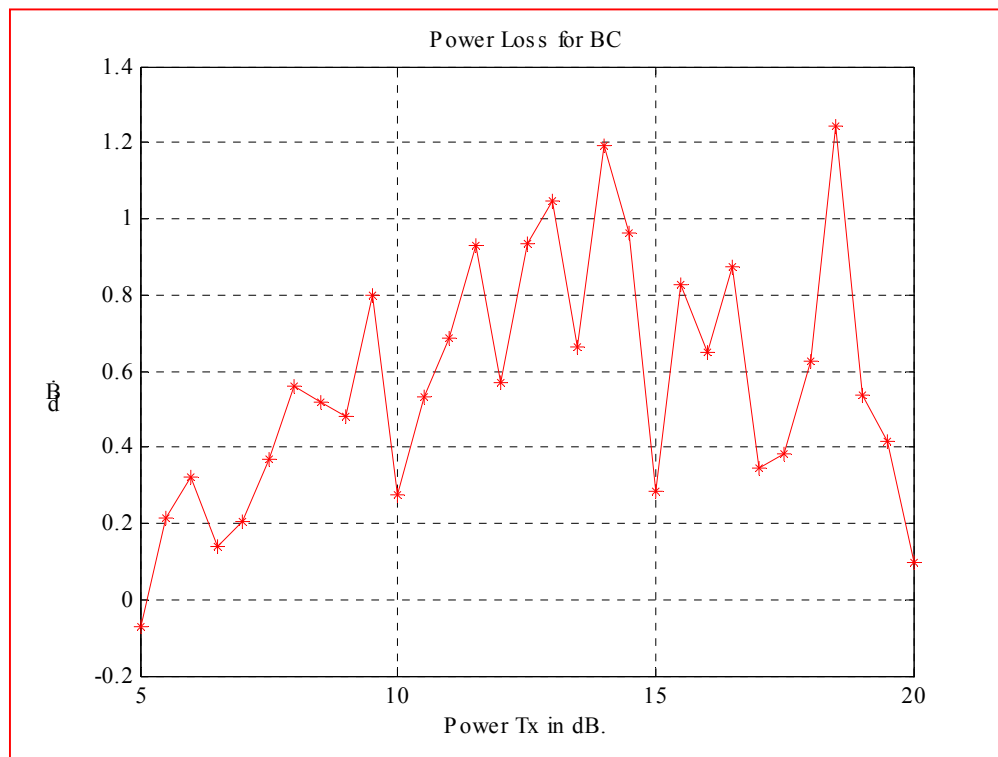
The uniform distribution within the limits of the basic modulus operation (thresholds set equal to the maximum plus half the symbol separation distance) provides a power equal to the square of the maximum divided by 3.



Modulation	Modulus at:		
Unit Power	In-phase	Quadrat.	Excess Power
BPSK	2	0	1.24 dB.
QPSK	$2/\sqrt{2}$	$2/\sqrt{2}$	1.24 dB.
8-QAM	$4/\sqrt{6}$	$2/\sqrt{2}$	0.45 dB.
16-QAM	$4/\sqrt{10}$	$4/\sqrt{10}$	0.28 dB.
32-QAM	$8/\sqrt{26}$	$4/\sqrt{10}$	0.11 dB.
64-QAM	$8/\sqrt{42}$	$8/\sqrt{42}$	0.06 dB.



Power loss for BC for global Tx power ranging from 5 to 20 dB. MIMO-BC for 3 users and 3 antennas at Tx. Note that it is always below 1.3 dB.



MIMO-BC Design Summary

For a given BER_{tar}

$$\underline{R}_0^{-1/2} \cdot \underline{H} = \underline{V}_H \cdot \underline{\Lambda}_H^{1/2} \cdot \underline{U}_H$$

$$\underline{\Lambda}_B = WF(\underline{\Lambda}_H)$$

$$QR\left(\left[\underline{V}_H \cdot \underline{\Lambda}_H^{1/2} \cdot \underline{\Lambda}_B^{1/2}\right]^H\right) = \underline{Q} \cdot \underline{R}$$

$$SNR_{dis} = diag(\underline{R})$$

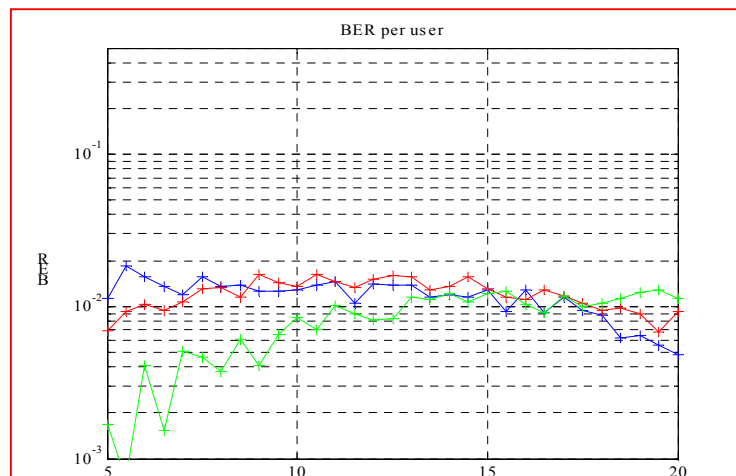
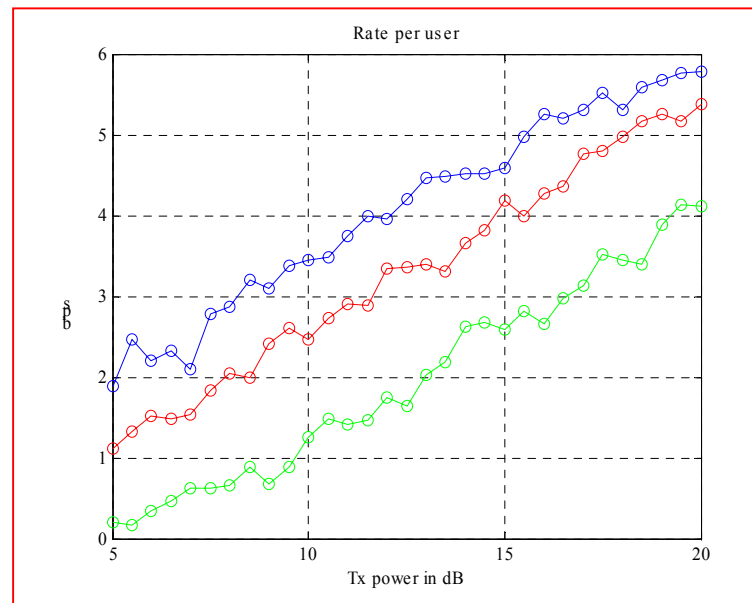
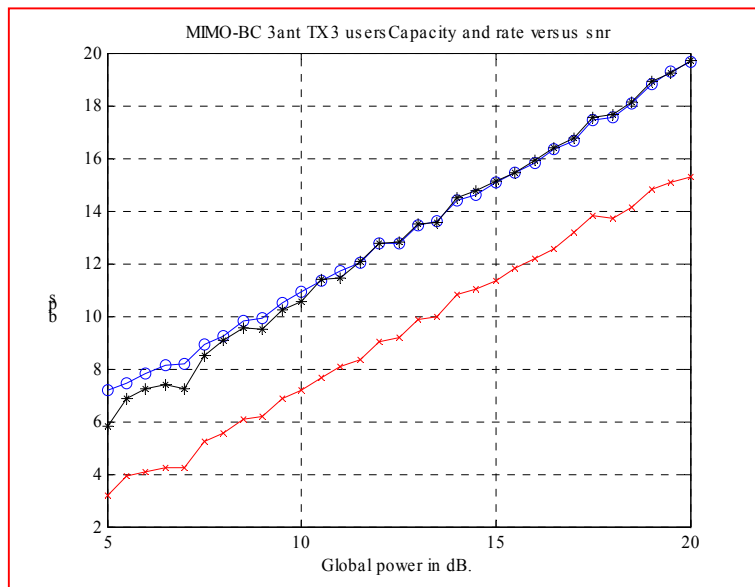
$$BA(BER_{tar}, SNR_{dis})$$

$$\underline{A} = \underline{R}_0^{-1/2} \cdot (diag(\underline{R}))^{-1}$$

$$\underline{C} = lower(\underline{R}^H)$$

$$\underline{B} = \underline{U}_H \cdot \underline{\Lambda}_B^{1/2} \cdot \underline{Q}$$

MIMO-BC: 3 users, Tx with 3 antennas, maximum constellation size 6 bits, 50 channel realizations. BER target equal to 10^{-2} .



Again note the effect of user labeling



As before, we resort to the generalized qr decomposition in order to remove the labeling effect

$$\underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{H}} \cdot \underline{\underline{U}} \cdot \underline{\underline{\Lambda}}_B^{1/2} = \underline{\underline{T}} \cdot \underline{\underline{R}}^H \cdot \underline{\underline{Q}}^H$$

The receivers take care of the transposition matrix as follows:

$$\underline{\underline{A}}^H = \text{diag}(\text{diag}(\underline{\underline{R}}))^{-1} \cdot \underline{\underline{T}}^H \cdot \underline{\underline{R}}_0^{-1/2}$$

Note that the BA does not need to be changed, just the streams that are loaded in natural order 1,2,... are delivered according to the transposition.

$$\underline{\underline{usuarios}} = \underline{\underline{T}}^H \cdot \underline{\underline{calidad / orden \quad natural}}$$



MIMO-BC: Design Summary

Given BER_tar target

$$\underline{R}_0^{-1/2} \cdot \underline{H} = \underline{V}_H \cdot \underline{\Lambda}_H^{1/2} \cdot \underline{U}_H$$

$$\underline{\Lambda}_B = WF(\underline{\Lambda}_H)$$

$$qr\left(\left[\underline{V}_H \cdot \underline{\Lambda}_H^{1/2} \cdot \underline{\Lambda}_B^{1/2}\right]^H\right) = \underline{Q} \cdot \underline{R} \cdot \underline{T}^H$$

$$SNR_dis = diag(\underline{R})$$

$$BA(BER_tar, SNR_dis)$$

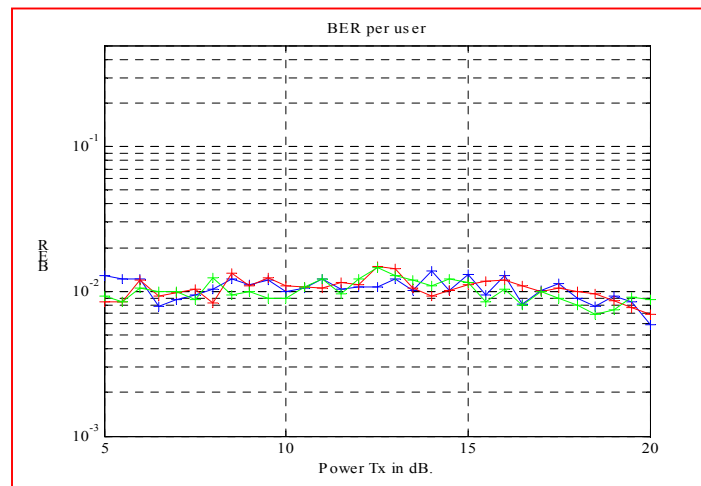
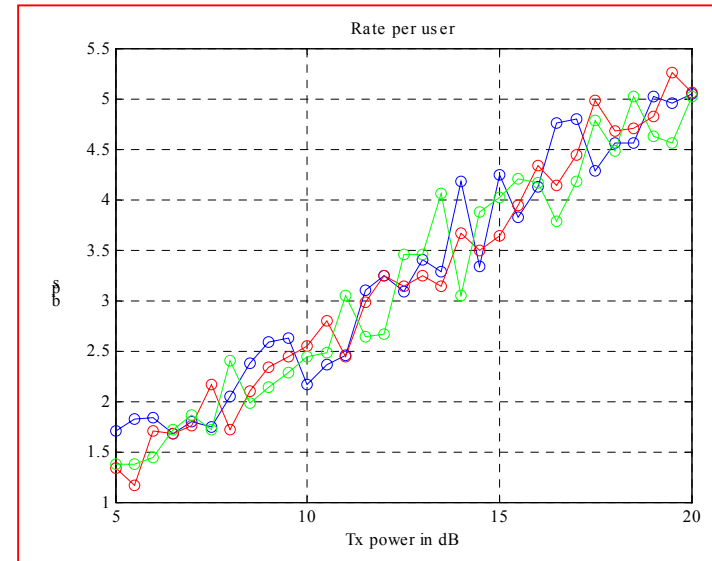
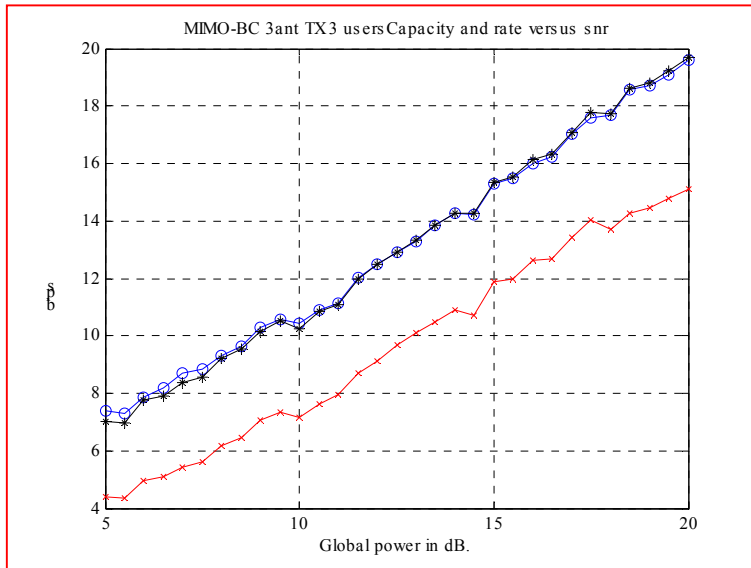
$$\underline{A} = \underline{R}_0^{-1/2} \cdot (diag(\underline{R}))^{-1}$$

$$\underline{C} = lower(\underline{R}^H)$$

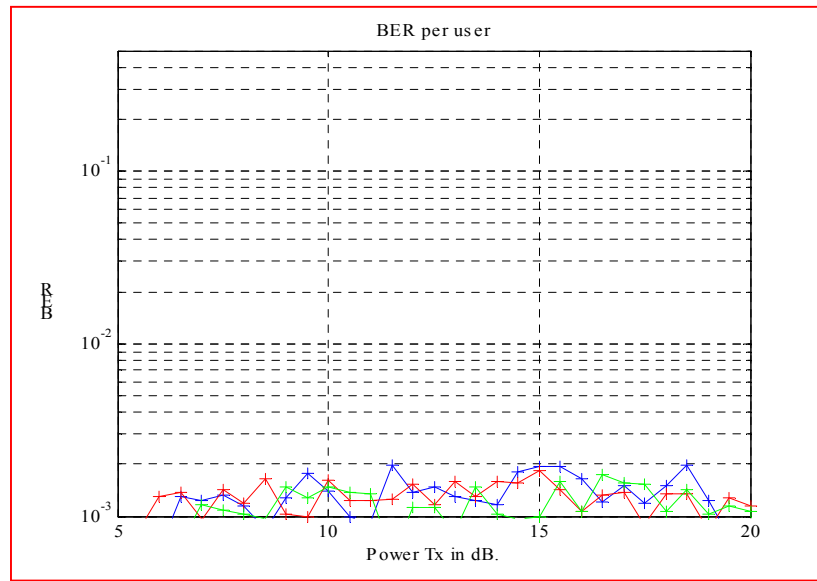
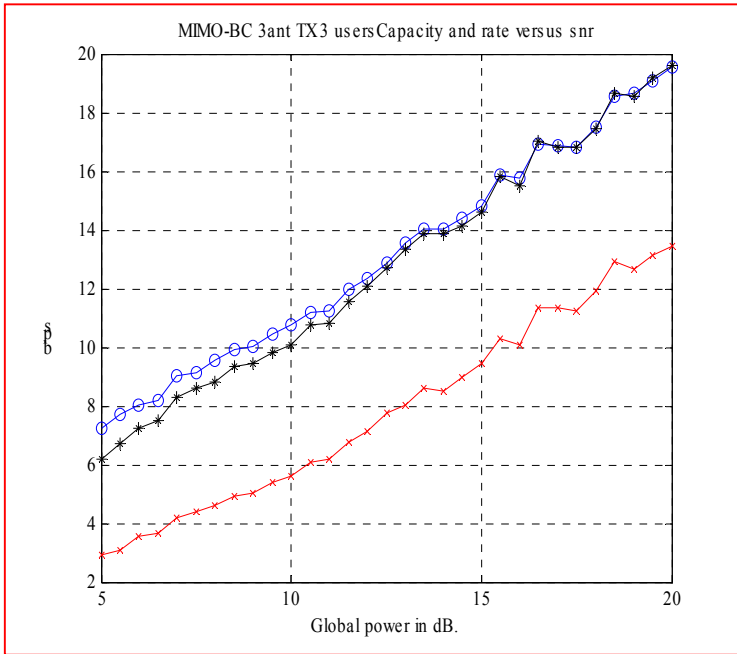
$$\underline{B} = \underline{U}_H \cdot \underline{\Lambda}_B^{1/2} \cdot \underline{Q}$$

$$users = \underline{T}^H \cdot outputs$$

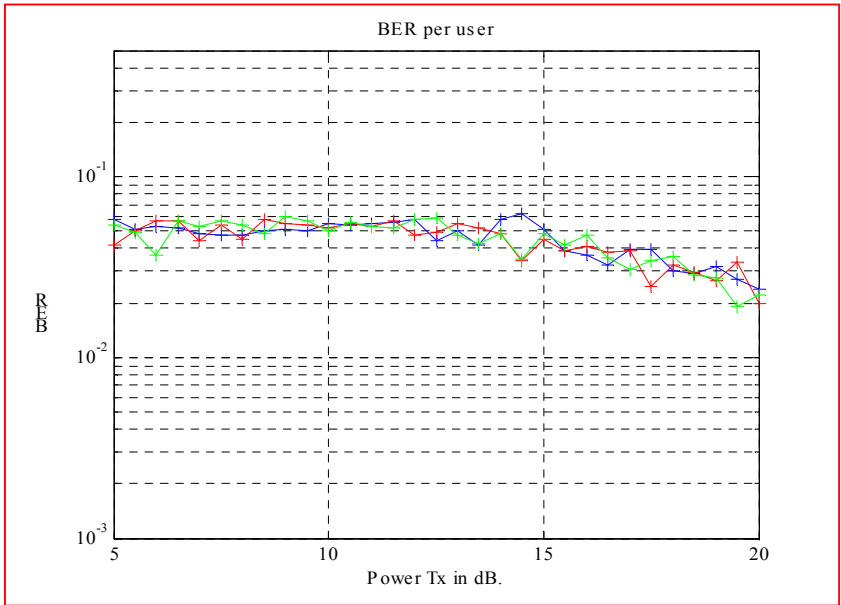
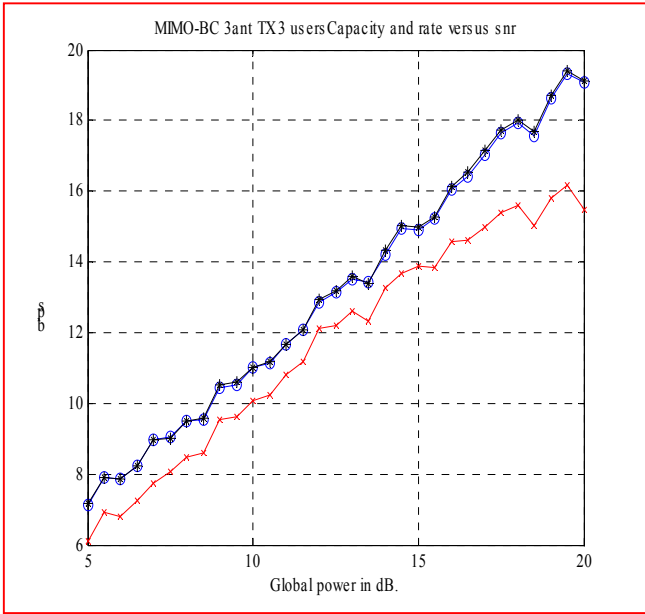
MIMO-BC: Broadcast 3 users from a BS with 3 antennas, BER target equal to 10^{-2} . Transposition and modulus operation at Tx and Rx, 50 channel realizations.



MIMO-BC: BER 10^{-3}



MIMO-BC: BER equal to $5 \cdot 10^{-2}$



Summary

- Max Capacity/ML receiver design
- Full CSIT MIMO-PTP, BC and MAC
- Proper use and handling of the qr decomposition
- All the schemes including Bit-Allocation
- Over-passing the problem of user labeling for MAC and BC scenarios
- Accurate quality control

