From point to point to Multiuser MIMO

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Contents

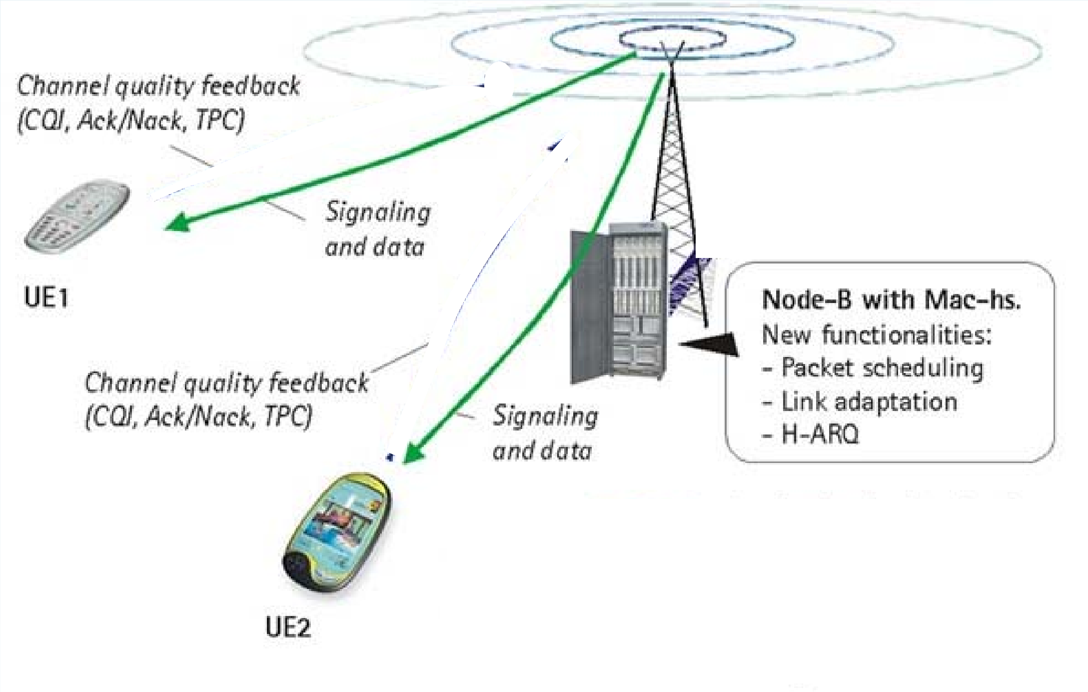
- Introduction
- Lessons learned form Information Theory
  - MAC channel
  - BC channel
  - Duality
  - Summary
    - *Scaling laws of BC-MIMO (pre-log factor)*
    - *Information theory design guidelines*
- BC channel: signal processing design
  - Aiming at DPC: Linear and non-linear precoders
  - Practical schemes: Multiuser diversity
    - Opportunistic beamforming schemes

*System issues

Multiuser MIMO Capacity Regions

Multiuser Scenario

A very interesting scenario to exploit the benefits of MIMO is the multiuser scenario, where a single Base Station (BS) communicates with several geographically separated users **Broadcast channel**



There is also the dual, the MAC (Multiple access) channel

Motivations for Going Multiuser

Multi-user makes certain things difficult:

- Dealing with users of unequal channel conditions (fairness issues)
- Mixing antenna filtering and scheduling problems into a harder problem
- Multiple users cannot cooperate as well as multiple antennas on a single device
- Leads to multiple (rather than single) power constraints
- In BC, in the absence of CSIT, user multiplexing is generally not possible

But others much easier: (we have the user dimension)

- Provides multi-user diversity (less reliance on antenna diversity)
- Provides decorrelation of spatial signatures
- Allows for user- (in addition to stream-) multiplexing with low complexity receivers
- Low rank channels no longer a problem but an advantage

The BC is the most challenging

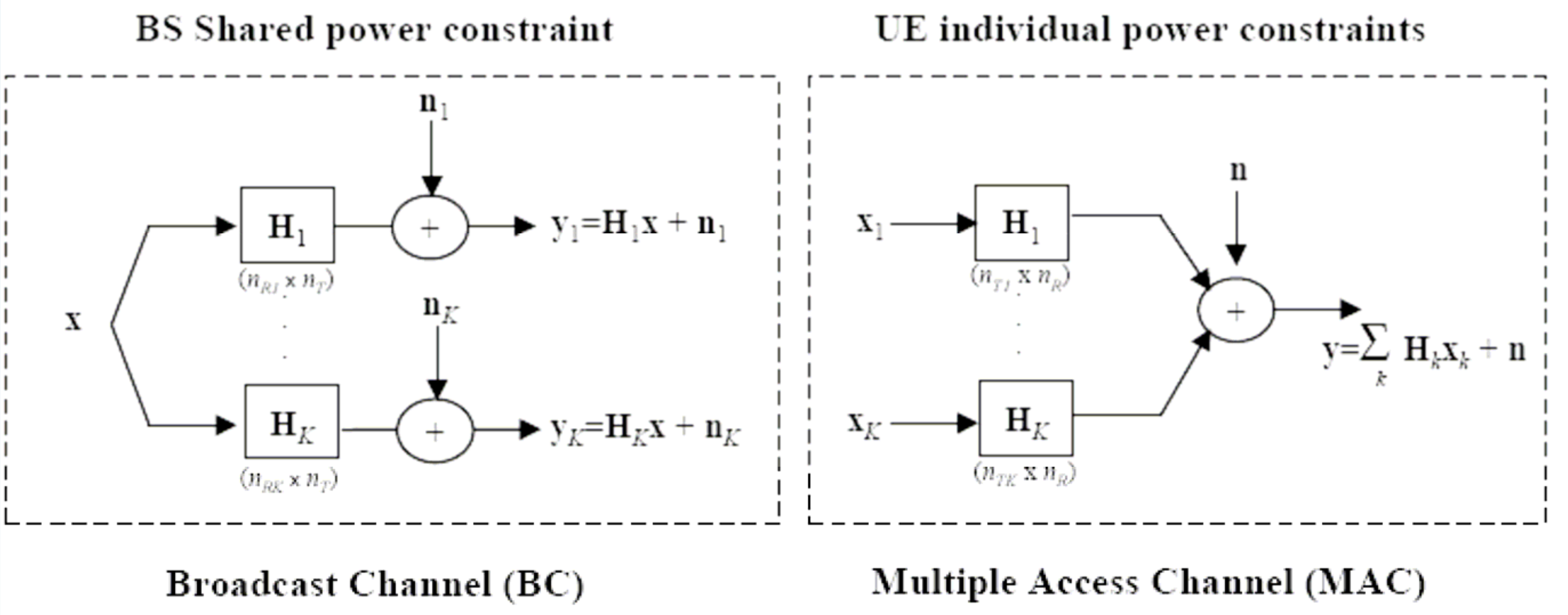
… because of downloads bottleneck

- Linear and non-linear precoding
- Channel State Information feedback
- Multiuser receivers
- User selection and Scheduling strategies
- Power control and other radio resource management

- Different sights
- From information theory point of view
- From signal processing point of view
- From network/protocol point of view

In single user SISO $C = \log(1 + SNR) \quad [bps/Hz]$

In MU-MIMO:



$$C = \sum_k \log(1 + SNIR_k) \quad [bps/Hz]$$

?

Remember

$$y = Hs + w \quad \Rightarrow \quad C = \max_{P_x} I(s,y) = \max_{P_x}\left(H(y) - H(y/s)\right) =$$

$$= \log\frac{|R_y|}{|R_{y/x}|} = \log\frac{|R_y|}{|R_w|}$$

$$\det(A.B) = \det(A).\det(B)$$

$$\det\left(I_m + AB^T\right) = \det\left(I_n + B^T A\right)$$

$$\det\left(I + ab^T\right) = 1 + b^T a$$

$$\det\left(X + ab^T\right) = \det(X)\left(1 + b^T X^{-1} a\right)$$

$$\det\left(\exp(A)\right) = \exp\left(tr(A)\right)$$

$$n = 1 \Rightarrow \det(A) = tr(A)$$

$$n = 2 \Rightarrow \det(A) = \frac{1}{2}\left(tr(A)^2 - tr\left(A^2\right.\right.$$

$$d\det(A) = \det(A).Tr(A^{-1}dA)$$

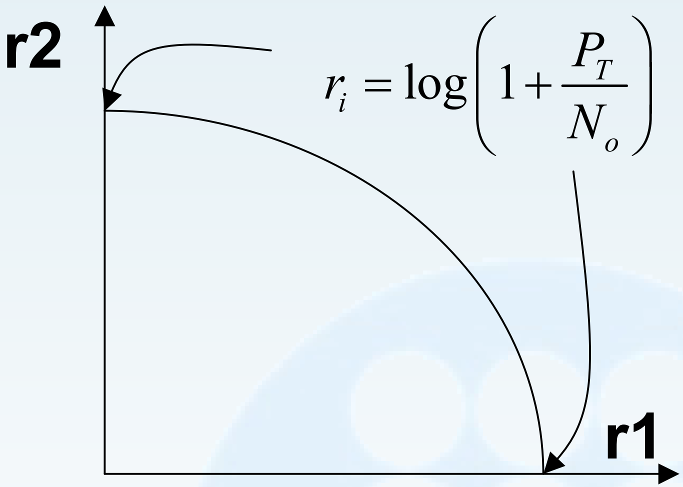
$$\det(I + \varepsilon X) = 1 + tr(X) + O(\varepsilon^2)$$

Also …

In the cooperative MIMO case

$$y = Hs + w \quad with \quad K = 2\ streams$$

$$\begin{cases} r_1 + r_2 < \log\left|I + \Lambda P\right| = \sum \log\left(1 + P_i \lambda_i\right) = \log\left(1 + P_1\lambda_1 + P_1\lambda_2 + P_1 P_2 \lambda_1 \lambda_2\right) \\ P_1 + P_2 = P_T \end{cases}$$



$$P_1\lambda_1 = \exp\left(r_1\right) - 1$$

$$\left(\exp\left(r_1\right) - 1\right)/\lambda_1 + \left(\exp\left(r_2\right) - 1\right)/\lambda_2 = P_T$$

$$r_1 = \log\left(1 + P_T\lambda_1 - \left(\exp\left(r_2\right) - 1\right)\lambda_1/\lambda_2\right)$$

$$If \quad \lambda_1 = \lambda_2 \Rightarrow r_1 = \log\left(2 + P_T - \exp\left(r_2\right)\right) \Leftarrow \exp\left(r_1\right) + \exp\left(r_2\right) = P_T + 2$$

Sato’s upper capacity bound

$$\lim_{SNR\to\infty} \frac{C_{MIMO}}{\log SNR} = \min(M, N$$

...

There may be several "capacities" in multi-user tx., Unlike single-user tx.

The capacity region is the tool in multi-user tx.

- It is the largest union (or convex hull) of all bps vectors

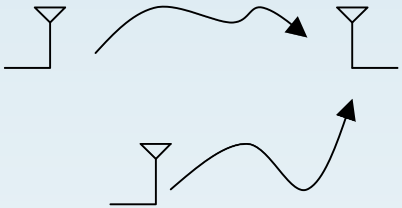
- Any point outside this region results in at least one of the receivers having a BER bounded away from 0 no matter what coding is used

MAC

For the single antenna case

K=2 users

$$y = \sum_i h_i s_i + w = \begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + w$$



$$r_1 < \log\left(1 + \frac{P_1 |h_1|^2}{N_o}\right) = I(s_1; y | s_2)$$

$$r_2 < \log\left(1 + \frac{P_2 |h_2|^2}{N_o}\right) = I(s_2; y / s_1)$$

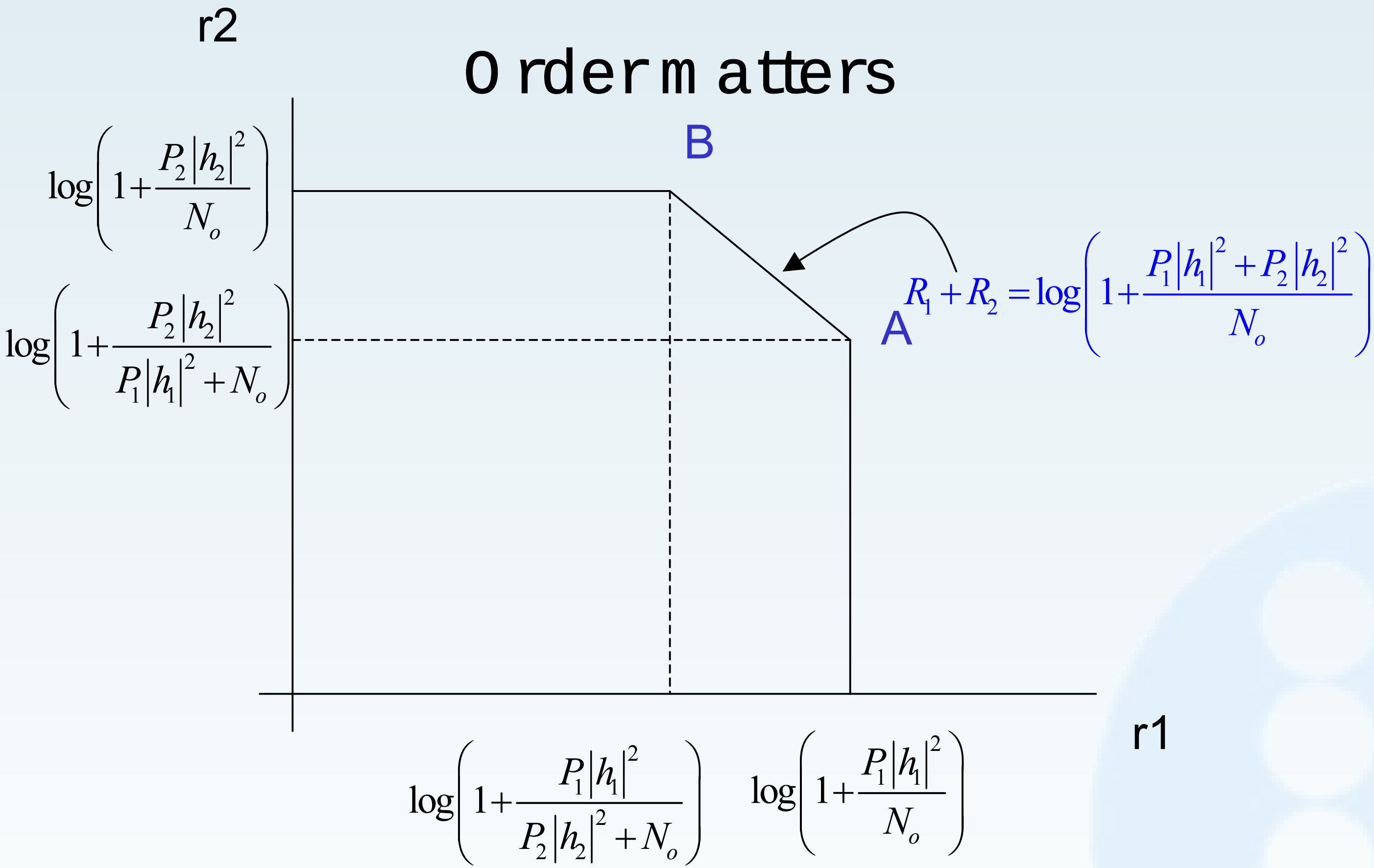
$$r_1 + r_2 < \log\left(1 + \frac{P_1 |h_1|^2 + P_2 |h_2|^2}{N_o}\right) =$$

$$= \log\left(1 + \frac{P_2 |h_2|^2}{N_o}\right) + \log\left(1 + \frac{P_1 |h_1|^2}{P_2 |h_2|^2 + N_o}\right) = I(s_2; y / s_1) + I(s_1; y)$$

Chain rule, SIC decoder, K! corners

Capacity region

Order matters



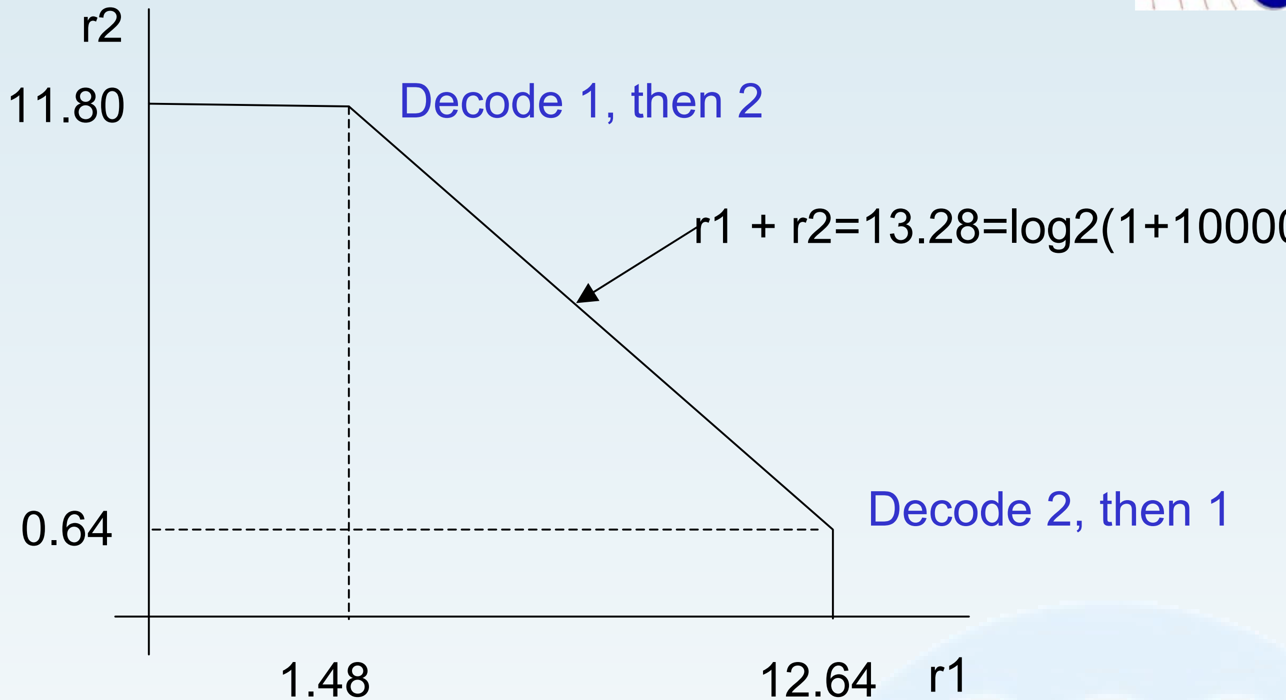
MAC

Example:

h1=0.8

h2=0.6

P1=P2=1



First alternative

$$r_2 = \log 2\left(1+\frac{.36\,.1}{.0001+.64}\right) = .64$$

$$r_1 = \log 2\left(1+\frac{.64\,.1}{.0001+.36}\right) = 1.48$$

Optimal alternative with SIC

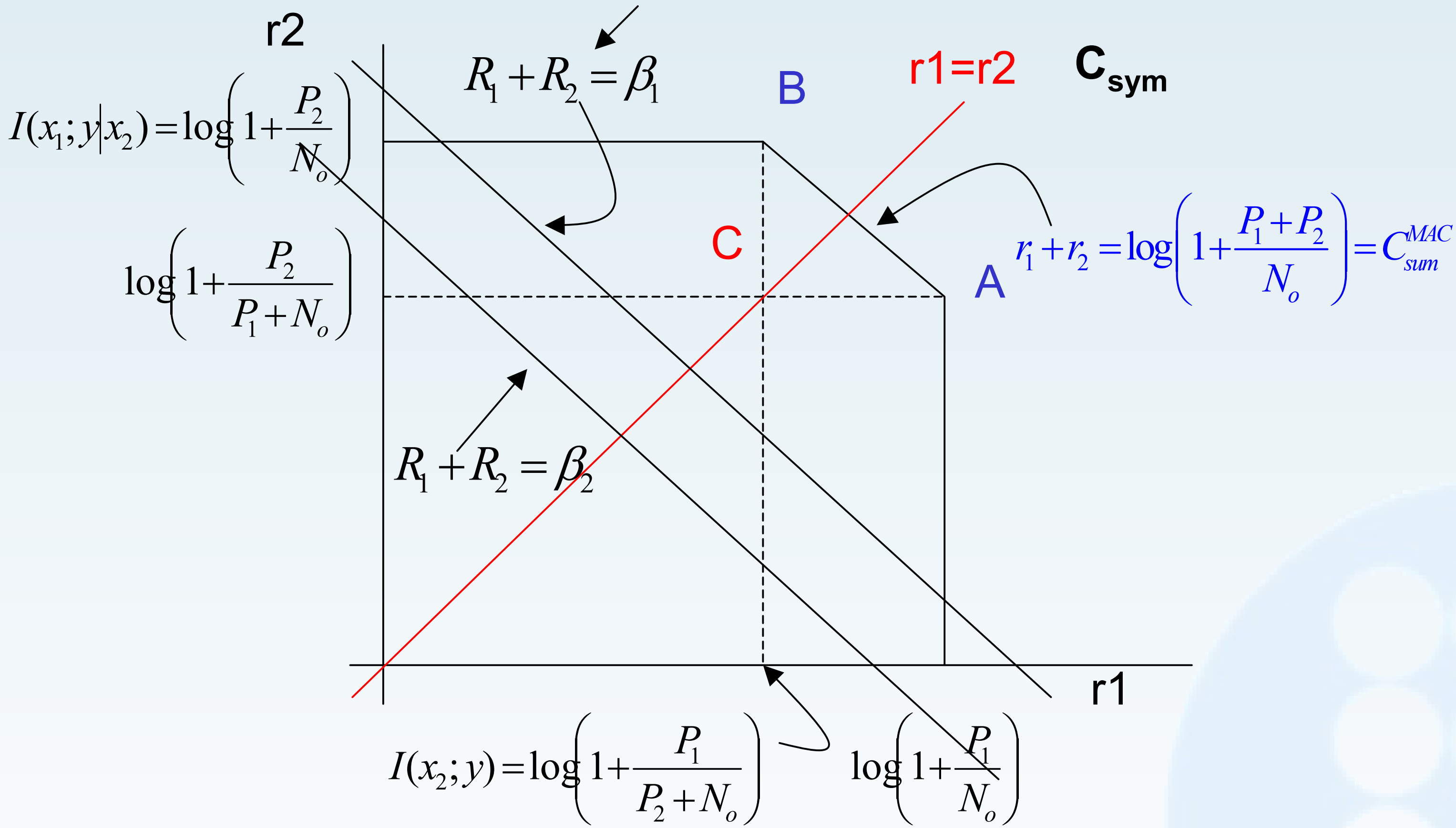
$$r_2 = \log 2\left(1+\frac{.36\,.1}{.0001}\right) = 12.64$$

$$r_1 = \log 2\left(1+\frac{.36\,.1}{.0001}\right) = 11.8$$

MAC

Capacity region

TDMA $C = \beta . r_1 + (1-\beta) . r_2$



FDMA:

$$r_1 < \beta \log\left(1 + \frac{P_1}{\beta N_o}\right) \qquad r_2 < (1-\beta) \log\left(1 + \frac{P_2}{(1-\beta) N_o}\right)$$

The best strategy:

All users should access simultaneously

Note that at low SNR: power limited

K users

Capacity region *C*: set of all achievable rate vectors

$$C=\left\{\sum_k \theta_k r_k ; \theta_k \le 1, \sum_k \theta_k = 1, r_k \in S, k \in \{1 \ldots K\}\right\} = convex\ hull\,\{S\}$$

$$S = \bigcup_{P_i \le P_T} \left\{ \sum_k r_k \le \log\left(1 + \frac{\sum_k P_i |h_i|^2}{N_o}\right), k \in \{1 \ldots K\} \right\}$$

Practical aspects: -chose decoding order

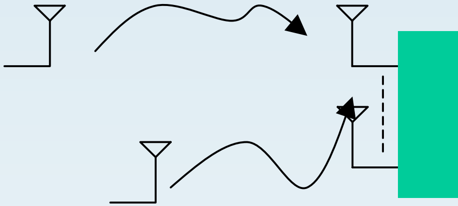
-chose subset of users S

MAC

For the multiple antenna case

K=2 users

$$\underline{y} = \left(\underline{h}_1 \quad \underline{h}_2\right)\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underline{w} = Hs + w$$



$$r_1 < \log\left(1 + \frac{P_1\left|\underline{h}_1\right|^2}{N_o}\right)$$

$$r_2 < \log\left(1 + \frac{P_2\left|\underline{h}_2\right|^2}{N_o}\right)$$

$$r_1 + r_2 < \log\left|I + \frac{1}{N_o} H R_s H^H\right| = \log\left|I + \frac{1}{N_o}\sum_{i=1}^{2} P_i h_i h_i^H\right|$$

$$\underset{R_s = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}}{=} \log\left(1 + \frac{P_1\left|\underline{h}_1\right|^2 + P_2\left|\underline{h}_2\right|^2}{N_o} + \frac{P_1 P_2}{N_o}\left|HH^H\right|\right) =$$

$$= \log\left(1 + \frac{P_2\left|\underline{h}_2\right|^2}{N_o}\right) + \log\left(1 + P_2 \underline{h}_2^H \left(N_o I + P_1 \underline{h}_1 \underline{h}_1^H\right)^{-1} \underline{h}_2\right) =$$

$$= \sum_{i=1}^{2} \log\left(1 + SNIR_i\right)$$

Interesting for practical designs

$$SNIR_1 = SNR_1 = \frac{P_1\left|\underline{h}_1\right|^2}{N_o}$$

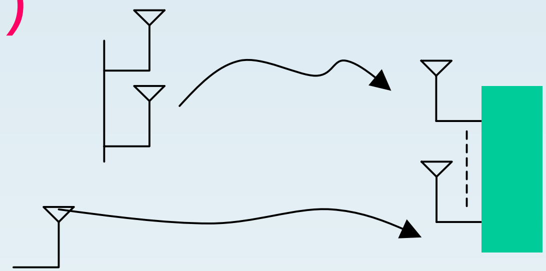
$$SNIR_2 = \frac{P_2\left|\underline{b}_2\right|^2}{N_o}$$

$$\underline{b}_2 = R_n^{-1/2}\underline{h}_2$$

MAC

For the MU-MIMO (K users, N, M)

$$\underline{y} = \begin{pmatrix} H_1 & H_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underline{w} = Hs + w$$



K users

$$r_k \le \log\det\left(\mathbf{I} + \frac{1}{N_o} \mathbf{H}\mathbf{R}_k\mathbf{H}^H \right) \quad k = 1 \ldots K$$

$$C_{sum}^{MAC} = \sum_{k=1}^{K} r_k \le \log\det\left(\mathbf{I} + \frac{1}{N_o} \sum_{k=1}^{K} \mathbf{H}\mathbf{R}_k\mathbf{H}^H \right)$$

$$Tr\left(R_k\right) \le P_k$$

The weaker are interfering

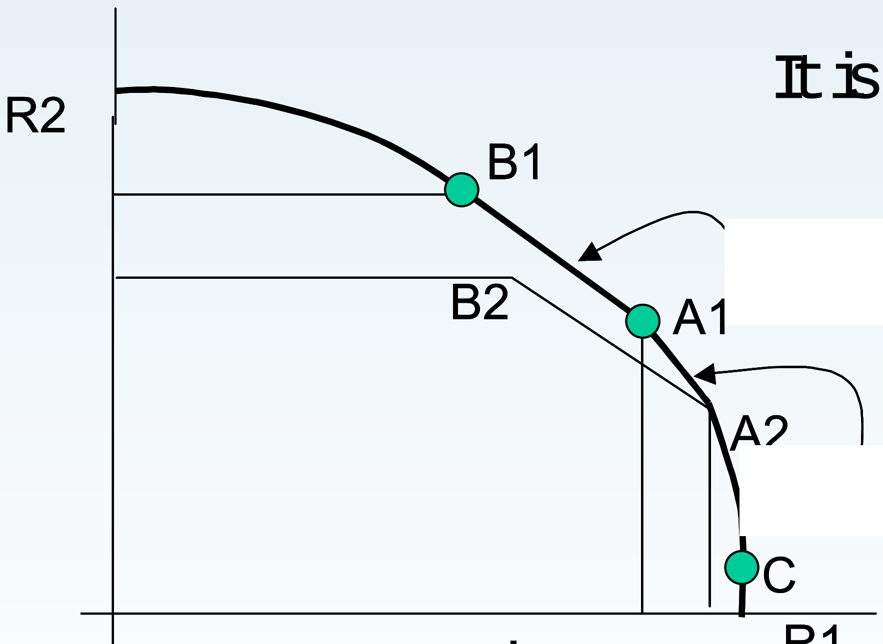
$$\sum_{i=1}^{K} r_i = \log\left| \mathbf{I} + \sum_{i=1}^{K} \mathbf{H_i R_i H_i^H} \right| = \log\left| \mathbf{I} + \mathbf{H}_1\mathbf{R}_1\mathbf{H}_1^{\mathbf{H}} \right| + \ldots + \log \frac{\left| \mathbf{I} + \sum_{i=1}^{j} \mathbf{H}_i\mathbf{R}_i\mathbf{H}_i^{\mathbf{H}} \right|}{\left| \mathbf{I} + \sum_{i=1}^{j-1} \mathbf{H}_i\mathbf{R}_i\mathbf{H}_i^{\mathbf{H}} \right|} + \ldots + \log \frac{\left| \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i\mathbf{R}_i\mathbf{H}_i^{\mathbf{H}} \right|}{\left| \mathbf{I} + \sum_{i=1}^{K-1} \mathbf{H}_i\mathbf{R}_i\mathbf{H}_i^{\mathbf{H}} \right|}$$

For the MU-MIMO

Capacity Region

$$C_{MAC} = co\left\{ \bigcup_{\{Tr(\mathbf{R}_i)\le P_i\ \forall i\}} \left\{ (r_1 \dots r_K) : \sum_{i\in S} r_i \le \log\left| I + \sum_{i\in S} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right| \quad \forall S \subseteq \{1 \dots K\} \right\} \right\}$$

It is concave on Ri



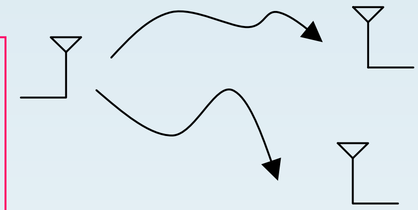
$$C_{MIMO-MAC}^{SUM} \approx \phi_0 . \min\left[n_R, \sum_{k=1}^{K} n_{Tk} \right]$$

$$\frac{C_{MIMO-MAC}^{SUM}}{C_{TDMA}} \approx \min\left[\frac{n_R}{n_T}, K \right]$$

CSIT and CSIR

For the single antenna case

$$y_i = h_i s + w_i \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \underbrace{(s_1 + s_2)}_{x} + w$$



K=2 users

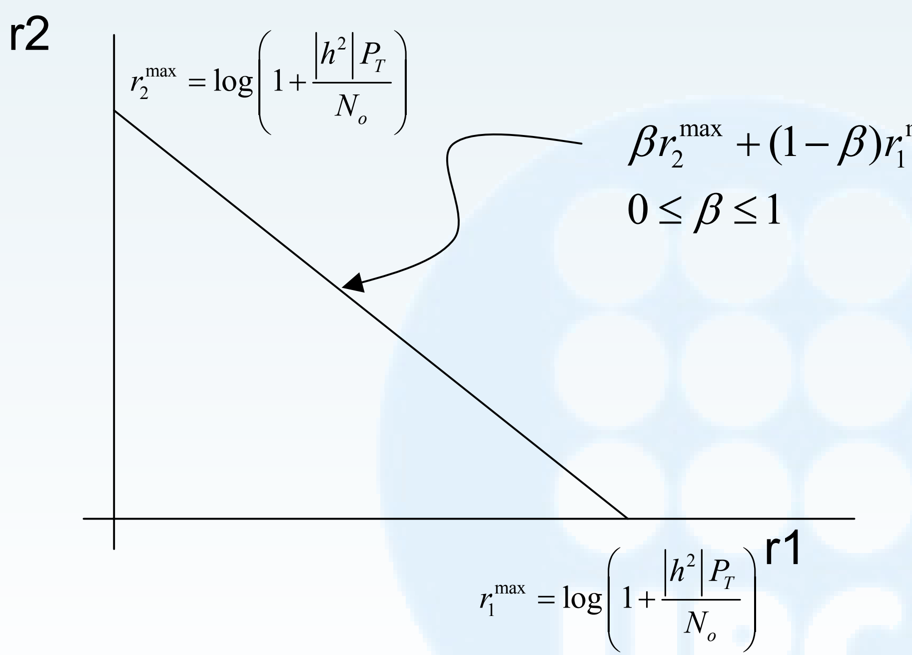
Degraded channel

Symmetric case: h1=h2=h

$$r_1 + r_2 < \log\left(1 + \frac{(P_1 + P_2)|h|^2}{N_o}\right)$$

$$P_2 + P_1 = P_T$$

$$r_2 = \log\left(1 + \frac{P_T}{N_o}|h|^2\right) - r_1$$



For the single antenna case

More about r1 and r2?

Note that
$$y_1 = (s_1 + s_2)h + w_1$$
$$y_2 = (s_1 + s_2)h + w_2$$

$$\sigma_1 < \sigma_2 \rightarrow w_2 = w_1 + w'$$

$$y_2 = y_1 + w'$$

y2 is a degraded version of y1

At each rx: the codeword intended for y2 can also be decoded by y1 and viceversa (SIC)

$$r_1 + r_2 < \log\left(1 + \frac{(P_1 + P_2)|h|^2}{N_o}\right) = \log\left(1 + \frac{P_2|h|^2}{P_1|h|^2 + N_o}\right) + \log\left(1 + \frac{P_1|h|^2}{N_o}\right)$$

For the single antenna case $\quad If \quad |h_2| > |h_1|$

At rx 2: SIC, the performance of user 2 (the strongest one) is then

$$r_2 = \log\left(1 + \frac{P_2 |h_2|^2}{N_o}\right)$$

At rx 1 (weak): the weak user can only decode its own signal and user 2 acts as interference

$$r_1 = \log\left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o}\right) = \log\left(1 + \frac{(P_1 + P_2)|h_1|^2}{N_o}\right) - \log\left(1 + \frac{P_2 |h_1|^2}{N_o}\right)$$

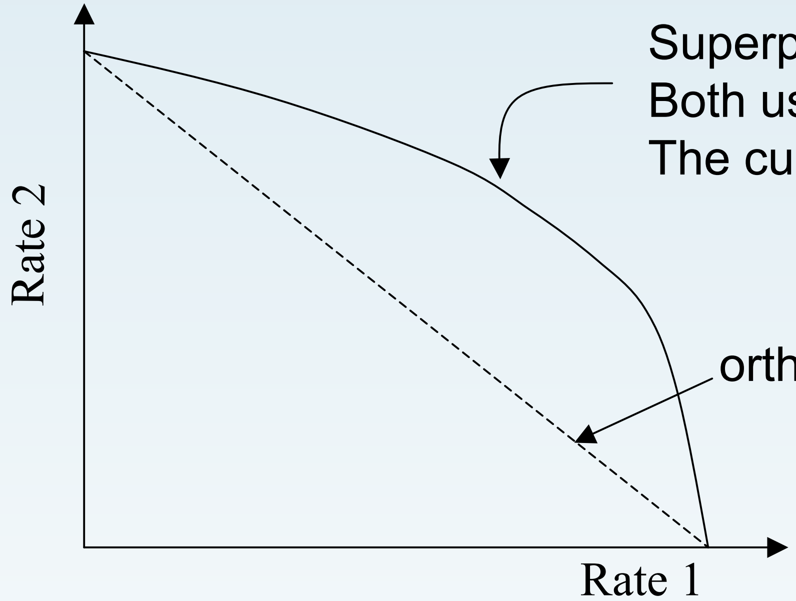
With respect to the MAC, users order is reversed: the strongest user has the better quality. Note that decoding order (1,2) produces smaller r2 and User 1 would have less energy to use than with order 2,1

Fairness problem

NOW, there is no closed expresión for r1+r2 ??

For the single antenna case

By simulating both cases for $P_1 \in [1:100]$, $P_2 = 100 - [1:100]$



Superposition coding and asymmetric channels. Both users transmit with total power constraint. The curve is due to the global power constraint

$$r_1 = \log\left(1 + \frac{P_T(1-\alpha)|h_1|^2}{P_T\alpha\ |h_1|^2 + N_o}\right) \quad bits/s/Hz$$

$$r_2 = \log\left(1 + \frac{\alpha P_T |h_2|^2}{N_o}\right) \quad bits/s/Hz$$

Best policy in BC-SISO if there is CSIT: one user at a time as we will see later on

However, SIC may present error propagation, therefore other multiuser detectors may be more practical or if CSIT is available Dirty Paper coding can be carried out, where s2 is coded in the already contaminated environment by s1

For the single antenna case

Other implementation: Dirty Paper Coding at tx

It is dual to SIC rx: interference cancellation at tx (CSIT)

$$tx_1 = s_1 \qquad tx_K = s_K - \sum_{i=1}^{K-1} cod(s_i)$$

$$tx_2 = s_2 - cod(s_1)$$

$$\vdots$$

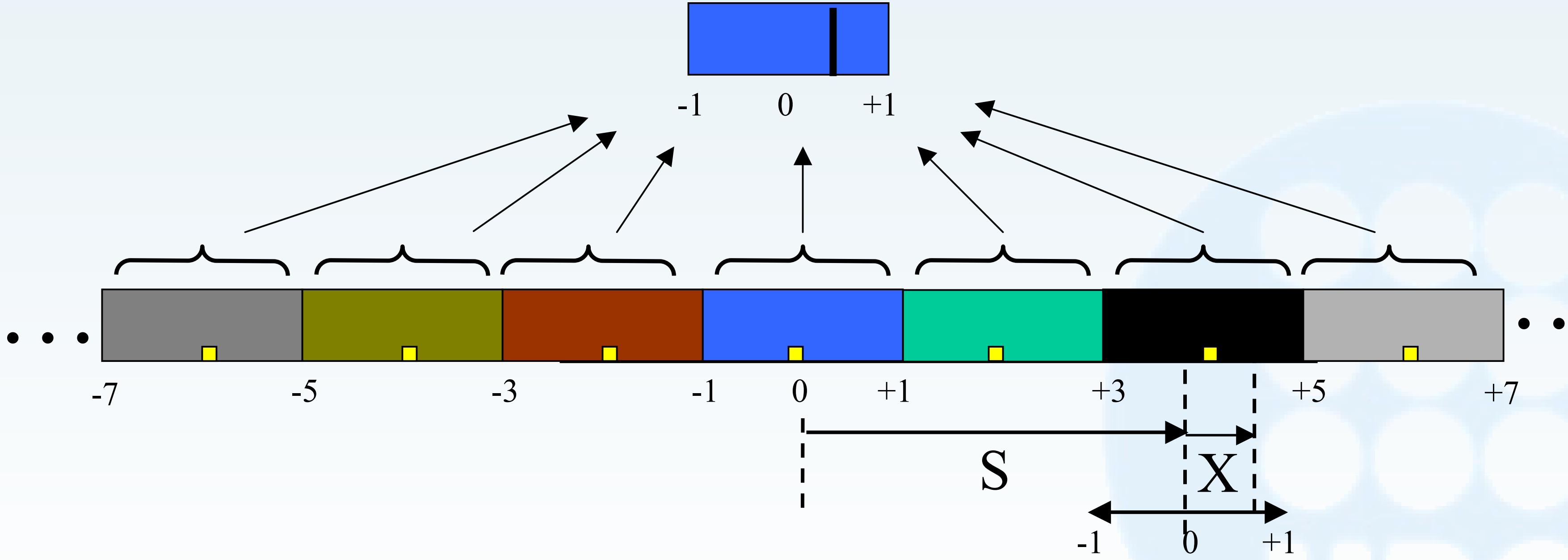
K=2

$$r_1 < \log\left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o}\right) \quad bits/s/Hz$$

$$r_2 < \log\left(1 + \frac{P_2 |h_2|^2}{N_o}\right) \quad bits/s/Hz$$

For the single antenna case. Example of DPC

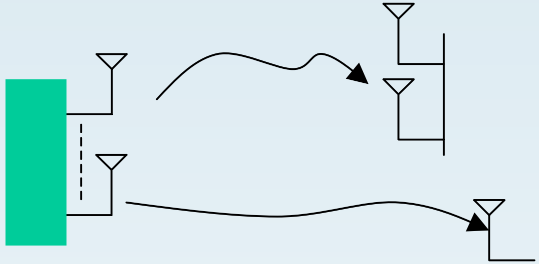
- **Received signal Y=X+S, -1≤X≤1**
  - S known to transmitter, not receiver
- **Modulo operation removes the interference effects**
  - Set X so that $\lfloor Y \rfloor_{[-1,1]}$=desired message (e.g. 0.5)
  - Receiver demodulates modulo [-1,1]



BC

For the multiple antenna case (Kusers, N, M)

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} H_1^H \\ H_2^H \end{pmatrix} \underbrace{\left(s_1 + s_2\right)}_{x} + w = H^H x + w$$



With CSIT and CSIR

$$r_i\left(\pi, R_k\right) = \log \frac{\left| \mathbf{H}_{\pi(\mathbf{i})} \left(\sum_{k=i}^{K} \mathbf{R}_{\pi(k)} \right) \mathbf{H}^{\mathbf{T}}_{\pi(\mathbf{i})} + \mathbf{I} \right|}{\left| \mathbf{H}_{\pi(\mathbf{i})} \left(\sum_{k=i+1}^{K} \mathbf{R}_{\pi(k)} \right) \mathbf{H}^{\mathbf{T}}_{\pi(\mathbf{i})} + \mathbf{I} \right|}$$

Non-convex → Difficult to obtain

$$Tr\left(\sum_i R_i \right) \le P_T$$

The stronger are interfering

BC comments

For the multiple antenna case

1.- $$x = \sum_k W_k s_k$$

$$y_k = H_k W_k s_k + \sum_{j \neq k} H_j W_j s_j + n_k \qquad R_k = W_k E\left\{ s_k s_k^H \right\} W_k^H$$

For Wk or rank 1 (N=1) $$x = \sum_k w_k s_k$$

$$r_i < \log\left(1 + \frac{P_i \left| h_i^H w_i \right|^2}{N_o + h_i^H \left(\sum_{j \neq i} P_j w_i w_i^H \right) h_i} \right) = \log\left(1 + SNIR_i^{DPC}\right)$$

$$C_{sum}^{DP} = \sum_{i=1}^{K} \log\left(1 + SNIR_i^{DP}\right)$$

Optimal order

Optimal user selection

Starting point for subopt. Strateg

MAC-BC Capacity Regions

- MAC capacity region known for many cases
  - Convex optimization problem
- BC capacity region typically only known for (parallel) degraded channels
  - Formulas often not convex
- Can we find a connection between the BC and MAC capacity regions?

Duality

For the multiple antenna case

2.- By duality

$$C_{sum}^{BC} = C_{sum}^{DP} = \sum_{i=1}^{K} R_i = \max_{Tr\left(\sum_k R_k^{MAC}\right)} \log\left|\mathbf{I} + \sum_{i=1}^{K} \mathbf{H_i^H} \mathbf{R}_i^{MAC} \mathbf{H_i}\right|$$

BC

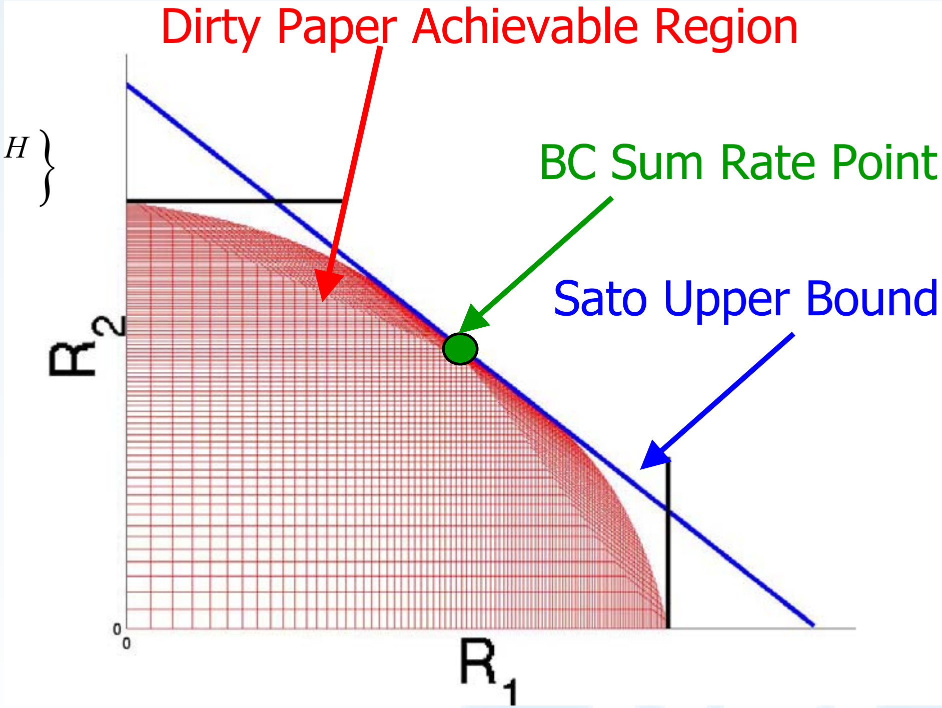
For the multiple antenna case

3.- For the Gaussian case

$$C_{BC}(P,\mathbf{H}) = Co\left(\bigcup_{\pi,\mathbf{R}} r(\pi,\mathbf{R})\right)$$

In addition, by duality

$$C_{BC}(\overline{P},\mathbf{H}) = \bigcup_{\{P_i\}_i^K : \sum_{i=1}^{K} P_i \le \overline{P}} C_{MAC}\left\{\mathbf{R}_1,\ldots,\mathbf{R}_K;\mathbf{H}^H\right\}$$



Flip the channel and

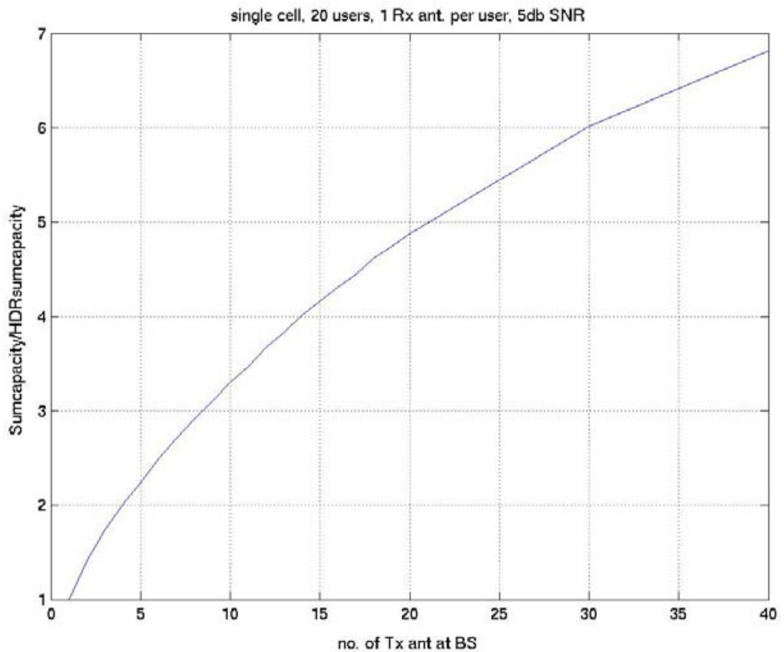
reverse the order

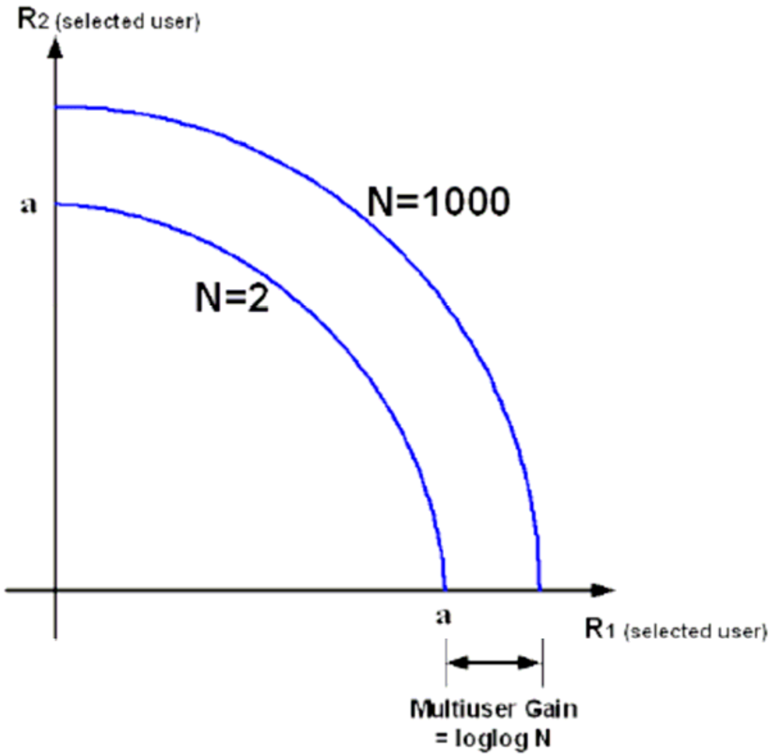
The union of convex reg.

For the multiple antenna case

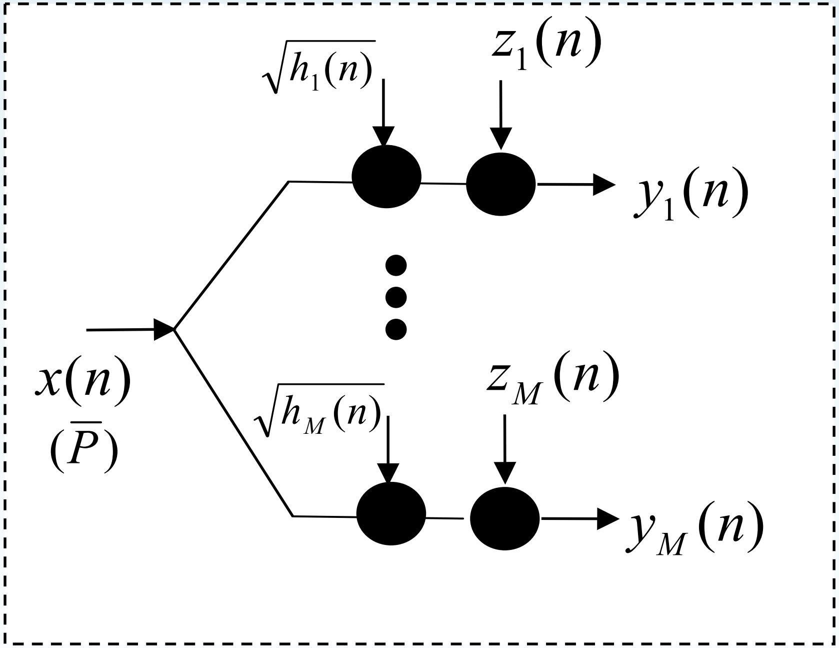
4.- Effect of K

DPC can be applied over K>nt

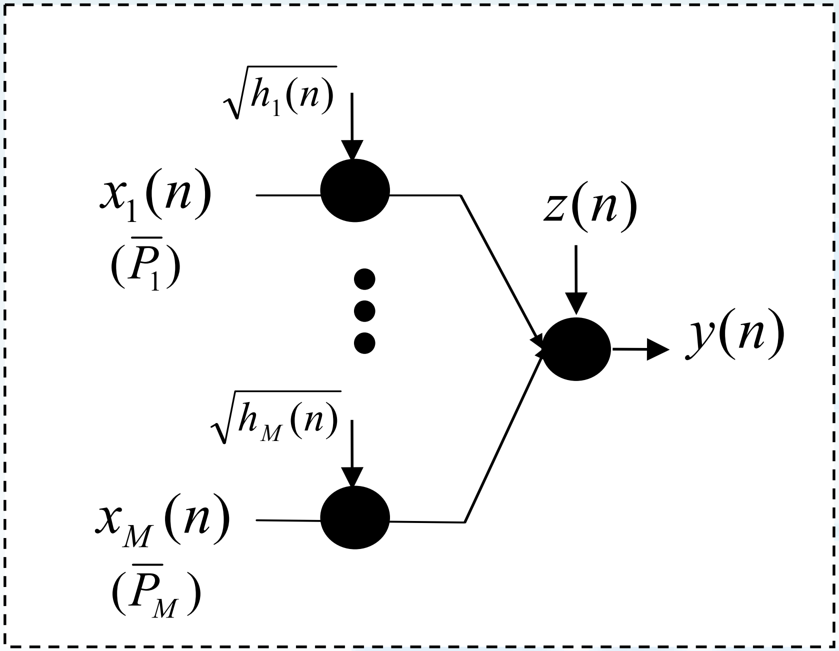




Gaussian BC and MAC with *same* channel gains and *same* noise power at each receiver



Broadcast Channel (BC)



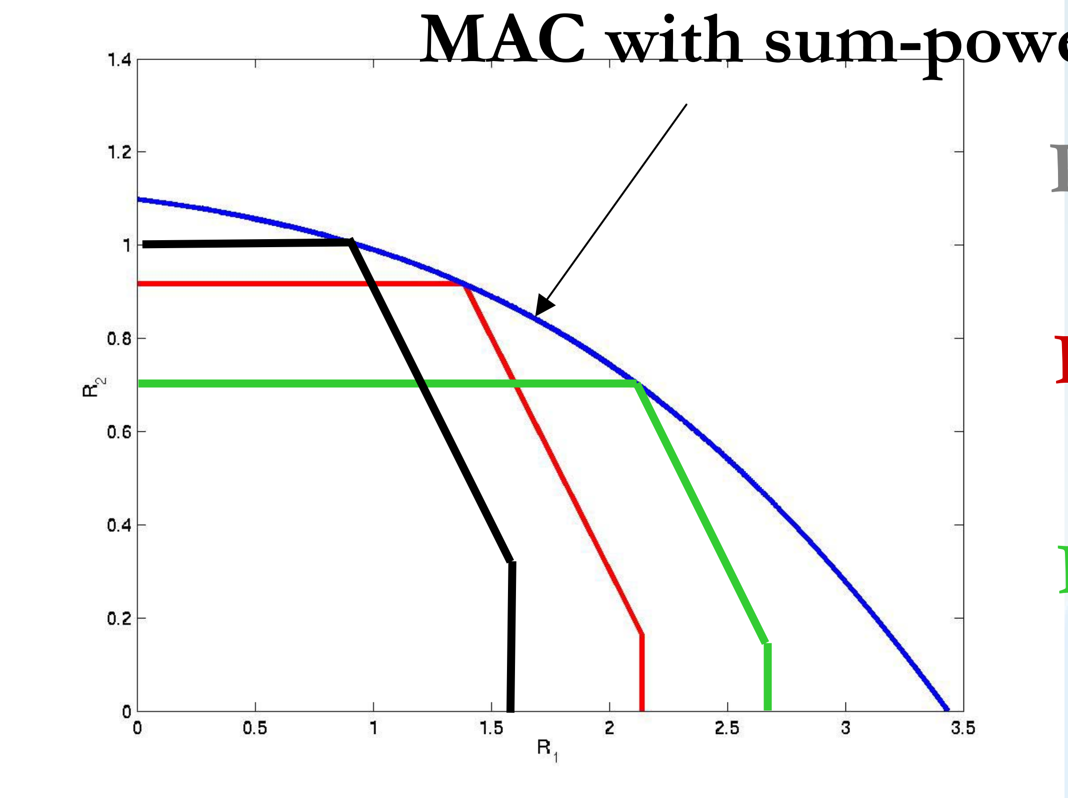
Multiple-Access Channel (MAC)

The BC from the MAC

$$C_{MAC}(P_1, P_2; h_1, h_2) \subseteq C_{BC}(P_1 + P_2; h_1, h_2)$$

$h_1 > h_2$

Blue = BC
Red = MAC



P_1=0.5, P_2=1.5

P_1=1, P_2=1

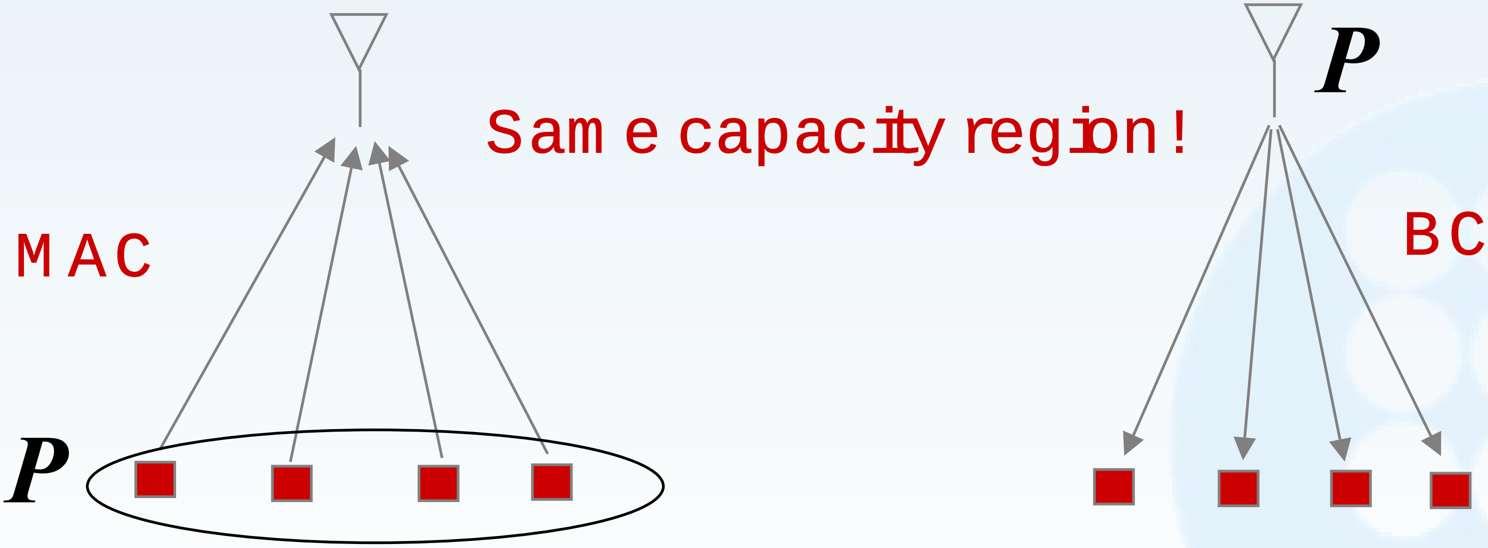
P_1=1.5, P_2=0.5

$$C_{BC}(P; h_1, h_2) = \bigcup_{0 \le P_1 \le P} C_{MAC}(P_1, P - P_1; h_1, h_2)$$

Sum-Power MAC

$$C_{BC}(P;h_1,h_2)=\bigcup_{0\le P_1\le P} C_{MAC}(P_1,P-P_1;h_1,h_2)\equiv C_{MAC}^{Sum}(P;h_1,h_2)$$

- MAC with sum power constraint
  - Power pooled between MAC transmitters
  - No transmitter coordination



MAC-BC duality

For the single antenna case

An Example

$$C_{BC}(P;h_1,h_2)=\bigcup_{0\le P_1\le P} C_{MAC}(P_1,P-P_1;h_1,h_2)$$

What is the relationship between the optimal tx strategies?

Equating rates and solve for powers:

$$r_1^{MAC}=\log\left(1+\frac{|h_1|^2 P_1^{MAC}}{N_o+|h_2|^2 P_2^{MAC}}\right)=\log\left(1+\frac{|h_1|^2 P_1^{BC}}{N_o}\right)=r_1^{BC}$$

$$r_2^{MAC}=\log\left(1+\frac{|h_2|^2 P_2^{MAC}}{N_o}\right)=\log\left(1+\frac{|h_2|^2 P_2^{BC}}{N_o+|h_2|^2 P_1^{BC}}\right)=r_2^{BC}$$

Opposite decoding order

Summary I

- Shannon capacity gives fundamental data rate limits for wireless channels
- Broadcast channels with ISI can use OFDM with near-optimality
- Duality and dirty paper coding are used to obtain the capacity of a broadcast MIMO channel.

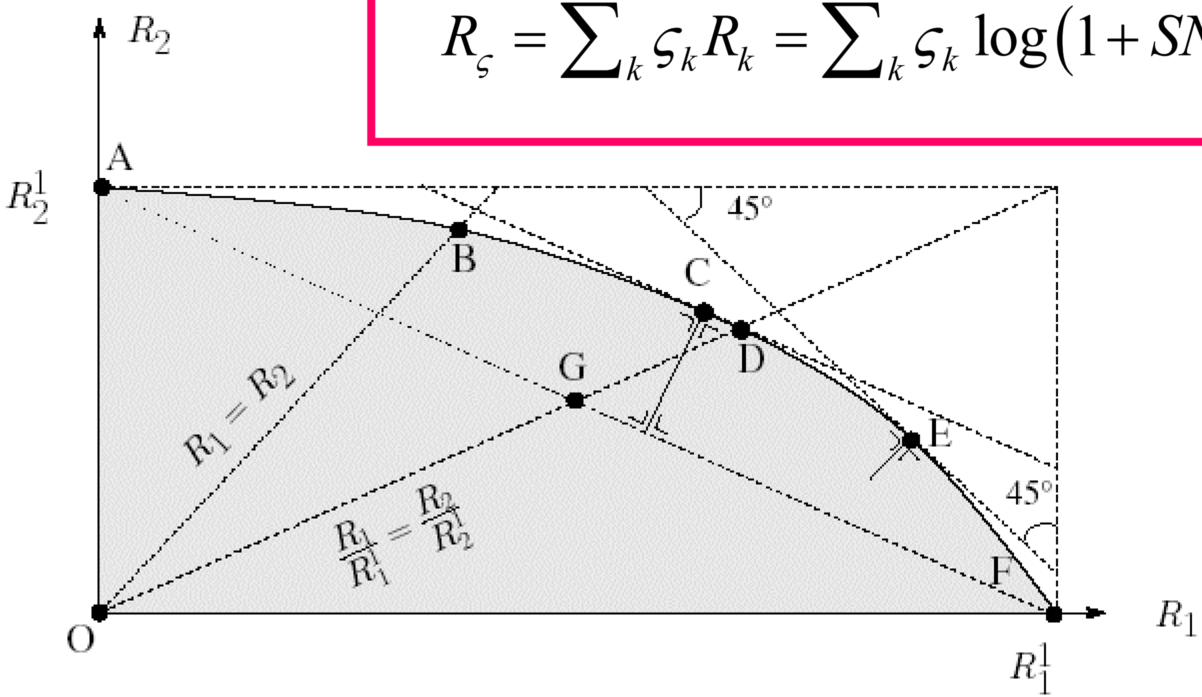
Summary II

Interestingly, both capacity regions are exactly the same (duality property) as soon as the power constraint is set on the total transmitted power.

The boundary of the global capacity region can be traced out by means of a set of relative priority coefficients $\sum_k \varsigma_k = 1$

Each boundary point of the capacity region maximizes the linear combination of the user rates

$$R_\varsigma = \sum_k \varsigma_k R_k = \sum_k \varsigma_k \log\left(1 + SNIR_k\right)$$



Summary III

The fundamental role played by the multiple antennas at either the BS or the users in expanding the channel capacity is best apprehended by examining how the sum rate scales with the number of users K

Capacity scaling laws in MIMO-BC or pre-log factor

Capacity scaling laws in MIMO-BC or pre-log factor

M tx, N rx, K users

Full CSIT and CSIR (impact of K)

$$\lim_{SNR\to\infty}\frac{C_{DP}}{\log SNR}=\min\left(M,\max\left(N,K\right)\right)$$

$$\lim_{K\to\infty}\frac{C_{DP}}{\log\log KN}=M$$

No CSIT and full CSIR (TDMA)

$$\lim_{SNR\to\infty}\frac{C_{DP}}{\log SNR}=\min\left(M,N\right)=\lim_{SNR\to\infty}\frac{C_{P2P-MIMO}}{\log SNR}$$

$$\lim_{K\to\infty}\frac{C_{DP}}{\log\log KN}=0$$

the BS selects and tx only to the user with max rate

$$\lim_{K\to\infty}\frac{C_{TS}}{\log\log K}=\min\left(M,N\right)$$

Summary V

Information theoretic design guidelines

-Capacity scaling laws advocate for SDMA. How many and which
sers should be served at any instant of time, and how much power, is
he problem addressed by the scheduler and resource allocation
trategy

-Unlike the P2P MIMO, spatial mux is possible with N=1

-The mux gain M comes at the condition of close to perfect
SIT, this is different from P2P MIMO, where the
symptotic capacity does not depende on the CSIT.
herefore, the importance of the feedback channel

.-Take advantage of Multiuser Diversity (big K)

.-Precoder design together with scheduling: M useres out of K

$$C = \sum_{k} \log\left(1 + SNIR_k\right) \quad \left[bps/Hz\right]$$

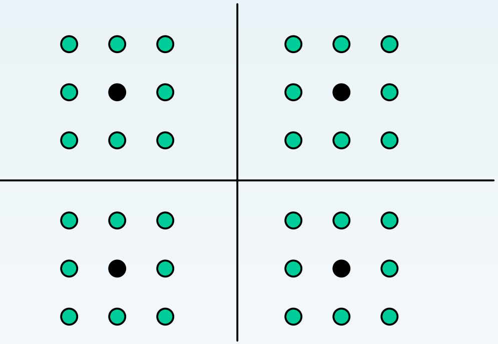
Contents

- Introduction
- Lessons learned form Information Theory
  - MAC channel
  - BC channel
  - Duality
  - Summary
    - *Scaling laws of BC-MIMO (pre-log factor)*
    - *Information Theory design guidelines*
- BC channel: signal processing design
  - Aiming at DPC: Linear and non-linear precoders
  - Practical schemes: Multiuser diversity
    - Opportunistic beamforming schemes
- System issues

For the single antenna case

In practice:

1.- For the general case: superposition coding achieves capacity
Superposition coding is a multiresolution technique



32-QAM con QPSK

QPSK → $$R_1 = \log\left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o}\right)$$

32-QAM → $$R_2 = \log\left(1 + \frac{P_2 |h_1|^2}{N_o}\right)$$

For the single antenna case

In practice:

2.-The weaker user

$$r_1 = \log\left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o}\right) \quad bits/s/Hz$$

The strongest user performs SIC at rx2

$$r_2 = \log\left(1 + \frac{P_2 |h_2|^2}{N_o}\right) \quad bits/s/Hz$$

For the single antenna case

On the other hand, orthogonal schemes achieve, for each power sp
P=P1+P2 and degree-of-freedom split a

$$R_1 = \beta \log\left(1 + \frac{P_1 |h_1|^2}{\beta N_o}\right)$$

$$R_2 = (1-\beta)\log\left(1 + \frac{P_2 |h_2|^2}{(1-\beta) N_o}\right)$$

ne can show that superposition coding is strictly better than the orth. Schemes
n these ones, a significant fraction of the degrees of freedom to the weak user a
eeded to achieve near single-user performance, degrading the strong user