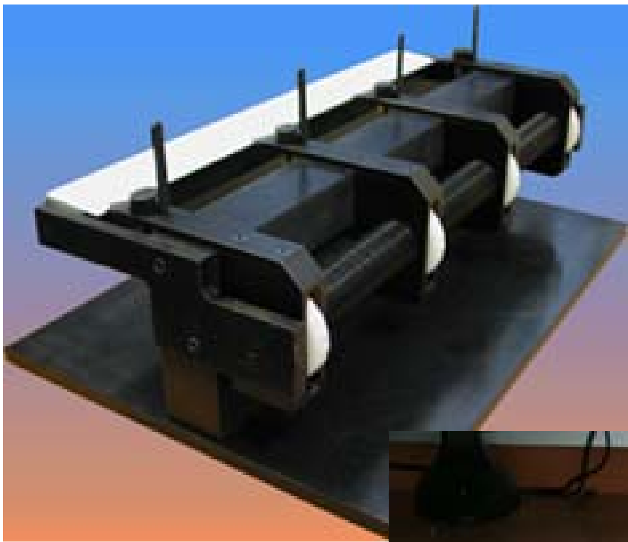
ARRAY PROCESSING II:

SPATIAL DIVERSITY IN RADIOCOMMUNICATIONS



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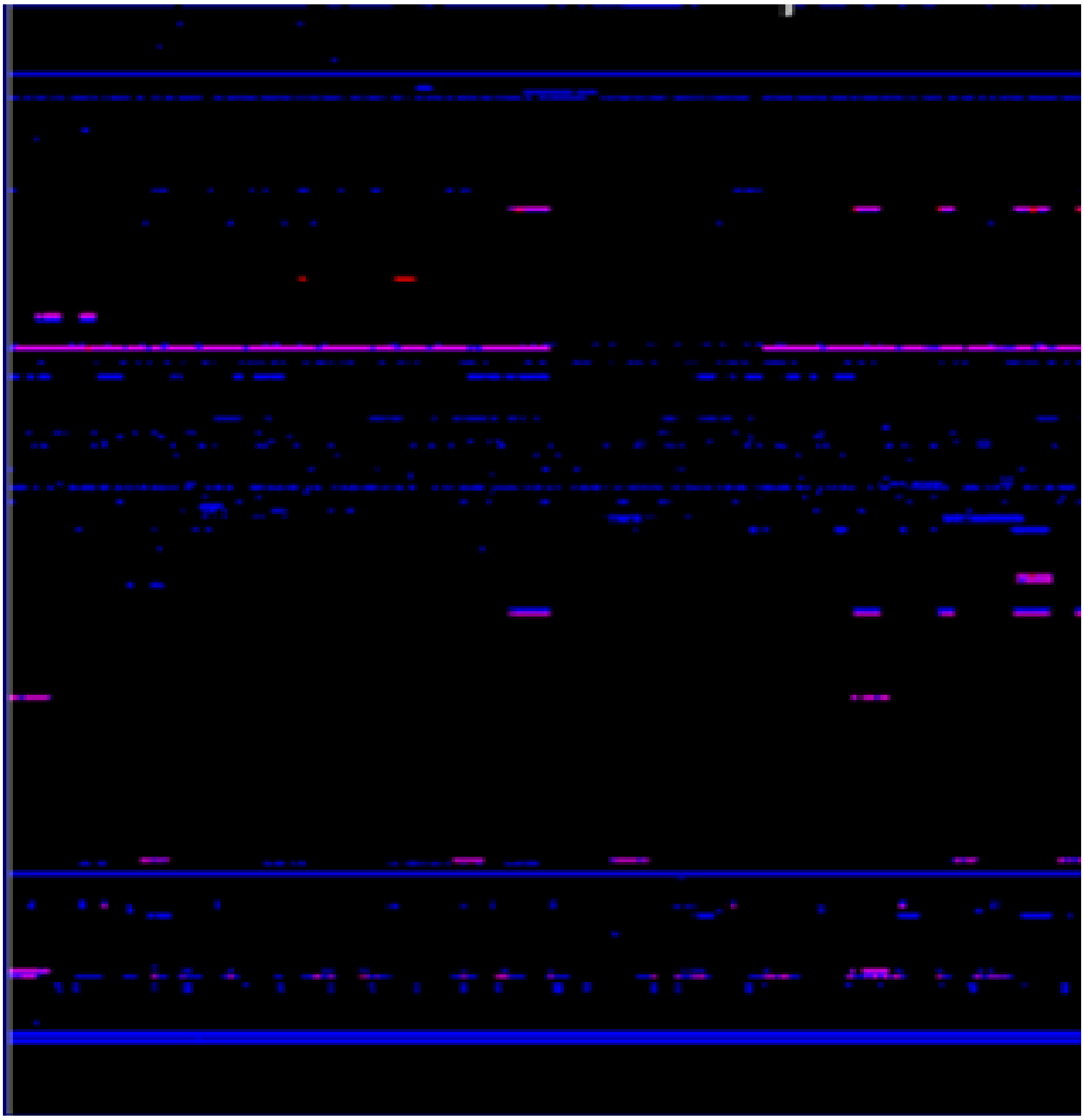
I.

I.INTRODUCTION



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I.1. CONTENT

The use of spatial diversity was introduced for the first time in radar and sonar systems with the purpose of improve the spatial resolution in transmission and reception. Later it was used in satellite systems, in the terrestrial sector. In all, the system was installed quickly, how we have said, both transmission and reception.

This high speed of their integration into radar, sonar and satellites is absolutely different with the apparent slowness of their integration into radio systems, both cellular and network systems. This is caused by two possible reasons. The first one is the so much complex channel (Rayleigh or Rice distribution instead of the, let's say optical channel, easy typed by steering vectors). The other one is the regulation and the standard belonging of the radio channels. This is structured in a layers model, ISO layers, in which the variations because of the spatial diversity are involved in variations in upper access and routing layers. For this second reason, any modification of the physical layer must be translated in a resources versus quality table, quality in broad terms which will include speed, BER, PER, etc.

These notes are structured for spatial diversity systems design in the telecommunication environment, but for radio channels. In many situations, the text will be related with the author's arrays course. Even though fourth section of the arrays course is related with communications, the course is designed, updated and recommended that the arrays one. In any case, the reader is not recommended to begin with this topic if the arrays course contents are not previously revised.

The course begins with a revision of the called array Broadband processing. As reader will see, in this chapter is possible to see the importance of revise the previous array course. In this topic is evaluated the border between broad and narrow bands in a radio environment. After an explanation of the basic process that takes part in the transmitter, this topic will show MIMO channel characteristics and its characterizations. Finally the availability of channel information, both transmission and reception, it will be described briefly in terms of complexity and sensibility of the non perfect design information.

The rest of chapters will describe MIMO systems in which the channel type or the channel information availability will serve as the connection thread of the chapters sequence and content.

It's worth mentioning that there won't be a barrier, as it's useful to find in some other texts, between linear process and space-time coding systems. Another novelty will be the general use of the optimal detector in all the presentations, but only when the complexity requires this, it will be replaced sometimes with an MSE (Mean Square Error) or ZF (Zero Forcer) type instant detector.

I.2. DIVERSITY PROCESSING

From now, let's consider n_R receiving antennas. Once again, as in the Array I course, the basic information will be the snapshot $\underline{X}_n$, the difference is that in this case it's not able to modelate as the sum of sources waveforms times their steering vectors.

A narrow band beamformer to the receiver would be a vector ($\underline{a}$), which will generate an output sample y(n).

$$y(n) = \underline{a}^H . \underline{X}_n \tag{I.1}$$

Even if we have a narrow band channel, the process of each antenna output signal into an equalizer or an adaptative filter modifies the simple architecture described by last equation. That is to say, when each antenna requires some different linear process –for instance, because of the propagation channel for each receiving antenna is different and requires an also different igualation–, the global space-time process requires a snapshot with bith diversity temporal/spatial. The collection of snapshots required will be called Xn.

$$\underline{\underline{X}}_n = \left[\underline{X}_{n+1} \quad .... \quad \underline{X}_{n+N} \right] \tag{I.2}$$

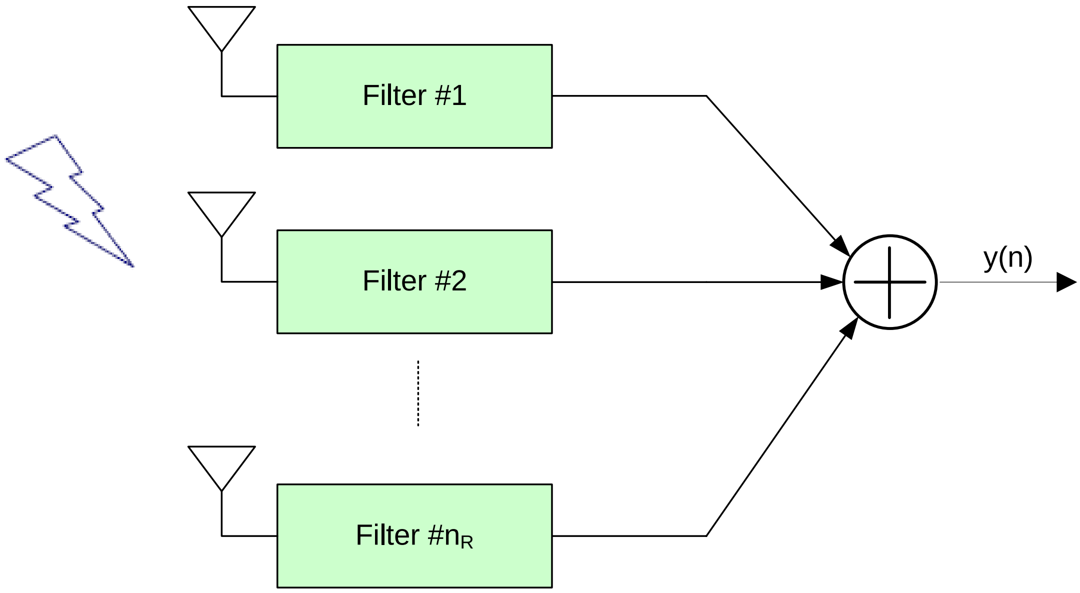

Figure I-1.- Receiving array with different processing in each antenna

A scheme like Figure I-1.- one it's easyble transformable to a broadband system, as we can see in Figure I-2.-. The broadband process it's the process of a packet of N snapshots that basically consists of take a different beamvector for each snapshot. That is, the first snapshot of the frame is processed by the beamformer $\underline{a}_1$, the second one by $\underline{a}_2$ and so on till the last snapshot, processed by $\underline{a}_N$. This is a form to set up the difference between the narrowband process and the broadband one. Also is considered the simultaneous use of different beamformers in spatial frame. The spatial-temporal processor output can be writed as follows:

$$y(n)=\sum_{m=1}^{N}\underline{a}_m^H.\underline{X}_{n+m} \tag{I.3}$$

It can be seen the use of multiple beamformers. Another filter-based formulation, with a double sum in the spatial axis ('q') and in the temporal axis ('m'):

$$y(n)=\sum_{q=1}^{n_R}\sum_{m=1}^{N}a_m(q).x_q(n+m) \tag{I.4}$$

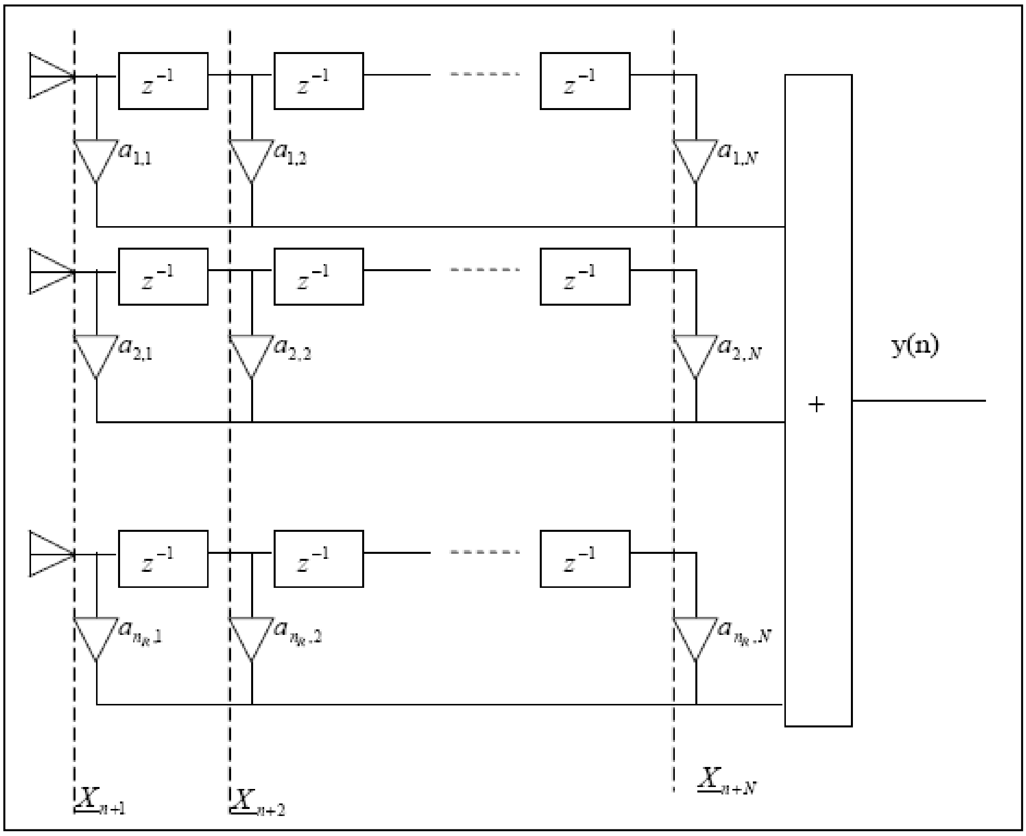

Figure I-2.- Space-Time Processir details

The most interesting processor formulation is based in the following definitions:

$$\underline{\underline{A}}=\left[\underline{a}_1 \quad \underline{a}_2 \quad \dots \quad \underline{a}_N\right]=\begin{bmatrix} a_{1,1} & a_{1,2} & .. & a_{1,N} \\ a_{2,1} & a_{2,2} & .. & a_{2,N} \\ .. & .. & .. & .. \\ a_{n_R,1} & a_{n_R,2} & .. & a_{n_R,N} \end{bmatrix} \tag{I.5}$$

$$\underline{\underline{X}}_n=\left[\underline{X}_{n+1} \quad \underline{X}_{n+2} \quad \dots \quad \underline{X}_{n+N}\right] \tag{I.6}$$

With these matrices it's easier to formulate the space-time processor output, as the trace of their product, it is:

$$y(n) = trace\left(\underline{\underline{A}}^H . \underline{\underline{X}}_n\right) \qquad (I.7)$$

Because of the simplicity of the last expression, this will be used for the receiver of a system with diversity.

I.3. SCHEME OF TRANSMITTER WITH SPATIAL DIVERSITY.

It will be considered the information to be transmitted for the communications system as a set of n_S bits grouped into a single vector, as we can see in the following equation.

$$\underline{I}_n = \begin{bmatrix} b(1) & b(2) & .. & b(n_S) \end{bmatrix} \quad con \quad b(i) = \{+1,-1\} \; \forall i = 1, n_S \qquad (I.8)$$

Firstly, the transmitter generates the symbols to be transmitted depending on the constellation used. This mapping is represented as the product of the streams vector with a matrix, $\underline{\underline{V}}$.

$$\underline{S}_n = \underline{\underline{V}}^H . \underline{I}_n \qquad (I.9)$$

In this stage, the designer must choose between compress to a single symbol or to transmit all streams directly in BPSK. For exemple, if we have four streams and we want to use QPSK, we will generate two symbols with the matrix:

$$\underline{\underline{V}}^H = \begin{bmatrix} 1 & j & 0 & 0 \\ 0 & 0 & 1 & j \end{bmatrix} \qquad (I.10)$$

And for the 16-QAM case we will generate one symbol with:

$$\underline{\underline{V}}^H = \begin{bmatrix} 1 & 3 & j & 3j \end{bmatrix} \qquad (I.11)$$

Take note that the number of possibilities is big. In more advanced parts of the course we will see which solution is the better. It is worth mentioning that there is, in any case, two situations to be studied separately: the case of one symbol and the case of multiple symbol (it will be shown later on).

From this, if the number of symbols to be transmitted is n_B, it's necessary to assign energy to all of them. This can be done through a diagonal matrix which elements are the square root of the energy to assign at each symbol.

$$\underline{\theta}_n = diag\left[E_1^{1/2}, .., E_{n_B}^{1/2}\right] . \underline{\underline{V}}^H . \underline{I}_n = \underline{\underline{P}} . \underline{\underline{V}}^H . \underline{I}_n \qquad (I.12)$$

Finally, the transmitter will use a different beamformer depending on the symbol (U), in order to generate the transmitted signal (I.13). Note that the beamformers matrix will have n_T x n_B size (it's like n_B beamformers).

$$\underline{X}_{T,n} = \underline{\underline{U}}.\underline{\underline{P}}.\underline{\theta}_n = \underline{\underline{U}}.\underline{\underline{P}}.\underline{\underline{V}}^H.\underline{I}_n \quad (I.13)$$

Without loss of generality it can be supposed that the beamformers matrix is unitary. If it is, the transmitted signal covariance matrix is given by (I.14) and total energy transmitted to the channel will be given by (I.15).

$$\underline{\underline{Q}} = E\left(\underline{X}_{T,n}.\underline{X}_{T,n}^H\right) = \underline{\underline{U}}.\underline{\underline{P}}.\underline{\underline{V}}^H E\left(\underline{I}_n.\underline{I}_n^T\right).\underline{\underline{V}}.\underline{\underline{P}}.\underline{\underline{U}}^H = \underline{\underline{U}}.\underline{\underline{P}}.\underline{\underline{V}}^H.\underline{\underline{V}}.\underline{\underline{P}}.\underline{\underline{U}}^H \quad (I.14)$$

$$E_T = Trace\left(\underline{\underline{Q}}\right) = Trace\left(\underline{\underline{V}}^H \underline{\underline{P}}^2.\underline{\underline{V}}\right) \quad (I.15)$$

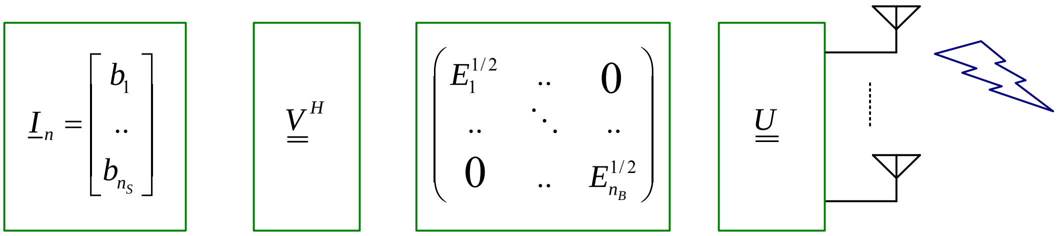

Figure I-3.-Transmitter design in three stages: Constellation, Power Allocation and Beamforming Matrix

Figure I-3 summarizes the exposition of this section. First stage is a process which generate the symbols to be transmitted from the initial bits. Second stage assign the energy desired to each symbol (partions of the total energy E_T). Finally, the spatial processor, that uses multiple beamformers (one for each symbol) to generate the transmitted signal.

I.4. THE MIMO CHANNEL.

There are two possibilities in order to modelate the channel since through the n_T transmitted signals arrive to the n_R received antennas. In fact there is a third possibility, the combination of the first and the second ones (this will be the best for any type of environment). The first possibility consider a deterministic (or ray) model in which each transmitter antenna signs with fixed steering vectors in the receiver opening. In this case, the channel matrix $\underline{\underline{H}}(n_R, n_T)$ will be given by:

$$\underline{\underline{H}} = \left[\underline{h}_1 \quad .. \quad \underline{h}_{n_T}\right] \quad (I.16)$$

It's possible to give in function of vectors from which transmitter is seen for the receiver opening:

$$\underline{\underline{H}} = \begin{bmatrix} \underline{h}_1^H \\ .. \\ \underline{h}_{n_R}^H \end{bmatrix} \qquad (I.17)$$

In both structures (I.16) or (I.17) there is a significative difference with the arrays I course, because now in any case can be considered steering vectors. This is one of the reasons of the spatial diversity study complexity in radio environments. In fact, to characterize the channel with steering vectors is a very particular and restricted case (so far from the reality in closed environments or with high number of scatters).

This channel representation is an instantaneous model and normally the behaviour in differents accesses won't be constant. Depending on coherence time the maximum assuming is that rest constant in all the used frame. In the following frames the channel matrix presents a random distribution for all their inputs. The most general model assume a fix component $\underline{\underline{H}}_f$ (mean) and a random component $\underline{\underline{H}}_a$(zero mean, covariance meaning) and also assume the inputs $h_{i,j}^a$ of which in phase and quadrature components are Gaussian.

$$\underline{\underline{H}} = \underline{\underline{H}}_f + \underline{\underline{H}}_a \qquad (I.18)$$

In not Vision Of Sight (VOS) between transmitter and receiver environments, only the random component is significative. Therefore $\underline{\underline{H}}$ inputs will be random viariables with zero mean and Gaussian distribution in their in phase and in quadrature components (Rayleigh channel).

If there are n_T transmitting and n_R receiving antennas, the number of available channels will be n_T x n_R. But ISI-free channels are a maximum of min(n_T, n_R), that is, the range of their matrix. The SVD descomposition of the channel matrix is:

$$\underline{\underline{H}} = \underline{\underline{V}}_h . \underline{\underline{\Gamma}}_h . \underline{\underline{U}}_h^H \quad con \quad \underline{\underline{\Gamma}} = diag\left[\gamma_{H1} \quad .. \quad \gamma_{H\min(n_T,n_R)}\right] \qquad (I.19)$$

In which we can see that in the best case, the number of channels is equal to min(n_T, n_R). These are called as the MIMO channel eigenmodes. If we work with these channels this implies a diagonalization of the channel matrix (from right by the transmitter spatial processor and from left by the receiver spatial processor). We will recover this idea in following chapters to analyze in detail.

As we will be able to see further on, normally the quality figures of MIMO transmission system make appear the channel as the product of the channel matrix (transposed) with the channel matrix. This new matrix will be decomposed as follows:

$$\underline{\underline{H}}^H . \underline{\underline{H}} = \sum_{q=1}^{n_R} \underline{h}_q . \underline{h}_q^H = \underline{\underline{U}}_h . \underline{\underline{\Sigma}}_h . \underline{\underline{U}}_h^H \quad con \quad \underline{\underline{\Sigma}}_h = \left[\lambda_{H1} \quad \lambda_{H2} \quad .. \quad \lambda_{H\min(n_T,n_R)}\right] \qquad (I.20)$$

Therefore, the MIMO channel eigenmodes describe the eigenvalues as their gain. It's important to note that, in contrast to the diagonal of the original matrix, the eigenvalues have a lot of dispersion and, in general, only the first and the second ones are significative, even if n_R and n_T is large.

Finally, there are two types of channels, attending at their frequency selectivity. The first case, called selective fading, in which the channel matrix depends to the frequency, $\underline{\underline{H}}(f)$. This situation is presented on radio DSSS systems (Spread Spectrum) or in UWB systems (pulsed version). On the contrary the second case, called flat fading, in which the channel is supposed flat, so this doesn't depend on the frequency. All multicarrier systems and OFDM, and also all the cases that can consider the channel as a constant belong to the second type.

I.5. CSI (CHANNEL STATE INFORMATION).

In radiocommunications the variability of the channel impose important restrictions to the attempts of optimizing the information transmission through it. The most adverse situation is the one in which anyone (transmitter and receiver) have information of the channel (CSI). In that case the design is done through differential techniques that in fact try to give hiddenly CSI to the receiver.

All coding techniques, specifically the space-time codes, have their origin in a MIMO system in which transmitter doesn't have CSI but the receiver has. The receiver obtain CSI by means of training sequences reemitted by transmitter.

The much technically complicated case is the one in which we assume that both transmitter and receiver have CSI.

For obtain CSI in MIMO systems are required n_T training sequences (or using the same but in different time slots). Obviously this reduces the capacity, but also the processing requeriments arise because n_R processors must identify the n_T channels that every receiving antenna can see. Most of the cases we will use the perfect CSI hypothesis when there is noise present (this noise will introduce errors into channel taps estimation). These errors are proportionally bigger in low energy taps so it's not recommended to use taps which energy is close or below the noise level. Moreover, the estimation errors have different consequences depending on the use (complexity). For instance the effect of noise in H estimation won't be the same if its eigenvectors are required or only an adaptive filtering is done.

If CSI in the transmitter is required, the difficulty of the problem grows. In theory the best way for transmitters to have CSI requires that receivers send the training sequences. This solution multiplies by two the commented problems. A notorious simplification typically used consists in make the receiver to send channel information to the transmitter through return channel.

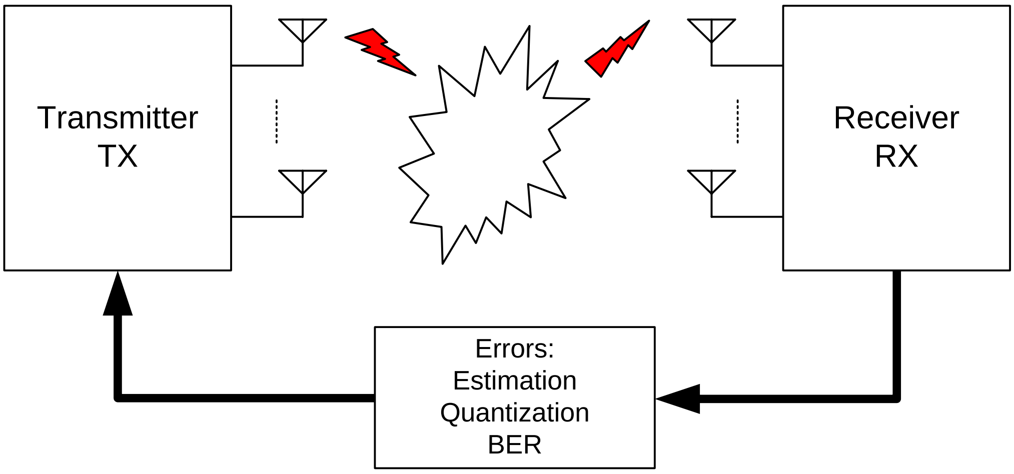

Figure I-4.- CSI sending from RX to TX (see also existing error sources).

As well as the loss of capacity, must bear in mind the quantification errors of adapting tabs into a digital transmission and the noise presence in the receiver that will cause detection errors. This problem have been studied in detail, because of that there exist some techniques as vectorial coding or complex clustering and differential coding systems in order to make lighter the quantification problems.

The second and more important errors source is the quick variability of the channel. Exists the possibility that since the time of the CSI estimation, the channel had changed. This can cause very unpredictable system behaviour. This can became a very serious problem when exists big asymmetry between transmitter and receiver data rates.

In summary, the reader can suspect the complexity of know as perfectly as possible the CSI, and won't consider strange that in most case, systems won't work with CSI in the transmitter. It's interesting to comment that the best techniques for MIMO channel were developed in DSL environments. In this systems, CSI availability is more simple and usually used. It's also important to remember to the reader that spatial diversity it's another link added to the difficulty of use frequency diversity in multicarrier systems or OFDM. The most important advantage of this architecture is that in MIMO system it's easy to find $\underline{\underline{H}}$ diagonal or almost diagonal. In radio case, this assumption is not true.

All these notes content is perfectly valid in any type of diversity but it's always destined to spatial diversity, because it's the most diffictult or that more challenges and expectation.

I.6. SUMMARY

In this chapter we have described the basic process in transmission and reception in a system with spatial diversity in RX, TX or both. In order to make the exposition clear, from now the transmitter three block processes will be assigned to the $\underline{\underline{B}}$ matrix.

The same thing happens with the receiver, characterized by $\underline{\underline{A}}$. In any case will be useful for the reader to keep remembering the steps in order to deduce these matrices.

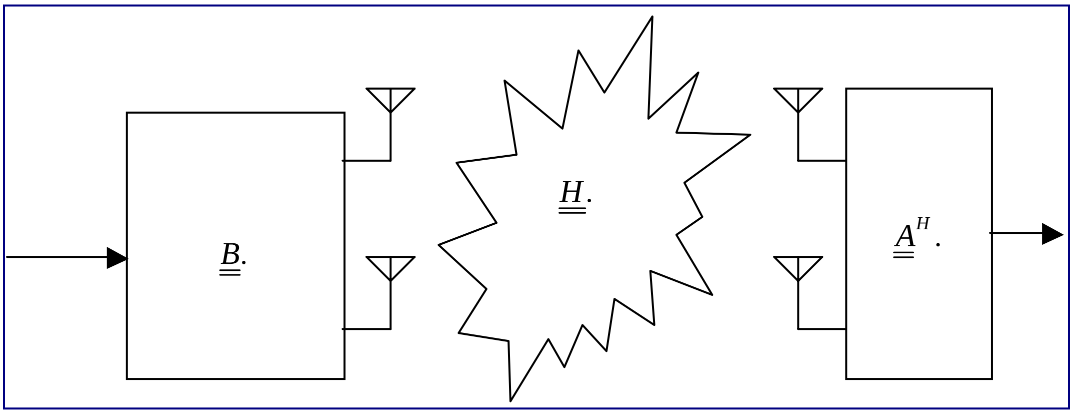

Figure I-5.- Generic scheme of MIMO transmission

After that, the MIMO channel has been described. The eigenmodes of the channel have been presented and their value as ISI-free channels (orthogonal). Channel flat or frequency selective will motivate different schemes. The tradition have made that in the literature it's more explained the flat fading case, but the appearances of new broadband systems without frequency canalization makes grow the interest in the frequency selective fading case.

Finally, technical problems related to the CSI availability have been commented. It has been seen the complexity of the process and the difficulties to have CSI in transmission. From this the reader, it's interesting to appreciate the value of the techniques which not required CSI in transmission.

Next chapter consider the case of an unique symbol of a n_S bits constellation. This will be transmitted through a flat fading MIMO channel. This will be the easier and for that it will be a great introduction to the following chapters. In fact this situation doesn't have practical interest but it works as an introduction.

II.

II. A SYMBOL OVER FLAT FADING CHANNEL

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II.1. THE ML DETECTOR

This chapter will tackle the transmission of a symbol s(n), which belongs to a n_S bits constellation, over a flat fading MIMO channel (not selective in frequency).

Assuming transmitted signal as the product of s(n) and the transmitter matrix $\underline{\underline{B}}$, we should design the optimum receiver to get, in reception, the symbol transmitted with the smaller error rate possible. Note that the number of $\underline{\underline{B}}$ rows must be n_T (transmitter antennas). The number of columns (N) implies N channel accesses to bring the symbol to the receiver. Obviously, when N is large, the greater the quality (system SNR), but also the greater the channel use inefficiency (N accesses for one symbol transmitted).

The system under study scheme is presented in Figure II-1.

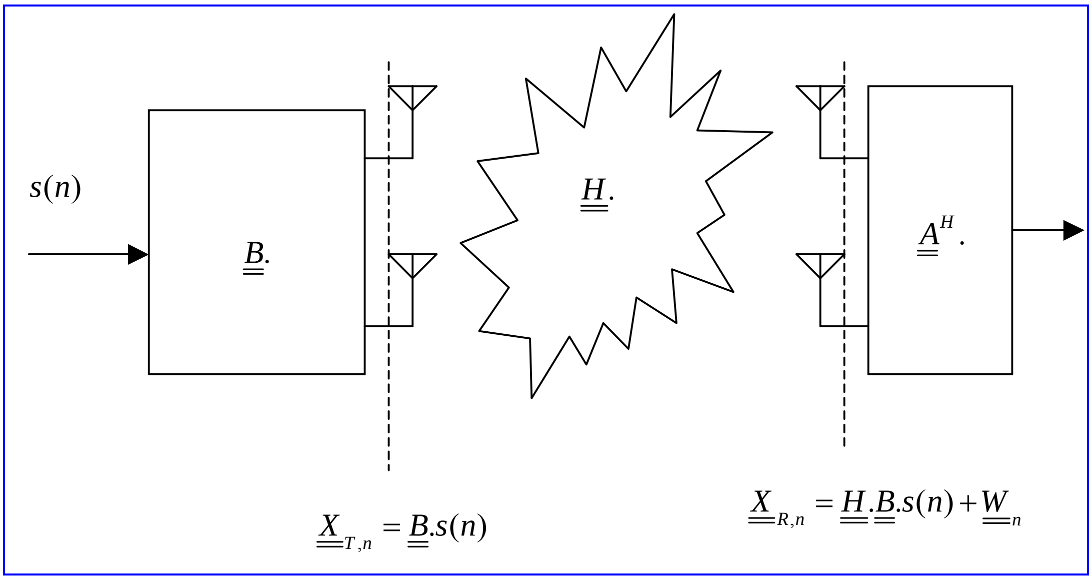

Figure II-1.- Receiving array with different processing in each antenna

Transmitted signal (corresponding to the information symbol) will have N snapshots, which agrupation create the transmitted matrix.

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n) \qquad \text{(II.1)}$$

Transmitted energy is (II.2), where square root of E_S is the distance between M-QAM constellation and the one who belongs transmitted symbol.

$$E_T = trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).E\left(|s(n)|^2\right) = \frac{E_s}{3.\left(2^{n_S}-1\right)}.trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right) \qquad \text{(II.2)}$$

In the receiver, received matrix will be the transmitted one after pass through MIMO channel, and also the noise.

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_n \qquad \text{(II.3)}$$

Where every matrix term is complex and their real and imaginary parts have zero mean and $N_0/2$ power.

The likelihood of a symbol s(n) will be the following:

$$\begin{aligned} \Lambda\left(s(n)\right) &= -\left|\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right|_F^2 = \\ &= -Trace\left[\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right).\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right)^H\right] = \\ &\rightarrow 2.\mathrm{Re}\left[s(n)^*.Trace\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{X}}_{R,n}\right)\right] - \left|s(n)\right|^2.Trace\left[\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right] \end{aligned} \qquad \text{(II.4)}$$

This equation shows that the receptor is defined only for the unique operation that likelihood requires from data.

$$\underline{\underline{A}} = \underline{\underline{H}}.\underline{\underline{B}} \qquad \text{(II.5)}$$

Note that the second term, except for a constant envelope modulation, is important to calculate the likelihood of every symbol. From now, general case will be presented. Correct s(n) symbol decision will imply a likelihood larger than other constellation symbols.

$$si \quad \Lambda\left(s(n)\right) > \Lambda\left(s(m)\right) \quad \forall m \neq n \quad decide \quad s(n) \qquad \text{(II.6)}$$

If correct symbol is s(n), that is, if received matrix is (II.3), then (II.6) must be verified. It's easy to check that the s(n) transmission implies in (II.6) that (II.7) must be verified, where ŝ(n) is the difference between correct symbol and s(m).

$$Trace\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^2\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Trace\left(\underline{\underline{W}}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right] \qquad \text{(II.7)}$$

If consider the error only will be produced in nearer constellation symbols, it's easy to check that right term is reduced to:

$$\gamma = 4.E_s.Trace\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right) \qquad \text{(II.8)}$$

The right term it's a random Gaussian variable, which has zero mean and variance equal to 4 times the square of last expression times noise one. In Figure II-2 can be shown that error probability is given by the following expression:

$$\Pr\left(s(n) \rightarrow s(m)\right) = P_e = Q\left(\sqrt{\frac{\gamma}{2.N_0}}\right) \qquad \text{(II.9)}$$

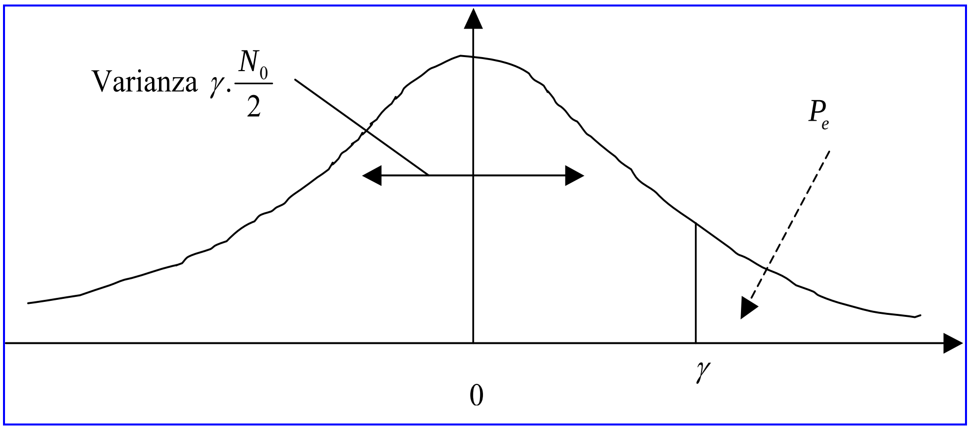

Figure II-2.- Error probability in ML detector.

This last expression shows the optimal transmitter process will be those which maximizes the trace inside threshold expression. Optimal design will be those which maximizes threshold with respect to transmitted energy. Finally, we obtain the following expression after use the circularity symmetric property of trace operator:

$$\begin{aligned} &4E_s.Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}.\underline{\underline{B}}^H\right)\Big|_{MAX} \\ &k_0.Trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right) = E_T \end{aligned} \qquad \text{(II.10)}$$

This completes system design because receiver is adapted to the channel and to the transmitter.

II.2. COMPLETE CSI AT THE TRANSMITTER

The solution of the transmitter design when a perfect CSI is available will be described in the following lines. From now the product of transposed channel matrix times itself will be expressed as one matrix.

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H.\underline{\underline{H}} \qquad \text{(II.11)}$$

Trace maximization in (II.10) is easy if the following property is known:

$$\begin{aligned} &Trace\left(\underline{\underline{A}}.\underline{\underline{B}}\right) \le \lambda_{\max}\left(\underline{\underline{A}}\right).Trace\left(\underline{\underline{B}}\right) \\ &Equal \quad if \quad \underline{\underline{B}} = \underline{e}_{\max}.\underline{g}^H \quad \forall \underline{g}\ (N,1) \end{aligned} \qquad \text{(II.12)}$$

That is to say the trace to maximize will be maxim when the spatial-temporal processor is a matrix of range one, consisting of, on the left, the maximum eigenvector of channel matrix and on the right, any N-dimensional vector (to verify that N channel accesses will be done). Next this premise will be revised.

With this, the threshold expression will be:

$$\gamma = E_s.Trace\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = E_s.\lambda_{\max}\left(\underline{\underline{R}}_H\right).Trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right) \quad \text{(II.13)}$$

This expression, restricted in transmitted energy, is:

$$\gamma = E_T.\lambda_{\max}\left(\underline{\underline{R}}_H\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right) \quad \text{(II.14)}$$

Error rate would be given by (II.15) or by Chernoff level (II.16). The two values are close for SNR reasonable values.

$$P_e = Q\left(\sqrt{\frac{2.E_T}{N_0}.\lambda_{\max}\left(\underline{\underline{R}}_H\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right) \quad \text{(II.15)}$$

$$P_e = k_1.\exp\left[-\frac{2.E_T}{N_0}.\lambda_{\max}\left(\underline{\underline{R}}_H\right).\frac{3}{2^{n_S}-1}\right] \quad \text{(II.16)}$$

Interesting details can be extracted. Note that MIMO channel gain is given by the channel matrix maximum eigenvalue. Note also, considering a QAM constellation, the unfavourable behaviour of n_S (number of streams) in effective SNR.

It's also interesting the transmitter and receiver structure.

$$\begin{aligned}\underline{\underline{B}} &= k_1.\underline{e}_{\max}.\underline{g}^H \\ \underline{\underline{A}} &= \underline{\underline{H}}.\underline{\underline{B}} = \lambda_{\max}^{1/2}.\underline{v}_{\max}.\underline{g}^H\end{aligned} \quad \text{(II.17)}$$

That is, both processors are range 1 and their first component is the H eigenvector from right (transmitter) and from left (receiver). k_1 constant adjust transmitted energy to its corresponding value.

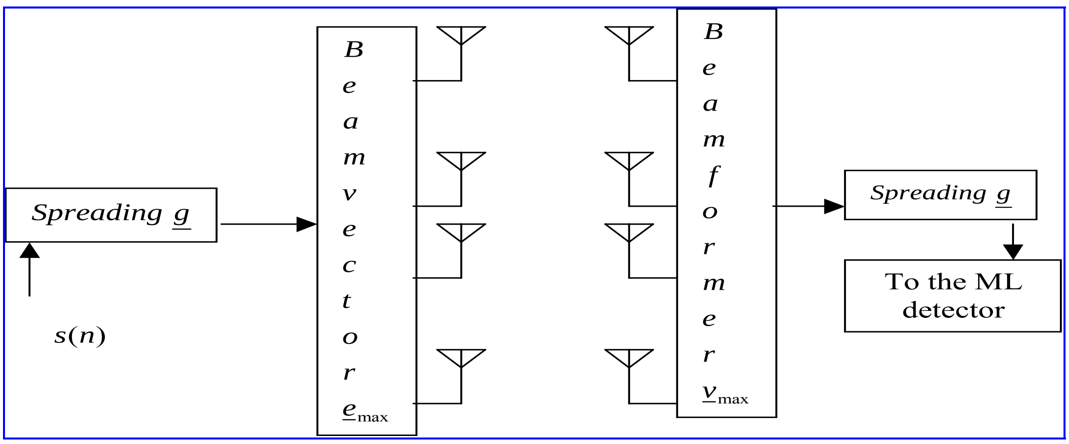

Figure II-3.- Optimal TX-RX for full-CSI at TX and flat fading channel.

At this point will be discussed the role of the second N-length vector. This vector implementation is equivalent to the use of a spreading sequence (enlarging original symbol). The advantage of use N accesses instead of one (full-rate system, i.e. one complex symbol for every channel access) is give access to users who use spreading sequences (synchronous CDMA) or give security to the transmission. Anyway, if any of these advantages are needed, the system can work in full-rate mode only replacing this vector to the unit vector.

Anyhow note the problem's optimal solution is basically a temporal processor (chosen by designer) followed by a beamformer which is equal to maximum from right eigenvector of MIMO channel matrix.

II.3. INSTANT DETECTION

Last section has described ML detector and complete transmitter and receiver design. In symbol likelihood appeared two terms:

$$\Lambda\left(s(n)\right) = 2.\mathrm{Re}\left[s(n)^{*}.Trace\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] - \left|s(n)\right|^{2}.Trace\left[\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{H}}.\underline{\underline{B}}\right] \qquad \text{(II.18)}$$

Reviewing this expression, receiver must search which constellation symbol gives a higher value and then decide that this is the correct symbol. Nevertheless, a detailed analysis reveals that search can be simplified in M-QAM constellations. In QPSK the simplification is still higher because it has a constant envelope and the second term is not important to get the maximum likelihood value. The detector will follow (II.7) to decide the two bits of this constellation.

$$\begin{aligned}
&\mathrm{Re}\left[Trace\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] \underset{<}{\overset{>}{}} 0 \\
&\mathrm{Im}\left[Trace\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] \underset{<}{\overset{>}{}} 0
\end{aligned} \qquad \text{(II.19)}$$

Although the triviality of reduce the optimal detector to an instant detector, it can be shown that in other scenarios or with other process techniques this simplification won't be allowed, especially with zero thresholds, that is to say without automatic gain control (AGC) in the receiver.

This instant detection has been possible because, as perfect CSI in transmitter is available, system works with the best channel mode. Even though we have multiple channels at one's disposal, we'll only work with one of them. These modes, eigenvectors in Tx and Rx, allow to work under ISI-free SISO channels, and this is the reason who allows to implement an instant detector keeping its optimal character up.

At the same time, the transmitter and receiver structure for a single symbol, reveal that with N accesses and with orthogonal spreading sequences, up to N users will be able to multiplex (in code). With a gradual degradation of the rest of modes, eigenvectors lower than maximum and therefore channels with lower gain, N symbol streams for every channel mode can be used. But their worse gain will force a reduction of the constellation size (adaptive modulation) or equalization, increasing transmitted power in these modes if we want to maintain a constant error rate in all used modes. Afterwards we will talk about these topics when multiple streams transmission will be explained.

II.4. TRANSMISSION WITHOUT CSI

In case of no CSI availability in receiver, the restriction maximization in transmitter will be more complicated. The design equations are:

$$\begin{aligned} &4E_s.Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}.\underline{\underline{B}}^H\right)\Big|_{MAX} \\ &k_0.Trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right)=E_T \end{aligned} \qquad \text{(II.20)}$$

Obviously maximization seems to have no sense if channel matrix is unknown.

A possibility is to approach the problem as a two player's game. One player, the designer, wants to maximize the trace (last equation). The other player, the channel, wants to minimize that. Both players are focused in same award: the trace. Game rules are, for the designer, maximum transmitted energy, and for the channel is not obvious.

In channel's move restrictions, both reality applied restrictions can be imposed:

$$Trace\left(\underline{\underline{R}}_H\right)\geq\rho \qquad \text{(II.21)}$$

$$\lambda_{\min}\left(\underline{\underline{R}}_H\right)\geq\vartheta \qquad \text{(II.22)}$$

Any of these would be valid and will prevent player to set the matrix to zero, which would mean a wined game against any other player.

Taking trace restriction (II.21) we will select a simple game, only a move for each player. The channel plays firstly, minimizing the trace (with trace restriction which will allow no to be zero).

$$\begin{aligned} &\min_H\left[Trace\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right)\right] \\ &\text{with}\quad Trace\left(\underline{\underline{R}}_H\right)\geq\rho \end{aligned} \qquad \text{(II.23)}$$

Last minimization is simple if the following property is known:

$$Trace\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right)\geq\lambda_{\min}\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).Trace\left(\underline{\underline{R}}_H\right) \qquad \text{(II.24)}$$

After this move, the designer plays trying to maximize right part of last expression. The solution is easy: search the maximum of the minimum matrix value, with trace -eigenvalues sum- restriction. Final solution is all equal:

$$\underline{\underline{B}} = \underline{\underline{\Pi}} \quad with \quad \underline{\underline{B}}.\underline{\underline{B}}^H = \underline{\underline{\Pi}}.\underline{\underline{\Pi}}^H = \underline{\underline{I}}_{n_T} \tag{II.25}$$

Where $\underline{\underline{\Pi}}$ is an unitary matrix.

Some details are interesting. Firstly, note that with these restrictions and with only one player movement, the game is a process type called mini-max. Minimum for the channel and maximum for transmitter design. Secondly, a constant when CSI isn't available, the best solution is uniform power to all channel modes (identity matrix), a solution callet UPA (Uniform Power Allocation). Thirdly, to build the n_T dimensional unity matrix there is no other solution than realize at least the same channel accesses ($N=n_T$) therefore it seems that it'll have low rate. This is not essentially true because some symbols can be simultaneously used if transmitter matrix is the appropriate.

For instance, to transmit two symbols s1(n) and s2(n) simultaneously, both real with spatial-temporal process matrices $\underline{\underline{\Pi}}_1$ and $\underline{\underline{\Pi}}_2$, with the restriction:

$$\underline{\underline{\Pi}}_1.\underline{\underline{\Pi}}_2^H = -\underline{\underline{\Pi}}_2.\underline{\underline{\Pi}}_1^H \tag{II.26}$$

This condition is given to avoid ISI between both symbols. The receiver for each of two symbols will be:

$$\begin{aligned} \hat{s}1(n) &= \mathrm{Re}\left[Trace\left(\underline{\underline{\Pi}}_1^H.\underline{\underline{H}}^H.\underline{\underline{X}}_{R,n}\right)\right] \\ \hat{s}2(n) &= \mathrm{Re}\left[Trace\left(\underline{\underline{\Pi}}_2^H.\underline{\underline{H}}^H.\underline{\underline{X}}_{R,n}\right)\right] \end{aligned} \tag{II.27}$$

Obviously (II.28) is guaranteed, which allows recovering perfectly each symbol without the interference of another.

$$\begin{aligned} &\mathrm{Re}\left[Trace\left(\underline{\underline{\Pi}}_1^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{\Pi}}_2\right)\right] = \mathrm{Re}\left[Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{\Pi}}_2\underline{\underline{\Pi}}_1^H\right)\right] = \\ &= -\mathrm{Re}\left[Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{\Pi}}_1\underline{\underline{\Pi}}_2^H\right)\right] \\ &= \left|\begin{matrix}\text{Equal to} \\ \text{real part} \\ \text{of conjugated}\end{matrix}\right| = \mathrm{Re}\left[Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{\Pi}}_1\underline{\underline{\Pi}}_2^H\right)\right] \\ &\text{The only possibility is that be} = 0 \end{aligned} \tag{II.28}$$

The property (II.21) is lower demanding than matrix orthogonality, which can give so much possibilities to find it. This property is called "friendly" matrices.

They can be also transmitted two complex symbols and detect the four real symbols $s_1(n)$, $s_2(n)$, $s_3(n)$ and $s_4(n)$ if the following equation is verified:

$$\underline{\underline{\Pi}}_3 . \underline{\underline{\Pi}}_4^H = \underline{\underline{\Pi}}_3 . \underline{\underline{\Pi}}_4^H \qquad \text{(II.29)}$$

This way, transmitted signal and the corresponding detector would be as indicate (II.30). Although every symbol fill multiple channel accesses, the system can turn into full-rate using friendly matrices, as many as number of accesses, which will be equal to transmitter antennas.

$$\begin{aligned}
\underline{\underline{X}}_{T,n} &= \underline{\underline{\Pi}}_1 s1(n) + \underline{\underline{\Pi}}_2 s2(n) + j.\underline{\underline{\Pi}}_3 s3(n) + \underline{\underline{\Pi}}_4 s4(n) \\
\hat{s}1(n) &= \mathrm{Re}\left[Trace\left(\underline{\underline{\Pi}}_1^H . \underline{\underline{H}}^H . \underline{\underline{X}}_{R,n} \right) \right] \\
\hat{s}2(n) &= \mathrm{Re}\left[Trace\left(\underline{\underline{\Pi}}_2^H . \underline{\underline{H}}^H . \underline{\underline{X}}_{R,n} \right) \right] \\
\hat{s}3(n) &= \mathrm{Im}\left[Trace\left(\underline{\underline{\Pi}}_3^H . \underline{\underline{H}}^H . \underline{\underline{X}}_{R,n} \right) \right] \\
\hat{s}4(n) &= \mathrm{Im}\left[Trace\left(\underline{\underline{\Pi}}_1^H . \underline{\underline{H}}^H . \underline{\underline{X}}_{R,n} \right) \right]
\end{aligned} \qquad \text{(II.30)}$$

The importance of design spatial-temporal codes is high, where friendly matrices positions will belong to a restricted alphabet.

Returning to one symbol case, there are two classic and very used transmitter matrices structures, which are Identity matrix and DFT matrix. Resulting scheme for the first case (identity) is called delay diversity, because same symbol is sent to different antennas in every access. Under second scheme (DFT), the reader will find quite similitude to FDSS or frequency diversity modulations, similar to OFDM but with the same symbol in every carrier.

Finally, it's interesting to recover the game situation when the channel restriction is changed to the minimum eigenvector. In this case the channel move will produce:

$$Trace\left(\underline{\underline{R}}_H . \underline{\underline{B}} . \underline{\underline{B}}^H \right) \geq \lambda_{\min}\left(\underline{\underline{R}}_H \right) . Trace\left(\underline{\underline{B}} . \underline{\underline{B}}^H \right) \qquad \text{(II.31)}$$

The transmitter only gives its power and nothing more. Such interesting thing is, if we restrict every antenna power instead of restrict total transmitter power, then the solution obtained will be the same as last case. For this reason it's recognized that design solution for no CSI always give to UPA.

II.5. WITH OR WITHOUT CSI COMPARATION

As we might think, working without CSI has some impact in detection quality. It has made clear that in both cases the optimal receiver can implement instantly without depending on transmitted constellation. It's left to examine the cost associated to reduce complexity when system doesn't have CSI.

Keeping in mind that process matrix, when no CSI is available, is orthonormal, transmitted energy restriction implies that:

$$Trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right) = k_2 \quad \text{which accomplish} \quad k_2.E_s.\frac{2^{n_S}-1}{3} = E_T \tag{II.32}$$

With this, the process matrix will be:

$$\underline{\underline{B}} = \underline{\underline{\Pi}}.\sqrt{\frac{k_2}{n_T}} \quad y \quad \gamma = 4E_s.Trace\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = \frac{4.E_s.k_2}{n_T}.Trace\left(\underline{\underline{R}}_H\right) \tag{II.33}$$

And error rate without CSI is:

$$P_e^{NO-CSI} = Q\left(\sqrt{\frac{2.E_T}{N_0}.\left(\frac{Trace\left(\underline{\underline{R}}_H\right)}{n_T}\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right) \tag{II.34}$$

We can appreciate differences between this expression and the one with CSI availability, that is:

$$P_e^{CSI} = Q\left(\sqrt{\frac{2.E_T}{N_0}.\lambda_{\max}\left(\underline{\underline{R}}_H\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right) \tag{II.35}$$

The effect can be explained as, with CSI, the system has the best channel, and without CSI the system has a mean channel which value is always lower than maximum. Only in two situations CSI is not important to transmitters:

- When channel has range unit. This is equivalent to a MIMO or SIMO system, that is to say, when one terminal (Tx or Rx) has no diversity.
- When channel matrix is orthonormal with all values equal. In this case there is no ISI therefore it's not necessary to remove them in transmission.

In all other situations, CSI knowledge will bring higher BER with regard to no CSI knowledge.

II.6. SUMMARY

In this chapter a one symbol system over flat fading channel has been described. Firstly with the optimal detector for one symbol with N MIMO channel accesses. We have seen also that error probability is controlled by the trace of channel and space-time transmitter processor product.

From error probability expression, transmitter has been designed for two cases: with or without CSI. The first case solution is to diagonalize the channel and work only over the best channel eigenmode. The processor is range unit with sharp separation between temporal and spatial processor. Temporal processor can be seen as a Spreading process and spatial processor consist on a classic beamformer that uses the associated channel eigenvector expression to maximum eigenvector.

It has been shown with brevity, despite chapter study only one symbol case, with and without CSI, that exist the possibility of provide access to other users. In case of CSI, this access can be done using less gain eigenmodes (smaller eigenvalues) or using different spreading sequences like synchronous CDMA.

If not CSI is available the processor has complete range, that is all eigenmodes are used because system doesn't know which is better. The access depends on the use of friendly process matrices for different users.

In error rate, with CSI is the maximum channel matrix eigenvalue and without CSI it can be used the mean of all eigenvalues. In this difference is the loss of not to have CSI in transmitter.

III.

III. MIMO CHANNEL CAPACITY

Miguel Ángel Lagunas, Ana I. Pérez-Neira

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III.1. INTRODUCTION

In last chapter it has been presented the transmission of one symbol over MIMO channels. The objectives, in this case, are to obtain the maximum quality in physical layer, that is to say in error probability terms. With the results our interest now is the way as the system scales the error-rate in function of number of bits or basic streams to transmit (n_S). Obviously this power scaling was independent on CSI availability because it depended on basic communications principles, specifically the way as mean energy transmitted for symbol increase, keeping a constant distance $\sqrt{E_S}$ between constellation points. Error rate expression is the following, in case of perfect CSI.

$$P_e = Q\left(\sqrt{\frac{2.E_T}{N_0}.\lambda_{\max}\left(\underline{\underline{R}}_H\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right) \tag{III.1}$$

Main aim will be to find the limits that MIMO channel has in terms of number of streams or bits for channel access. Once found it will be discussed the design of systems which work in these limits, adapting transmitter to channel.

Two concepts are fundamental. Firstly, it's the first time that spatial-temporal processor design visualizes to reader the three phases mentioned in chapter I: constellation design, power assignation and spatial processor. Secondly, the importance of spatial-temporal constellation made in the transmitter. Specifically it can be concluded that pack all streams inside one symbol, as in last chapter, isn't the best procedure won't in error rate or in information speed.

The chapter begins with a notion on mutual information in communications to continue with the analysis, in some steps, of MIMO channel capacity.

III.2. MUTUAL INFORMATION

Considering Figure III-1.- transmission system, where vector X is the transmitted signal of n_T components, matrix H is the one of MIMO channel considering flat fading of $n_R \cdot n_T$ inputs and Y is the received signal, mutual information between transmitted signal and received one is the following.

$$I\left(\underline{X},\underline{Y}\right) = \iint f\left(\underline{X},\underline{Y}\right).Ln\left[\frac{f\left(\underline{X},\underline{Y}\right)}{f\left(\underline{X}\right)\cdot f\left(\underline{Y}\right)}\right].d\underline{X}.d\underline{Y} \tag{III.2}$$

Where f(·) means probability distribution.

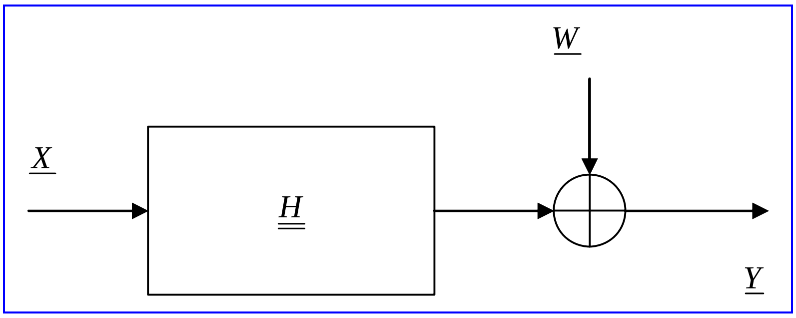

Figure III-1.- MIMO transmission system scheme.

The mutual information expression is difficult to manipulate, from distributions to physical parameters, if Gaussian distribution is not assumed for transmitted signal and noise. Obviously the supposition to have Gaussian zero-mean noise it's not far from reality, but it's not obvious than transmitted signal has also this same distribution. Only when we have a linear process or not linear temporally from original streams this supposition it's correct. In any case, the difficulty to operate with another supposition forces transmitter to work with Gaussian coding. It's also true than conclusions and design procedures derived from here have a lot of validity, even if transmitted signal distribution is not Gaussian. One more interesting thing is that it possible to prove than mutual information is maximal when distribution is Gaussian, so then an upper value is found, denominated MIMO channel capacity (C).

This way transmitted signal and noise Gaussianity are supposed, with the following covariance matrices for each signal:

$$\underline{X} = G\left(\underline{0}, \underline{\underline{Q}}\right) \quad \underline{W} = G\left(\underline{0}, \underline{\underline{R}}_0\right) \tag{III.3}$$

Note the two basic terms that compose a Gaussian distribution. In transmitted signal one, there is a term corresponding to covariance matrix inverse that doesn't depend on integration variables and another which depend on inside the exponent.

$$\Pr\left(\underline{X}\right) = \det\left(\underline{\underline{Q}}^{-1}\right).\exp\left\{-\left(\underline{X}^H.\underline{\underline{Q}}.\underline{X}\right)\right\} \tag{III.4}$$

This gives way to two terms corresponding to determinant and exponent. In this expression it has been applied that inverse of determinant is equal to determinant of the inverse and also that quotient removes all non crossed terms between transmitted and received signal.

$$C = \iint f\left(\underline{X},\underline{Y}\right).Ln\left[\frac{\det\left[\underline{\underline{Q}}\right].\det\left[\underline{\underline{R}}_{yy}\right]}{\det\begin{pmatrix}\underline{\underline{Q}} & \underline{\underline{R}}_{xy}\\ \underline{\underline{R}}_{yx} & \underline{\underline{R}}_{yy}\end{pmatrix}}\right].d\underline{X}.d\underline{Y} + \qquad \text{(III.5)}$$
$$-\iint f\left(\underline{X},\underline{Y}\right).\left[\underline{X}^{H}.\underline{\underline{R}}_{xy}.\underline{Y}+\underline{Y}^{H}.\underline{\underline{R}}_{yx}.\underline{X}\right].d\underline{X}.d\underline{Y}$$

First term finally is the logarithm of determinants quotient and second term is zero because both terms have zero mean.

$$C = Ln\left[\frac{\det\left[\underline{\underline{Q}}\right].\det\left[\underline{\underline{R}}_{yy}\right]}{\det\begin{pmatrix}\underline{\underline{Q}} & \underline{\underline{R}}_{xy}\\ \underline{\underline{R}}_{yx} & \underline{\underline{R}}_{yy}\end{pmatrix}}\right] \qquad \text{(III.6)}$$

Assuming the following property:

$$\det\begin{pmatrix}\underline{\underline{Q}} & \underline{\underline{R}}_{xy}\\ \underline{\underline{R}}_{yx} & \underline{\underline{R}}_{yy}\end{pmatrix} = \det\left[\underline{\underline{Q}}\right].\det\left[\underline{\underline{R}}_{yy} - \underline{\underline{R}}_{yx}.\underline{\underline{Q}}.\underline{\underline{R}}_{xy}\right]$$

The capacity expression can be simplified as we can see in (III.7), where, understanding determinant as an energy or power measure of their corresponding matrix, there is a relationship between received signal and the part of received signal which doesn't corresponds to transmitted signal. This SNR notion in capacity expression will be revised afterwards.

$$C = Ln\left[\frac{\det\left[\underline{\underline{R}}_{yy}\right]}{\det\left[\underline{\underline{R}}_{yy} - \underline{\underline{R}}_{yx}.\underline{\underline{Q}}.\underline{\underline{R}}_{xy}\right]}\right] \qquad \text{(III.7)}$$

As noise is used to be uncorrelated with transmitted signal, the received signal covariance and also crossed covariances can be written as follows.

$$\begin{aligned}\underline{\underline{R}}_{yy} &= \underline{\underline{H}}.\underline{\underline{Q}}.\underline{\underline{H}}^{H} + \underline{\underline{R}}_{0}\\ \underline{\underline{R}}_{xy} &= \underline{\underline{Q}}.\underline{\underline{H}}^{H}\\ \underline{\underline{R}}_{yy} &= \underline{\underline{H}}.\underline{\underline{Q}}\end{aligned} \qquad \text{(III.8)}$$

The capacity denominator is, exactly, a measure of received noise. With all it's possible to simplify even more the capacity expression, see (III.8).

$$C = Ln\left[\frac{\det\left[\underline{\underline{R}}_0 + \underline{\underline{H}}.\underline{\underline{Q}}.\underline{\underline{H}}^H\right]}{\det\left[\underline{\underline{R}}_0\right]}\right] = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{\underline{Q}}.\underline{\underline{H}}^H\right]\right) \qquad \text{(III.9)}$$

This last expression, the easiest to handle till now will be used in the rest of chapter. It's interesting to note than capacity depends only on covariance matrices of transmitted signal and noise and channel matrix. In general, except multiuser environments or when co-channel interference is present, we'll suppose white noise with N_0 energy. We will also consider, in transmitted signal, both matrices normalized to their trace, that is,

$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \left(\frac{E_T.Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}\right)}{N_0}\right).\ddot{\underline{\underline{H}}}.\ddot{\underline{\underline{Q}}}.\ddot{\underline{\underline{H}}}^H\right]\right), \qquad \text{(III.10)}$$

Where three points indicate normalization, that is, $\ddot{\underline{\underline{Q}}} = \dfrac{\underline{\underline{Q}}}{Trace\left(\underline{\underline{Q}}\right)} = \underline{\underline{Q}}/E_T$.

Finally it's interesting to write capacity in terms of receiver aperture dimension instead of transmission one. Thus the next two expressions are valid and can be used independently.

$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{\underline{Q}}.\underline{\underline{H}}^H\right]\right) = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \underline{\underline{H}}^H\underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{\underline{Q}}\right]\right) \qquad \text{(III.11)}$$

Looking the second expression, we can see that white noise suppose is not necessary if we consider than channel has inside the noise contribution.

$$\underline{\underline{H}} \rightarrow \underline{\underline{R}}_0^{-1/2}.\underline{\underline{H}} \qquad \underline{\underline{R}}_H = \underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}} \qquad \text{(III.12)}$$

This consideration is always true in all topics explained in this course. Thus, till now, always white noise will be considered, and if there isn't, then we must include their contribution as indicated in the last expression.

III.3. MIMO CHANNEL CAPACITY

MIMO channel capacity expression gives a very interesting vision about benefits, in terms of rate, offered by diversity use in one or both ends of transmission channel. Next this vision will be examined. In order to simplify the presentation and with the purpose of establish a comparation base, we will suppose than transmitter uses UPA (Uniform Power Allocation) directly to streams to transmit, that is, we will suppose than transmitter covariance matrix (normalized to transmitted signal) is equal to unity matrix divided by number of transmitter antennas n_T. This way, the capacity expression that will be used is:

$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \frac{\gamma}{n_T}\underline{\underline{H}}.\underline{\underline{H}}^H\right]\right)$$
$$with \quad \gamma = \frac{E_T.Trace\left(\underline{\underline{H}}^H.\underline{\underline{H}}\right)}{N_0} \qquad \text{(III.13)}$$

With the parameter γ, next comparison is normalized.

First reference case will be a SISO channel, that is to say without spatial diversity in both ends of transmission system. In this case, capacity will be:

$$C_{SISO} = Ln\left[1+\gamma\right] \qquad \text{(III.14)}$$

If we analyze same expression for a MISO or a SIMO channel we will obtain:

$$C_{MISO} = Ln\left[1+\frac{\gamma}{n_T}\underline{h}^H.\underline{h}\right]$$
$$C_{SIMO} = Ln\left[1+\gamma.\underline{h}^H.\underline{h}\right] \qquad \text{(III.15)}$$

Surprising conclusions arise although they must be clarified. For constant transmitted energy and channel conditions, and with regard to SISO channel, capacity decreases when transmitter antennas increase and it doesn't change when receiver antennas increase. Therefore it seems that diversity in one of the ends isn't good for capacity. There are two different meaning notes which clarify this desolating conclusion. The first one, little reasonable, is to consider if we have, let's say, four antennas, four times power is transmitted, what causes a draw between MISO and SIMO channels respecting to SISO one. Then, for the same energy by antenna and same channel in its trace:

$$C_{MISO} = C_{SIMO} = Ln\left[1+\gamma\right] \qquad \text{(III.16)}$$

This meaning isn't very correct because one advantage of use spatial diversity is to reduce environment impact, which is not to modify total energy. So it seems logical to keep (III.15) because we don't want to increase transmitted power.

Second meaning note is so much important and very reasonable. Trace normalisation is not right because four physical channels must have a trace four times greater. With this, capacities in (III.15) become (III.17) ones. Then the conclusion is that there is no profit if we increase transmitter antennas and the profit arrives when we increase receiver antennas, profit increasing logarithmically. If reader just think about it will see as much logical is this conclusion.

$$C_{MISO} = Ln\left[1+\gamma\right]$$
$$C_{SIMO} = Ln\left[1+\gamma.n_R\right] \qquad \text{(III.17)}$$

The scenario changes if both transmitter and receiver have spatial diversity. In this case, channel matrix $\underline{\underline{R}}_H$ will have range different to one and up to $\min\left(n_T, n_R\right)$ not zero eigenvalues. Now capacity, with the condition of all eigenvalues sum, will be:

$$C = \sum_{q=1}^{\min(n_T,n_R)} Ln\left[1+\frac{E_T.\sum\lambda_p}{N_0.n_T}.\lambda_q\right] \qquad \text{(III.18)}$$

Suposing all eigenvalues equal, capacity have an expression which denotes the use of diversity in both ends.

$$C = \min\left(n_T, n_R\right).Ln\left[1+\frac{E_T.\min\left(n_T, n_R\right)}{N_0.n_T}\right] \qquad \text{(III.19)}$$

The expression reveals the importance to obtain maximum range for channel matrix. In fact, to increase minimum number of antennas, the better case is equal in both ends, linearly increases link capacity, all without increase transmission bandwidth. Without question, in second end and except two-dimension polarization, any other type of diversity obtains so much improvement. This represents a scope without frontier in future communication systems.

III.4. CAPACITY WITH CSI

This section explains the design of transmitter processor to obtain the maximum possible capacity. Hereafter the expression to maximize, for a given energy. Of these expressions we'll use the last one because of presentation clarity.

$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_T} + \underline{\underline{Q}}.\underline{\underline{H}}^H\underline{\underline{R}}_0^{-1}\underline{\underline{H}}\right]\right) = Ln\left(\det\left[\underline{\underline{I}}_{n_R} + \underline{\underline{H}}^H\underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{\underline{Q}}\right]\right) =$$
$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_T} + \underline{\underline{Q}}.\underline{\underline{R}}_H\right]\right) \qquad \text{(III.20)}$$

Before continue, we'll consider that channel matrix has $\min\left(n_T, n_R\right)$ different non zero eigenvalues and also has the following SVD decomposition:

$$\underline{\underline{R}}_H = \underline{\underline{U}}_H.\underline{\underline{D}}_H.\underline{\underline{U}}_H^H \quad \text{with} \quad diag\,\underline{\underline{D}}_H = \frac{1}{N_0}\left[\lambda_{H1} \quad \dots \quad \lambda_{H\min(n_T,n_R)}\right] \qquad \text{(III.21)}$$

As we'll see, in all multi-stream MIMO channel designs exist three basic steps. The first one, let's say, structural, refers to spatial processor design. The second one refers to PA (Power Allocation) and the third and last one will be the constellation design, also said AM (Adaptive Modulation). Every design determines <u>U</u>, <u>P</u>, and <u>V</u> matrices described in first chapter. See Figure III-2.

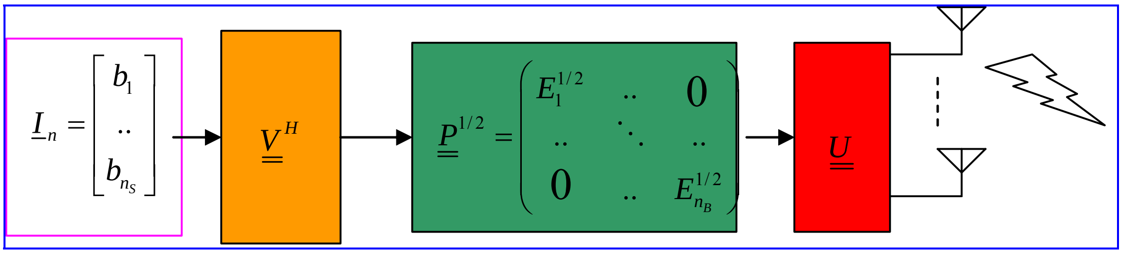

Figure III-2.- Spatial-temporal processor of MIMO transmitter

The structural design refers to spatial processor. In order to design this we must remember following property. The determinant of a positive defined matrix is always lower or equal than product of principal diagonal elements.

$$\det\left(\underline{\underline{A}}\right) \leq \prod A_{ii} \qquad \text{(III.22)}$$

Both terms are equal, that is to say determinant is maximum, when matrix is diagonal. Adapting this property to capacity maximization, determinant maximum is obtained when, by design, transmitted signal covariance matrix diagonalizes the channel. Therefore if <u>B</u> is the transmitter processor matrix, then:

$$\underline{\underline{Q}} = \underline{\underline{B}}.\underline{\underline{B}}^H \qquad \text{(III.23)}$$

As process matrix consists of three steps in Figure III-2, (III.24) will be verified.

$$\underline{\underline{Q}} = \underline{\underline{B}}.\underline{\underline{B}}^H = \left(\underline{\underline{U}}.\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H\right).\left(\underline{\underline{U}}.\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H\right)^H = \underline{\underline{U}}.\underline{\underline{P}}.\underline{\underline{U}}^H \qquad \text{(III.24)}$$

It can be seen, as a conclusion, that the transmitter spatial processor consists of channel matrix eigenvectors. That is, the processor diagonalizes the channel and reduces that to eigenmodes or channels without ISI, with a gain determined by correspondent eigenvalue.

After decide U design as:

$$\underline{\underline{U}} = \underline{\underline{U}}_H \qquad \text{(III.25)}$$

The capacity expression will be:

$$C = Ln\left(\det\left[\underline{\underline{I}}_{n_T} + \frac{1}{N_0}\underline{\underline{U}}_H.\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H.\underline{\underline{V}}.\underline{\underline{P}}^{1/2}.\underline{\underline{D}}.\underline{\underline{U}}_H^H\right]\right) =$$
$$C = Ln\left(\det\left[\underline{\underline{I}}_{\min(n_T,n_R)} + \frac{1}{N_0}\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H.\underline{\underline{V}}.\underline{\underline{P}}^{1/2}.\underline{\underline{D}}_H\right]\right) \qquad \text{(III.26)}$$

Although apparently innocuous, this step has structurally jeopardized the transmitter process. The process matrix, U, must be channel matrix eigenvectors. Otherwise, and related to constellation matrix, we must demand this to be diagonal, moreover, if we consider all power allocation realized in P matrix, the structural maximization of capacity also demands (III.27).

$$\underline{\underline{V}}^H.\underline{\underline{V}} = \underline{\underline{I}}_{n_T} \tag{III.27}$$

After these steps, the maximum capacity design is now restricted to a determinant of a diagonal matrix which contains P entries by D_H entries, with the restriction of constant transmitted energy.

$$C = Ln\left(\det\left[\underline{\underline{I}}_{\min(n_T,n_R)} + \frac{1}{N_0}\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H.\underline{\underline{V}}.\underline{\underline{P}}^{1/2}.\underline{\underline{D}}_H\right]\right) = \\ = \sum_{q=1}^{\min(n_T,n_R)} Ln\left[1+\frac{1}{N_0}z(q).\lambda_H(q)\right]\Bigg|_{MAX} \tag{III.28}$$

with the restriction $\sum_{q=1}^{\min(n_T,n_R)} z(q) = E_T$

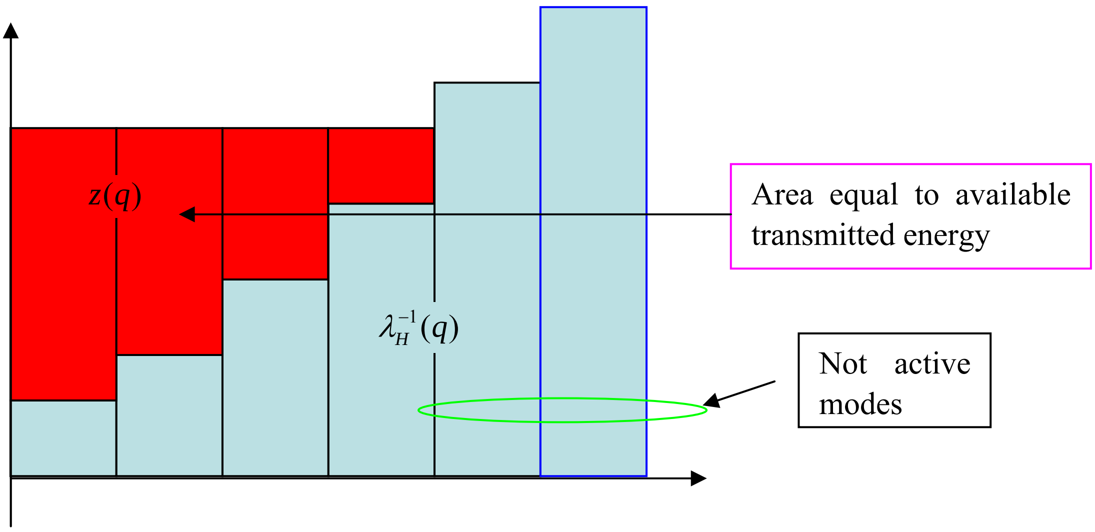

Figure III-3.- Water-Filling algorithm (optimal power distribution by eigenmode)

The solution to this problem is given by minimizing the Lagrangian function with energy restriction:

$$\Delta = \sum_{q=1}^{\min(n_T,n_R)} Ln\left[1+\frac{1}{N_0}z(q).\lambda_H(q)\right] - m1.\left(\sum_{q=1}^{\min(n_T,n_R)} z(q) = E_T\right)$$
$$\partial\Delta = 0 = \frac{1}{N_0}\frac{\lambda_H(q)}{1+z(q).\lambda_H(q)} - m1 \quad \Rightarrow \quad z(q) = \left[\mu - \frac{1}{\lambda_H(q)}\right]^+ \qquad \text{(III.29)}$$

Where $[.]^+$ means a function which takes its argument value when positive and zero when negative. The μ parameter assures the actived z(q) sum to available energy.

From founded solution, the reader can extract that all channel eigenmodes won't be used. In fact, only when available transmitted energy was high, then all modes will activate. There are two situations with an special interest. The first one, when only one mode is activated, the one corresponding to maximum eigenvector. The capacity in this case will be:

$$\mu = E_T + \frac{1}{\lambda_{H,\max}}\ ;\ z(0) = E_T$$
$$C_{BEAMFORMING} = Ln\left[1+\frac{E_T}{N_0}.\lambda_{H,\max}\right] \qquad \text{(III.30)}$$

The most interesting is that, as power matrix only has one non zero element, only one column of $\underline{\underline{U}}$ matrix will activate. That is, the transmitter processor is simply the maximum eigenvector, and structurally is only a single beamformer (see the capacity subscript index). This situation is done when next condition is verified:

$$\lambda_2 < (n_T - 1).\left[E_T + \frac{1}{\lambda_1}\right] \qquad \text{(III.31)}$$

It's important, before finish this section, to indicate, for reasonable transmitted energy values in radiocommunications (not in DSL), if MIMO channel presents moderated or low levels of correlation, the solution will be UPA (Uniform Power Allocation). Only in case of very correlated MIMO channel, with angular spread lower than eight degrees, the optimal solution is the beamforming, that is one mode activated. UPA goodness for CSI information or not will be shown in next section.

III.5. CAPACITY WITHOUT CSI IN TX

When no CSI is available, the solution shall be searched in the same way as chapter II. As we've seen, the problem will be shown as a game between channel and transmitter. In this case, the game will have multiple moves by both players. Imagine, although this is not important for the final solution, that designer makes a determinate power distribution. The channel, in its move, see this distribution and will make a reverse water-filling, that is, give more gain to the modes with less power assigned.

Then plays the designer and will make direct water-filling. Both will continue making their moves until balanced situation. It's clear that this final situation will reach when one of both arrives to uniform distribution. This will be the solution for this case, as usually. Therefore, when there is no channel knowledge, optimal distribution is the uniform one, called from now as UPA.

There exist intermediate solutions between perfect and none channel knowledge. One of them, which have low complexity of needed feedback for receiver to transmitter, consists in indicate which is the best antenna to use. This technique is called "Antenna selection". For n_T antennas, only $\log_2(n_T)$ bits are needed in return channel.

The best antenna decision is easy. If $\underline{h}_1$ is the transmitter antenna channel with index 1, then the contribution of this antenna to capacity will be (III.32), where E_T is the antenna assigned energy.

$$C(1) = Ln\left[1+\left(\underline{h}_1^H.\underline{h}_1\right)\frac{E_T}{N_0}\right] \tag{III.32}$$

The complexity of the problem increase when the two best ones are needed. Capacity for two transmitter antenna, using a diagonal matrix for transmitter energy will be (III.33) and the solution will be the use of W-F over channel norm seen by each transmitted antenna, once two best ones chosen according to their norm.

$$C(2) = Ln\left[1+\left(\underline{h}_1^H.\underline{h}_1\right)\frac{E_1}{N_0}\right] + Ln\left[1+\left(\underline{h}_2^H.\underline{h}_2\right)\frac{E_2}{N_0}\right] \tag{III.33}$$

More complex is the design in case of use the complete opening and the receiver wants to select the antenna. In this case, we must imagine that receiver has selected m antennas and wants to select or not the m+1 one. As (III.34) is verified for every vector seen by every receiver antenna of transmitter opening:

$$\underline{\underline{Q}}.\left(\underline{\underline{H}}_m^H.\underline{\underline{H}}_m + \underline{h}_{m+1}.\underline{h}_{m+1}^H\right) = \underline{\underline{Q}}.\left(\sum_{q=1}^{m}\underline{h}_q.\underline{h}_q^H + \underline{h}_{m+1}.\underline{h}_{m+1}^H\right) \tag{III.34}$$

Choosing:

$$\begin{aligned}\underline{\underline{A}} &= \underline{\underline{I}}_{n_T} + \underline{\underline{Q}}.\sum_{q=1}^{m}\underline{h}_q.\underline{h}_q^H \\ \underline{\underline{B}} &= \underline{\underline{Q}}.\left(\underline{h}_{m+1}.\underline{h}_{m+1}^H\right)\end{aligned} \tag{III.35}$$

And applying the determinant law,

$$\det\left[\underline{\underline{A}}+\underline{\underline{B}}\right] = \det\left[\underline{\underline{A}}\right].\det\left[\underline{\underline{I}}+\underline{\underline{A}}^{-1}.\underline{\underline{B}}\right] \tag{III.36}$$

We obtain:

$$C(m+1)=C(m)+Ln\left[1+\left(\underline{h}_{m+1}\cdot\left(\underline{\underline{I}}_{n_T}+\underline{\underline{Q}}.\sum_{q=1}^{m}\underline{h}_q.\underline{h}_q^H\right)^{-1}.\underline{\underline{Q}}\underline{h}_{m+1}^H\right)\right] \qquad \text{(III.37)}$$

Changing last matrices roles, that is the definition of $\underline{\underline{A}}$ and $\underline{\underline{B}}$, a new alternative expression is found in (III.38), when a new receiver antenna is added.

$$C(m+1)=Ln\left[1+\underline{h}_{m+1}.\underline{\underline{Q}}\underline{h}_{m+1}^H\right]+Ln\left(\det\left(\underline{\underline{I}}_{n_T}+\underline{\underline{Q}}_1.\sum_{q=1}^{m}\underline{h}_q.\underline{h}_q^H\right)\right)$$

where (III.38)

$$\underline{\underline{Q}}_1=\underline{\underline{Q}}.\left(\underline{\underline{I}}_{n_T}-\frac{\underline{h}_{m+1}.\underline{h}_{m+1}^H.\underline{\underline{Q}}}{1+\underline{h}_{m+1}^H.\underline{\underline{Q}}\underline{h}_{m+1}}\right)$$

This expression allows determining the improvement when a new antenna in the receiver is added, as is (III.37). Notwithstanding, the method doesn't consider redesigning the transmitter covariance matrix, what nowadays represent an still opened problem.

Inside opened problems it's interesting to note the case where only is available the CSI relative to every channel module (not the phase). This problem has very interest because phase estimation is much more unstable than module one. That's why this problem is interesting, when only the module is fiable. Only for 2x2 MIMO case the problem has a closed solution. If reader resorts to references, will check than at least for 2x2 MIMO case and in the most part of situations, it leads again to UPA. Then the most important conclusion is, together with next section, the Uniform Power Allocation covers the majority of situations except the instant and perfect channel knowledge one (normally difficult in radio environments).

III.6. ERGODIC AND OUTAGE CAPACITY

Till now, used capacity is established depending on available channel. In a frame transmission system, in each frame with corresponding channel, a different capacity is had. It's interesting to pose how the mean will be or, which is the probability that capacity was under an specific value.

The first concept brings to mean capacity measure. This is called Ergodic Capacity and it corresponds to:

$$C_{Ergodica}=E_{\underline{\underline{H}}}\left\{Ln\left[\det\left(\underline{\underline{I}}_{n_R}+\underline{\underline{H}}.\underline{\underline{Q}}.\underline{\underline{H}}^H\right)\right]\right\} \qquad \text{(III.39)}$$

The maximization of ergodic capacity over gaussian MIMO channel (Rayleigh) or any other distribuition invariant to rotation is another time unform, that is, UPA.

$$C_{ERG}(UPA) \geq C_{ERG}\left(\underline{\underline{Q}}\right) \qquad \text{(III.40)}$$

Obviously the existence of LOC between transmitter and receiver invalidate last conclusion and the solution must perform water-filling over LOS channel eigenmodes.

The use of Outage Capacity is much more interesting. This is the capacity which reach the channel with a determinate probability.

$$C_{outage} = \left\{ \text{Capacity reached when} \quad \Pr = 1-\varepsilon \right\} \qquad \text{(III.41)}$$

The maximization of outage probability is complex because the problem isn't convex. We can convert this if we use outage probability in a discrete way (0,1). If we mention $C_{objective}$ to desired capacity and t_i is defined as

$$t_i = \begin{cases} 1 \quad \text{outage} \quad \text{with} \quad \underline{\underline{H}}_i \\ 0 \quad \text{no outage} \quad \text{with} \quad \underline{\underline{H}}_i \end{cases} \qquad \text{(III.42)}$$

Then the following formulation converts the problem into convex:

$$\begin{aligned} & t_i = \in_{discrete} (0,1) \\ & Trace\left(\underline{\underline{Q}}\right) \leq E_T \\ & \rho_i = \log\left[\det\left(\underline{\underline{I}}_{n_R} + \underline{\underline{H}}_i . \underline{\underline{Q}} . \underline{\underline{H}}_i^H\right)\right] \geq C_{objective} . \left(1 - t_i\right) \\ & \min_{\underline{\underline{Q}}, t_i} \sum_{i=1}^{n} \rho_i . t_i \end{aligned} \qquad \text{(III.43)}$$

There isn't a closed solution for last problem but its formulation guarantee the existence of a single solution, which can be found with a convex optimization technique.

III.7. CONCLUSIONS

In this chapter it has been shown the capacity expression for Gaussian distribution, same for transmitted and received signal. After an asymptotic capacity values, it has been revised the design in case of CSI perfect.

The solution when transmitter knows perfectly the channel is Water-Filling, a procedure which establishes that, to obtain capacity is necessary to distribute power keeping constant the sum of this times the inverse of channel gain. The solution (W-F) is totally opposite to a equality/fairness criterion because it assigns more power to the best channel.

It is very important the role, whenever CSI is null, of Uniform Power Allocation (UPA). Even if UPA is not the optimal solution, the difference between complex methods as W-F and UPA is insignificant, all the more since if we take into account the complexity difference between UPA or another alternative.

Afterwards, procedures which work with partial CSI have been described. The most popular is the selection of antenna method, more accurately, the acceptation of progressive increase in antenna number in transmitter and/or receiver. With all, there aren't transmitter design methods to select antennas and all the procedures assume UPA in transmitter to implement the acceptation or rejection procedure.

Finally the ergodic and outage capacity have been presented. The last one, very interesting in practical situations but difficult to maximize. At any case, new procedures valuation in terms of outage capacity is the best way to value contributions in this field.

III.8. APENDIX

Linear time-invariant Gaussian channels

$$\text{SIMO: } C = \log\left(1 + \frac{P|\mathbf{h}|^2}{N_0}\right) \qquad \text{(III.44)}$$

$$\text{MISO: } C = \log\left(1 + \frac{P|\mathbf{h}|^2}{N_0}\right) \qquad \text{(III.45)}$$

But requires full CSIT

MIMO: Y=Hx+w

$$C = \sum_{i=1}^{n_{\min}} \log\left(1 + \frac{P_i^{opt}\lambda_i^2}{N_o}\right) \qquad P_i^{opt} = \left(\mu - \frac{N_o}{\lambda_i^2}\right) \qquad \text{(III.46)}$$

At high SNR Poptimal is P/K (where K is equal to the number of non-zero eigenvalues)

$$C \approx \sum_{i=1}^{K} \log\left(1 + \frac{P\lambda_i^2}{KN_o}\right) \approx k \log SNR + \sum_{i=1}^{K} \log\left(\frac{\lambda_i^2}{K}\right) \qquad \text{(III.47)}$$

More specifically, by Jensen's inequality

$$\frac{1}{K}\sum_{i=1}^{K}\log\left(1+\frac{P\lambda_i^2}{KN_o}\right)\leq\log\left(1+\frac{P}{KN_o}\frac{1}{K}\sum_{i=1}^{K}\lambda_i^2\right)$$
$$\sum_{i=1}^{K}\lambda_i^2 = Tr[\mathbf{H}\mathbf{H}^H] = \sum_{j,i=1}^{K}\left|h\right|_{i,j}^2 \qquad \text{(III.48)}$$

Therefore, the less spread out the singular values, the larger the capacity in the high SNR regime: well-conditioned channel matrices facilitate communication in the high SNR regime.

At low SNR

$$C\approx\frac{P}{N_o}\left(\max_i \lambda_i^2\right)\log_2 e \qquad \text{(III.49)}$$

IV.

IV. INSTANT DETECTION (MSE AND ZF) WITH CSI IN TRANSMISSION

Miguel Ángel Lagunas, Ana I. Pérez-Neira

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IV.1. INTRODUCTION

In chapter II ML detector was presented. This detector presents an expression which will have a great importance in this chapter. The expression was:

$$Trace\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^2\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Trace\left(\underline{\underline{W}}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right] \qquad \text{(IV.1)}$$

The interest of this expression is in the matrix that, depending on channel, appears in the first term. As the matrix which follows represents the distance of transmitted symbol to nearer ones, the first matrix must interpret as the one which conditions how signs the distance between symbols in receiver statistic.

When instead of work with a single symbol the MIMO channel transmit n_S symbol streams at the same time -at least with a value of n_T, number of transmitter antennas-, last expression changes to

$$Trace\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\underline{\tilde{I}}_0.\underline{\tilde{I}}_0^H\right|^2\right] > 2.\mathrm{Re}\left[Trace\left(\underline{W}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}.\underline{\tilde{I}}_0\right)\right] \qquad \text{(IV.2)}$$

Where $\underline{\tilde{I}}_0$ is the difference between really transmitted vector of symbols $\underline{I}_0$ and the vector $\underline{I}_e$ of symbols which will make error if last equation doesn't verify.

It's convenient to remember in the structure of transmitter process matrix, $\underline{\underline{B}}$, are involved three matrices completely different, which are:

$$\underline{\underline{B}} = \underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H \qquad \text{(IV.3)}$$

The first one ($\underline{\underline{U}}$) performs spatial processing, has dimensions (n_T, n_0), where n_0 is an integer up to n_T. Every column can be interpreted as a unity norm spatial beamformer. The number of beamformers, n_0, is still undefined exactly.

The second one ($\underline{\underline{P}}^{1/2}$) determine the energy level given to each beamformer. This matrix performs the Power Allocation over beamformers.

And the third one ($\underline{\underline{V}}$) should be understood as a modification of initial symbol constellation. It converts the initial constellation of n_S symbols to a new vector containing n_0 symbols, adapted in order to distribute each of them in their respective beamformers and with an energy level to decide.

$$\underline{\phi} = \underline{\underline{V}}^H.\underline{I}_0 \quad and \quad \underline{\tilde{\phi}} = \underline{\underline{V}}^H.\underline{\tilde{I}}_0 \qquad \text{(IV.4)}$$

Now, the transmitter scheme is detailed:

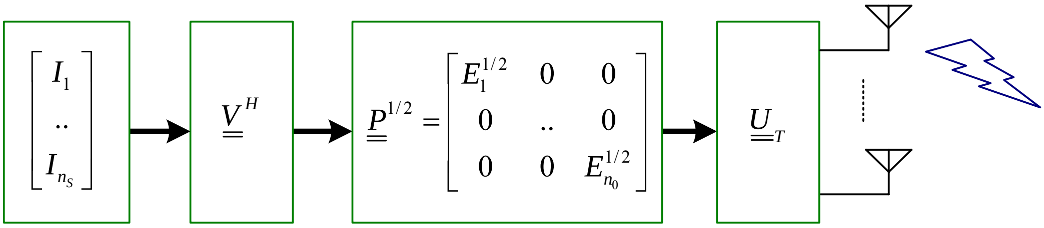

Figure IV-1.- Detailed structure of the transmitter

For the moment, and as we can see in the scheme, we put the design which modify the constellation temporally aside. In other words, the detector can be written as (IV.5), which separate perfectly the spatial and temporal transmitter process.

$$Trace\left[\left(\underline{\underline{U}}_T^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{U}}_T\right).\left|\underline{\underline{P}}^{1/2}.\underline{\underline{V}}^H.\underline{\tilde{I}}_0.\underline{\tilde{I}}_0^H.\underline{\underline{V}}.\underline{\underline{P}}^{1/2}\right|^2\right] > 2.\mathrm{Re}\left[Trace\left(\underline{W}_n^H.\underline{\underline{H}}.\underline{\underline{B}}.\underline{\tilde{I}}_0\right)\right] \qquad \text{(IV.5)}$$

If the first matrix of this expression isn't diagonal the system presents spatial inter symbol interference, i.e. spatial channels formed by each pair of antennas interferes from one another. This way, if matrix isn't diagonal the detector must be ML. Even more, proposing the low complexity of receiver and posing a MSE design (Minimum Square Error) or ZF (Zero Forcing) is clear that the best thing transmitter can do is to diagonalize the channel.

Define new constellation

Pow Alloca

As we've seen, this chapter will develop the design of this low complexity receiver, reducing the exhaustive search of ML detector, using design criterions as MSE or ZF. Note once we've decided to diagonalize, and taking care than the other matrix adjust the number of initial vectors to the number of beamformers, only rest to design each beamformer power allocation. It's also important to note when transmitter diagonalize the channel, a complete CSI situation is assumed.

In summary, the chapter will describe, under MSE or ZF criterion, the transmitter and receiver design assuming complete CSI in transmitter. Then, transmitter must diagonalize the channel and initial constellation must adapt also to this diagonalization. In a next chapter partial or null CSI (in transmitter) situation will be analyzed. The reader will note, when ML detector is left, other interfering environments in which noise and interferences matrix isn't diagonal may be described. Anyway, remember that a noise matrix different from identity can be done by changing the channel expression as shown in (IV.6) from here. If ML detector is needed, as well as this change it must whiten received vector in the receiver before introduce it into ML detector.

$$\underline{\underline{H}} \rightarrow \underline{\underline{H}}.\underline{\underline{R}}_0^{1/2} \qquad \text{(IV.6)}$$

IV.2. MSE AND ZF DETECTOR

MSE receiver design entails the minimization of difference between receiver output and what is really transmitted. The difference with regard to Wiener classic filter is that this last one is usually presented as a one channel filter and in this case the filter will be multi-channel.

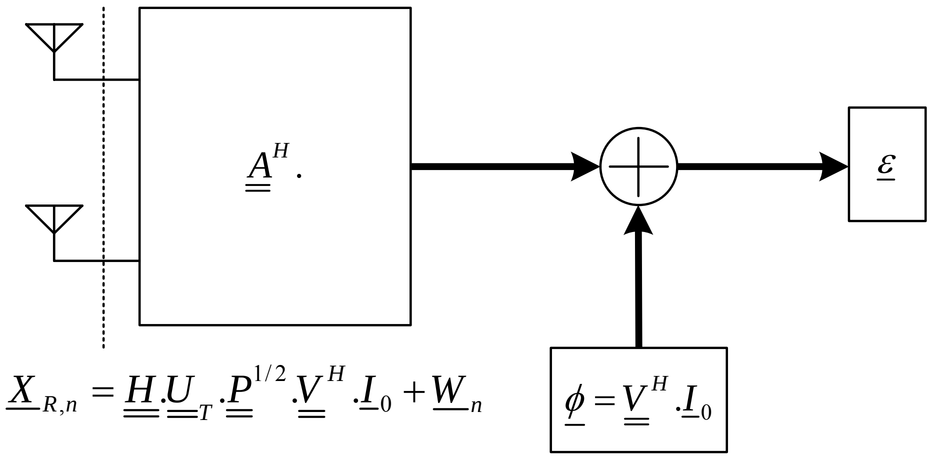

Figure IV-2.- MSE receiver (or multi-channel Wiener filter) design

This way, the design criterion will consists of a minimization of some function associated to error covariance matrix, defined as follows.

$$\underline{\underline{E}} = E\left[\underline{\varepsilon}.\underline{\varepsilon}^H\right] = E\left[\left(\underline{\underline{A}}^H \underline{X}_{Rn} - \underline{\phi}\right).\left(\underline{\underline{A}}^H \underline{X}_{Rn} - \underline{\phi}\right)^H\right] \quad \text{(IV.7)}$$

Note the differences between this formulation and the classic design mono-channel (ML detector).

$$E\left[\Im\left(\underline{I}_0\right)\right] = E\left[\left(\underline{X}_{Rn} - \underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}\underline{\phi}\right)^H.\underline{\underline{R}}_0^{-1}.\left(\underline{X}_{Rn} - \underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}\underline{\phi}\right)^H\right] \quad \text{(IV.8)}$$

Recovering expression (IV.7), if we apply inside it the received snapshot expression, we obtain:

$$\begin{aligned}\underline{\underline{E}} &= \underline{\underline{A}}^H.\left[\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right].\underline{\underline{A}} + \underline{\underline{I}}_{n_R} \\ &- \underline{\underline{A}}^H.\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2} - \underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H.\underline{\underline{A}}\end{aligned} \quad \text{(IV.9)}$$

This expression can be written alternatively which visually shows which is the optimal receiver design.

$$\underline{\underline{\mathrm{E}}} = \left[\underline{\underline{A}} - \left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right)^{-1}.\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}\right].\left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right).$$
$$\left[\underline{\underline{A}} - \left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right)^{-1}.\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}\right]^H +$$
$$\underline{\underline{I}}_{n_R} - \underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H.\left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right)^{-1}\underline{\underline{H}}.\underline{\underline{U}}.\underline{\underline{P}}^{1/2}$$
(IV.10)

In last formula, the third line doesn't depends on desired design. At the same time, the design depending terms, in two first lines, can turn to zero. This way, (IV.11) completes receiver design.

$$\underline{\underline{A}}_{MSE} = \left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right)^{-1}.\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2} \quad \text{(IV.11)}$$

Moreover, if we apply the MSE detector expression inside error covariance, we obtain the corresponding minimum value.

$$\underline{\underline{\mathrm{E}}}_{\min}^{MSE} = \underline{\underline{I}}_{n_R} - \underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H.\left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H + \underline{\underline{R}}_0\right)^{-1}\underline{\underline{H}}.\underline{\underline{U}}.\underline{\underline{P}}^{1/2} \quad \text{(IV.12)}$$

An alternative expression is found using the inverse lemma over this last one, and finally it gives way to error covariance matrix more compact expression.

$$\underline{\underline{\mathrm{E}}}_{\min}^{MSE} = \left(\underline{\underline{I}}_{n_R} + \underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H\,\underline{\underline{H}}.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}}^H\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}\right)^{-1} \quad \text{(IV.13)}$$

It's easy to check dual expressions are dual, in case of ZF, to expressions (IV.11) and (IV.13), are:

$$\begin{aligned}\underline{\underline{A}}_{ZF} &= \left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}.\underline{\underline{R}}_0^{-1}.\underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H\right)^{-1}.\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2} \\ \underline{\underline{\mathrm{E}}}_{\min}^{ZF} &= \left(\underline{\underline{H}}.\underline{\underline{U}}_T.\underline{\underline{P}}^{1/2}.\underline{\underline{R}}_0^{-1}.\underline{\underline{P}}^{1/2}.\underline{\underline{U}}_T^H.\underline{\underline{H}}^H\right)^{-1}\end{aligned} \quad \text{(IV.14)}$$

At this point, only left to go the thing more simple in theory: the transmitter design. The MSE case will be taken as a reference because it's completely similar to ZF case. From now we will use expression (IV.13). Relating to this expression, note the inclusion of constellation change matrix is avoided, and the beamformers one, as we might think, is related to MIMO channel. Therefore the transmitter spatial processor design and the optimal energy distribution for every beamformer of the spatial processor will be described.

IV.3. OPTIMUM TRANSMITTER WITH CSI

The optimum transmitter design contains two different designs. The first one corresponds to spatial processor and after, the second one consist on power assignation to every beamformer.

As in the introduction has been explained, the best aid the transmitter can give to the system is to diagonalize the channel or, in practical terms, to remove ISI, trying to obtain spatial channels without interference between them. The formality which supports this intuitive decision is, because of covariance matrix is positive defined, all values outside diagonal will increment the given function, that is to say increments MSE in every symbol (diagonal values). In conclusion, due to practical and formal reasons, any minimum pursued by means of a function defined over matrix components will be always lower if the matrix is previously diagonalized.

Thus, assuming than transmitter has CSI, its optimum design is based on identify its spatial processor with MIMO channel eigenvectors.

$$\begin{aligned}
&\textit{if} \quad \underline{\underline{R}}_H \equiv \underline{\underline{H}}.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}}^H = \underline{\underline{U}}.\underline{\underline{D}}.\underline{\underline{U}}^H \\
&\textit{with} \quad \underline{\underline{D}} = diag\left(\lambda_1,...,\lambda_{n_0}\right) \quad \textit{and} \quad n_0 \le \min\left(n_T, n_R\right) \qquad \text{(IV.15)} \\
&\textit{then} \quad \underline{\underline{U}}_T = \underline{\underline{U}}
\end{aligned}$$

Therefore, beamformers are directly the channel matrix eigenvectors and their number will be its range. Note also than maximum number of available channels must be up to the lower number of antennas used in transmitter or in the receiver, except if strong correlation is present in one or both ends.

This design, as indicated, converts MIMO channel in n_0 channels without ISI, which gain is just the corresponding eigenvalue.

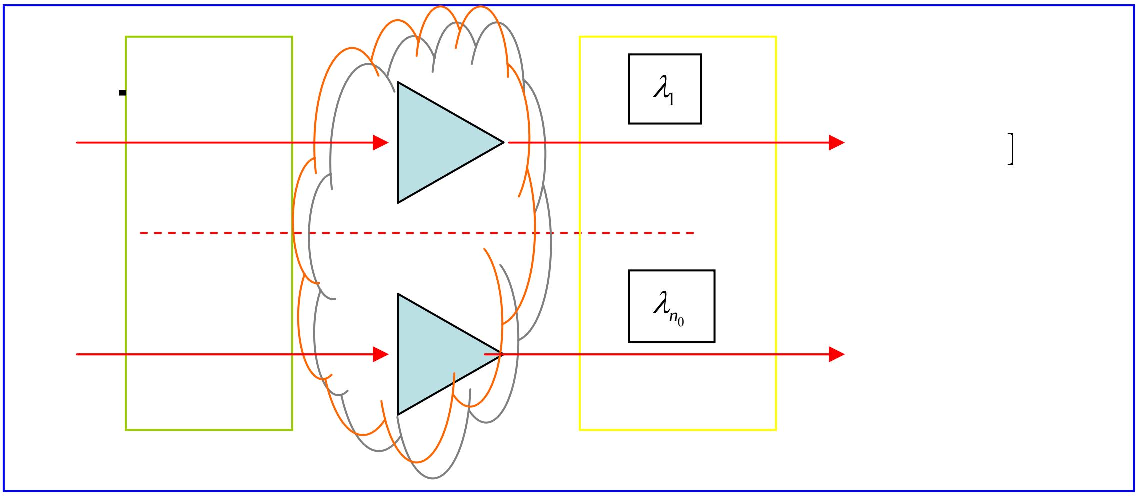

Figure IV-3.- Diagonalized MIMO channel: eigenmodes.

Now there is still left the optimal assignation of power to each eigenmode depending on their eigenvalues (eigenmode gain).

IV.4. POWER ALLOCATION

Once diagonalized, error matrix presents the following form:

$$\underline{\underline{\mathrm{E}}}_{\min}^{MSE} = \left(\underline{\underline{I}}_{n_R} + \underline{\underline{P}}^{1/2}.\underline{\underline{D}}.\underline{\underline{P}}^{1/2}\right)^{-1} = \begin{bmatrix} \frac{1}{1+\lambda(1).\beta(1)} & 0 & .. \\ 0 & .. & 0 \\ .. & 0 & \frac{1}{1+\lambda(n_0).\beta(n_0)} \end{bmatrix} \quad \text{(IV.16)}$$

It is also worth mentioning that, always and only if covariance matrix is diagonalized, SNR by stream is valid using the mono-channel relationship between MSE and SNR, as follows:

$$SNR = \frac{1}{MSE} - 1 \quad SNR(q) = \lambda(q).\beta(q) \quad \text{(IV.17)}$$

Besides SNR we can talk about error probability of every stream symbol, by means of Q(·) function or using the Chernoff level, as expressed in (IV.18), where k_0 depends on used constellation.

$$BER(q) = k_1.\exp\left(-k_0.SNR(q)\right) \quad \text{(IV.18)}$$

Here appears the bigger difference between one stream case. While in one stream, minimize the error is equivalent to maximize SNR and minimize BER, in multi-channel case this is not direct. Next it’s possible to see than similar criteria for these three objectives don’t produce equivalent designs. Not only same functions produce different designs, but in some cases the differences give surprising results.

First analyzed design, because of their intuitive behaviour, is the MSE arithmetic mean. Formally, before diagonalisation, the objective is

$$\Psi = f\left(\underline{E} = diag\left[\underline{\underline{\mathrm{E}}}\right]\right)\frac{1}{n_R}\sum_{q=1}^{n_R}\mathrm{E}_{qq} \quad \text{(IV.19)}$$

The formal diagonalisation necessity is explained.

Since function f(·), in this case arithmetic mean, is a Schur-concave function (see Appendix IV.A), then any vector $\underline{x}$ which majorize initial vector makes lower the function, that is:

$$\begin{aligned} & f(.) \quad es \quad Schur-concave \\ & if \quad \underline{E} \prec \underline{x} \quad then \\ & f(\underline{E}) \geq f(\underline{x}) \end{aligned} \tag{IV.20}$$

Because of eigenvalues always majorize diagonal values, then the eigenvalues arithmetic mean, f(·), will be lower than the original function. In consequence, it's necessary to diagonalize and then minimize. This is the most formal, not intuitive, way to prove the diagonalization necessity.

One diagonalized, the expression to minimize and its restriction (total emitted power):

$$\begin{aligned} & \left.\sum_{q=1}^{n_0} \frac{1}{1+\lambda(q).\beta(q)}\right|_{MIN} \\ & \sum_{q=1}^{n_0} \beta(q) \leq E_T \end{aligned} \tag{IV.21}$$

The solution to this problem using Lagrange (see appendix IV.B) leads to the following solution:

$$\beta(q) = \left\lfloor \mu\lambda(q)^{-1/2} - \lambda(q)^{-1} \right\rfloor^{+} \tag{IV.22}$$

Where $\lfloor . \rfloor^{+}$ is zero for a negative argument and keeps equal for a positive argument. The constant μ is used to adjust transmitted energy level.

This solution shows that there are some MIMO channel eigenmodes which is better not to use them. Only in case of high transmitted energy all modes should activate.

Now we want to minimize MSE geometric mean. Another time the function is S-C and the problem to solve is:

$$\begin{aligned} & \left.\prod_{q=1}^{n_R} \left(\frac{1}{1+\lambda(q).\beta(q)} \right)\right|_{MIN} \\ & \sum_{q=1}^{n_0} \beta(q) \leq E_T \end{aligned} \tag{IV.23}$$

Using the same procedure as in arithmetic mean case, it can be found the solution given by next equation. As reader will remember from last chapter, this is the optimal distribution in terms of capacity. Let's say, minimize MSE geometric mean is equivalent to maximize MIMO channel capacity.

$$\beta(q) = \left\lfloor \mu - \lambda(q)^{-1} \right\rfloor^{+} \tag{IV.24}$$

There is another very interesting interpretation. Geometric mean takes profit of a perfect channel (error zero) and automatically rules out all others. Another time we observe that capacity maximization is the more unequal proposal, because it gives more to the one which is better. It's important to remark that following this criterion, there can appear streams with very low error and other streams with a BER of 0.5. It's for this fact that all the streams mustn't be used and it must pack all the initial symbols inside a single symbol using $\underline{\underline{V}}$ matrix, still available if needed.

Another option is the MSE harmonic mean. In this case the objective is:

$$\sum_{q=1}^{n_0} \frac{1}{E_{qq}} \rightarrow \sum_{q=1}^{n_0} \left(1+\lambda(q).\beta(q)\right)\Big|_{MAX}$$
$$\sum_{q=1}^{n_0} \beta(q) \leq E_T \qquad \text{(IV.25)}$$

The solution of this problem consists in give all available energy to the best channel, and after all the use of a single beamformer, which is the maximum eigenvector.

The following alternative is the most, let's say, social with the eigenvectors because it pretends to minimize the maximum MSE. This function is Schur-convex, therefore its minimum is raised when the uniform solution is chosen (all MSE equal). Obtain the same MSE in all streams can be done in two different ways. The first one consists on force directly to (IV.26) without use the constellation matrix $\underline{\underline{V}}$ in the design.

$$\frac{1}{1+\lambda(q).\beta(q)} = \alpha^{-1} \quad \Rightarrow \quad \beta(q) = \frac{\alpha-1}{\lambda(q)}$$
$$\text{with} \quad \sum_{q=1}^{n_0} \beta(q) = E_T \quad \text{is obtained} \quad \beta(q) = \frac{E_T}{\sum_{p=1}^{n_0} \lambda(q)^{-1}} \frac{1}{\lambda(q)} \qquad \text{(IV.26)}$$

This is a kind of logarithmic W-F.

The second alternative involves the constellation matrix. The function of this matrix must be to distribute the n_0 channels error in the n_S original streams, that is to say it implements the equilibrium demanded by minimum solution. Then the best procedure should be selected (it is the arithmetic mean one) and design the constellation matrix in order to each original stream has the same error. Because of in a next chapter the constellation matrix design will be analyzed, since then we won't show its design.

This section will continue in a similar way (with same goals) but with SNR or BER. We'll see quickly these cases in order to compare them.

If SNR arithmetic mean is minimized, the solution consists surprisingly in gives all energy to the best channel, solution called, obviously, beamforming. Note the way as MSE arithmetic mean changes with regard to SNR mean.

If SNR geometric mean is minimized, the solution consists in UPA, exactly the same who minimize the ergodic capacity and the same who W-F converges for high energies in transmission.

The SNR harmonic mean is also Schur-convex and the solution is the same as in MSE case. In SNR minimax, just like BER minimax, the solution is the same because this function leads Schur-convex in all cases.

The BER geometric mean brings also to beamforming, and finally, the best one, the BER arithmetic mean doesn't have a closed solution. This last one is the best in terms of quality although its implementation is complex. Probably the best solution, if we look for a good performance between quality and complexity, was MSE arithmetic mean.

MSE mean	$\beta=\left[\mu\lambda^{-1/2}-\lambda^{-1}\right]^{+}$	SNR mean	Beamforming	BER mean	The best, without closed solution
MSE geometric.	W-F	SNR geometric.	UPA	BER geometric.	Beamforming
MSE harmonic.	Beamforming	SNR harmonic.	Minimax	BER harmonic.	Minimax
MSE, SNR, BER Minimax		Constellation and $\beta=\left[\mu\lambda^{-1/2}-\lambda^{-1}\right]^{+}$			

This table summarizes all power assignation possibilities available.

IV.5. CONCLUSIONS

In this chapter has been presented the MSE and ZF of the receiver and, once done, the transmitter design has been done in its three stages: in terms of Constellation Design, in terms of Power Allocation and in terms of Spatial Processor.

Once important action is to diagonalize the channel in order to change the MIMO channel with ISI to a number, less or equal, of channels isolated one from each others, without ISI and with gains which majorize the original ones. The fact that the gains were the eigenvalues in diagonalized channels, called MIMO eigenmodes, makes easy to work with ISI problem but makes higher the distance between good and bad channels. In any case the diagonalization, if CSI allows that, benefits the quality and capacity of the link.

The differences between the mono-channel processing and multi-channel one has been presented. Not only the expression of multi-channel Wiener filter is considerably different than in one channel case, even it's amazing that identical criterions in one channel situation became completely different, and sometimes opposite, in multi-channel environment. This fact has arisen when techniques to assign power to MIMO channel have been presented. Multi-stream MSE, SNR and BER criterions produce such different solutions depending on the quality parameter which apply. The final conclusion is that MSE arithmetic mean is a solution which presents a good quality/complexity agreement in front of the best results given by the BER mean in every stream.

Throughout the entire chapter it has been tried to simplify the involvement of the constellation matrix into the MIMO channel design. In ML over MIMO chapter this matrix will recover the value which corresponds to, probably the most important, transmitter design. In this chapter, the constellation matrix has appeared two times. The first time, when some eigenmodes aren't used, as well as when capacity was maximized, this matrix is the one who restructure initial streams to active modes the best way it can. Note that this way, adaptive modulation concept is implicit inside constellation matrix. The second time the matrix arises was in fairness criterions, which are minimax ones. This matrix, following again ideas related to adaptive modulation is destined to distribute initial streams over the best MIMO channel in order to have the same degradation or improvement from the beginning to the end. The constellation matrix is the key in transmitter temporal process when space-time codes (presented later on) are designing.

Probably the main and most valuable idea of this chapter is the importance, let's say, the need to diagonalize, or try to diagonalize the channel in transmitter.

IV.A. APPENDIX

Power assignation in communication channels with diversity means to use vectors with positive variables. Because of that, these variables can be put in order and the mathematical concept of majorization and minorization is interesting in the design.

Imagining a vector $\underline{x}$ of components x(i); i=1,…,N; the vector $\underline{y}$ majorize the vector $\underline{x}$, if:

$$\begin{aligned}&\sum_{q=1}^{m} x(q) \geq \sum_{q=1}^{m} y(q) \quad \forall m = 1, N-1\\&\textit{with}\\&\sum_{q=1}^{N} x(q) = \sum_{q=1}^{N} y(q)\end{aligned} \tag{IV.27}$$

And

$$\underline{x} \prec \underline{y} \tag{IV.28}$$

This notion allows to establish two sets of functions. The union of both doesn't cover all functions and their intersection isn't null, but most of functions of interest in power allocation problems belong to one of this two categories. This set of functions are called Schur concave (S-Co) and Schur convex (S-Cx). The definition is as follows:

$$\begin{aligned}&f(.) \quad \textit{Function} \quad S-Ca\\&\textit{if} \quad \underline{x} \prec \underline{y} \quad \textit{then} \quad f(\underline{x}) \geq f(\underline{y})\\&f(.) \quad \textit{Function} \quad S-Cx\\&\textit{if} \quad \underline{x} \prec \underline{y} \quad \textit{then} \quad f(\underline{x}) \leq f(\underline{y})\end{aligned} \tag{IV.29}$$

One important detail to emphasize is that a vector with the same components, UPA solution in terms of power allocation, minorize any other vector, therefore:

$$\begin{aligned}&\textit{as} \quad \underline{1} \prec \underline{x} \quad \forall \underline{x}\\&\textit{then}\\&f_{S-Ca}(\underline{1}) \geq f_{S-Ca}(\underline{x})\\&\textit{and}\\&f_{S-Cx}(\underline{1}) \leq f_{S-Cx}(\underline{x})\end{aligned} \tag{IV.30}$$

That is, the UPA solution is the maximum for S-Ca functions and the minimum for S-Cx ones. The last one is interesting in maximin problems, because is a S-Cx function. Probably in this formal basis we can look for the UPA's magic in a lot of design approaches.

One property very interesting in MIMO situations is to remember than for a given positive defined matrix, the eigenvalues vector always majorize the vector which contains the diagonal values.

IV.B. APPENDIX

In general, MIMO design face up to problems formulated as the search of extrems of a function, subjected to two types of restrictions. One of them establish frontiers to possible solutions and the other one establish equality restrictions. In formulation, described problem can be written as (IV.B.1).

$$\begin{aligned}&\min_{\underline{x}}\left(f_0(\underline{x})\right)\\&\text{with the restrictions:}\\&f_i(\underline{x})\le 0 \quad i=1,m\\&h_j(\underline{x})=0 \quad j=1,k\end{aligned} \tag{IV.31}$$

The Lagrangian of this minimization problem with restrictions is given to:

$$L\left(\underline{x},\underline{\lambda},\underline{\mu}\right)= f_0(\underline{x})+\sum_{i=1}^{m}\lambda(i).f_i(\underline{x})+\sum_{j=1}^{k}\mu(j).h_j(\underline{x}) \tag{IV.32}$$

Obvioulsy the Lagrangian function is convex respecting to multipliers.

The idea is, instead of search minimum of function with restrictions, to search the minimum of Lagrangian. If we define function g(·,·) called dual function as indicate (IV.B.3) it can be established (IV.B.4).

$$g\left(\underline{\lambda},\underline{\mu}\right)=\inf_{\underline{x}}\left[L\left(\underline{x},\underline{\lambda},\underline{\mu}\right)\right] \tag{IV.33}$$

$$f_0(\underline{x})\ge L\left(\underline{x},\underline{\lambda},\underline{\mu}\right)\ge g\left(\underline{\lambda},\underline{\mu}\right) \tag{IV.34}$$

The minimum of original problem is therefore delimited by the maximum of dual function.

$$\min_{\underline{x}}\left(f_0(\underline{x})\right)\ge \max_{\underline{\lambda},\underline{\mu}} g\left(\underline{\lambda},\underline{\mu}\right) \tag{IV.35}$$

Then, searching for the maximum of dual function in dual variables or multipliers we shell obtain a minimum level of original function. The difference between both values, the duality gap, is zero if KKT conditions are verified which are:

$$\begin{aligned}
&\lambda(i).f_i(\underline{x}_{opt}) = 0 \\
&h_j(\underline{x}_{opt}) = 0 \\
&\lambda(i) \geq 0 \\
&\nabla_{\underline{x},\underline{x}=\underline{x}_{opt}} L = \underline{0}
\end{aligned} \qquad \text{(IV.36)}$$

V.

V. MIMO OVER FREQUENCY SELECTIVE CHANNELS

Miguel Ángel Lagunas, Ana I. Pérez-Neira

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V.1. INTRODUCTION

In all precedent chapters an assumption has been done: the flat fading channel. This hypothesis is valid if a narrow bandwidth is used or the transport system selects total bandwidth in lower ones, in which the response is almost constant. The most popular system which works following that is the multicarrier one, in its generalized version, OFDM. For these systems all things exposed since chapter 1 are valid. Despite all this, there exist new transport systems as the UWB system, in which the channel is whole considered. In these systems, MIMO channel is frequency selective and also the spatial processing, both in transmission/reception must be also like this.

Model used in transmitter and receiver. All signals will be characterize in frequency, by means of their Fourier Transform. This way, input signal I(f) is shown in (V.1), where T indicates the separation between symbols to transmit.

$$I(f) = \sum i(n).\exp(-j2\pi nTf) \qquad \text{(V.1)}$$

This signal passes through transmitter opening by means of a wide band beamformer, see

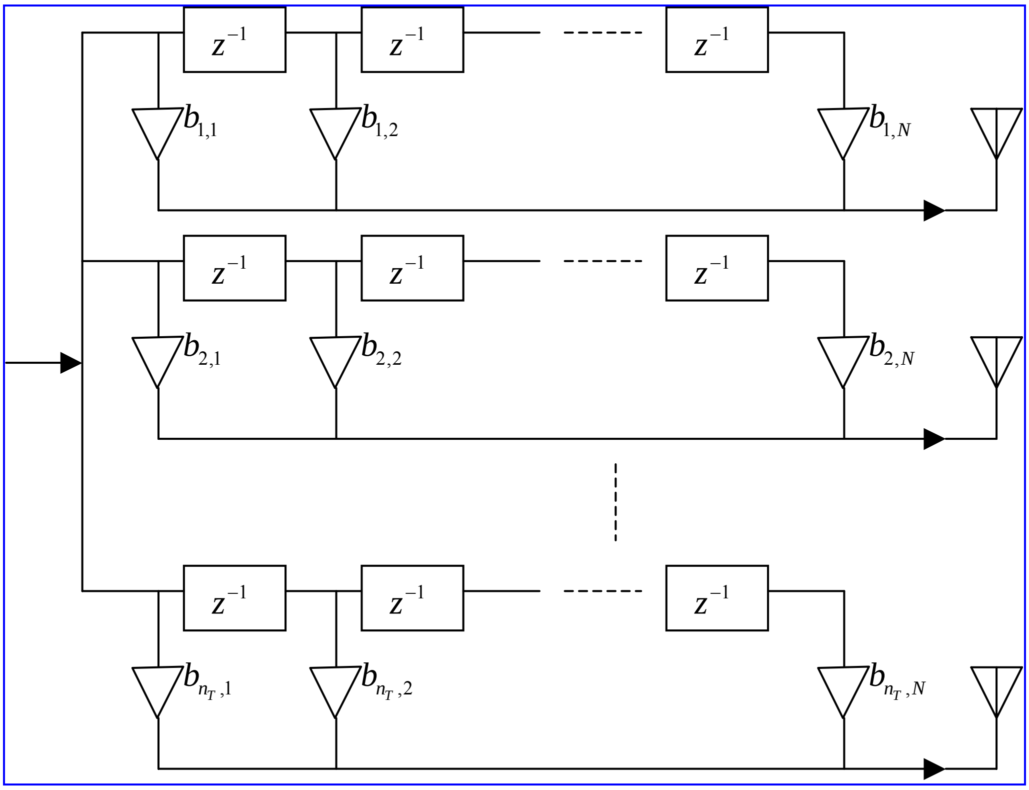


Figure V-1.- Scheme of spatial-temporal processor for frequency selective channel.

The frequency response of this beamformer and transmitted signal vector is given by (V.2).

$$\begin{aligned} \underline{B}(f) &= \sum_{n=0}^{N-1} \underline{b}_n.\exp(-j2\pi nfT) \\ \underline{X}_T(f) &= \underline{B}(f).I(f) \end{aligned} \qquad (V.2)$$

As well for received signal we obtain:

$$\underline{X}_D(f) = \underline{A}^H(f).\underline{X}_R(f) = \underline{A}^H(f).\left[\underline{\underline{H}}(f).\underline{B}(f).I(f) + \underline{W}(f)\right] \quad (V.3)$$

This formulation will be used to analyze different detectors and their quality.

V.2. TRANSMITTER DESIGN WITH CSI

V.2.1 ZF DETECTOR

Even for non selective channel, firstly the receiver is designed and then, assuming CSI availability, the transmitter.

The condition to be a ZF will be (V.4). These must verify for every frequency inside working bandwidth.

$$\underline{A}^H(f).\underline{\underline{H}}(f).\underline{B}(f) = 1 \quad \forall f \subset B_T \qquad (V.4)$$

With this consideration, the SNR in detection is given by

$$SNR = \frac{B_T}{N_0}.\frac{1}{\int \underline{A}^H(f).\underline{A}(f).df} \qquad (V.5)$$

In order to maximize SNR the following inequality will be used:

$$\begin{aligned} &\left|\underline{A}^H(f).\underline{\underline{H}}(f)\underline{B}(f)\right|^2 = 1 \leq \\ &\leq \left(\underline{A}^H(f).\underline{A}(f)\right)\left(\underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f)\right) \\ &\text{or equivalently} \\ &\left(\underline{A}^H(f).\underline{A}(f)\right) \geq \frac{1}{\left(\underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f)\right)} \end{aligned} \qquad (V.6)$$

This way, the maximum SNR for the receiver is:

$$SNR = \frac{B_T}{N_0} \cdot \frac{1}{\int \frac{1}{\left(\underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f) \right)} .df} \tag{V.7}$$

The optimum receiver is obtained forcing the equality condition inside the inequality (5.6). It's easy to check that equality condition is verified whenever (5.8) verifies.

$$\underline{A}_{ZF}(f) = \frac{\underline{\underline{H}}(f)\underline{B}(f)}{\left(\underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f) \right)} \tag{V.8}$$

It remains the SNR maximization with the appropriate transmitter design. Maximize SNR is equivalent to minimize its denominator integral. These integral is minimal if it's maximized for all frequency margin, the denominator of its integrand. In short, and with transmitted energy restriction, the design can be formulated as:

$$\begin{aligned} &\int \underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f).df \Big|_{MAX} \\ &\int \underline{B}^H(f).\underline{B}(f).df = E_T \end{aligned} \tag{V.9}$$

If we bear in mind the following inequality, already used in case of non selective channel:

$$\underline{B}^H(f).\underline{\underline{H}}^H(f)\underline{\underline{H}}(f)\underline{B}(f) \le \lambda_{MAX}\left(\underline{\underline{H}}^H(f)\underline{\underline{H}}(f) \right)\underline{B}^H(f).\underline{B}(f) = \lambda_{MAX}(f)\left|\underline{B}(f)\right|^2 \tag{V.10}$$

What produces maximum SNR for a ZF structure:

$$SNR_{ZF} = \frac{B_T}{N_0} \cdot \frac{1}{\int \frac{df}{\left(\lambda_{MAX}(f).\left|\underline{B}\right|^2 \right)}} \tag{V.11}$$

The transmitter design which obtains last SNR value is given by maximum channel matrix eigenvector multiplied by a frequency depending constant.

$$\underline{B}(f) = \underline{e}_{MAX}(f).\beta(f) \tag{V.12}$$

This constant is obtained from the restriction of transmitted energy and it results,

$$\left|\beta(f)\right|^2 = \frac{1}{\lambda_{MAX}^{1/2}(f)} \cdot \frac{E_T}{\int \frac{df}{\lambda_{MAX}^{1/2}(f)}} \tag{V.13}$$

And finally the maximum SNR value for ZF is:

$$SNR_{ZF} = \frac{E_T}{N_0} \cdot \frac{B_T}{\left| \int \frac{df}{\lambda_{MAX}^{1/2}(f)} \right|^2} \qquad \text{(V.14)}$$

Before next sub-section it's interesting to note that constant in (V.13) is basically the power loading that is, the power distribution over all frequencies inside working bandwidth. Note also than the sum of energy level and gain channel (maximum eigenvalue) in logarithmic scale, will remain constant.

$$10.\log\left[|\beta|^2\right] + 5.\log\left[\lambda_{\max}\right] = \text{ct.} \qquad \text{(V.15)}$$

As it was predictable system assigns more power to worst channel. This is a discouraging characteristic because in the most part of applications these systems must follow a mask of maximum levels transmitted in frequency. As we can see in Figure V-2, not only the best modes use less power but, moreover, if total available power is low these modes can be not used.

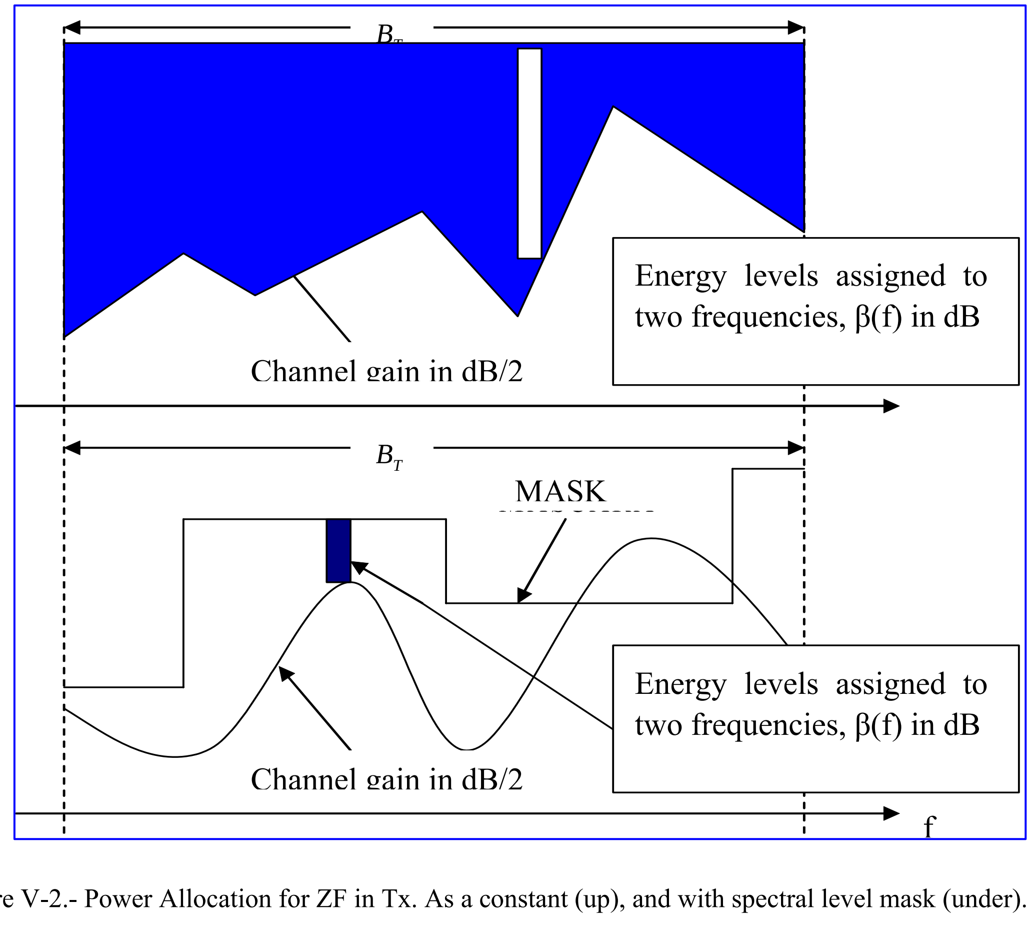


Figure V-2.- Power Allocation for ZF in Tx. As a constant (up), and with spectral level mask (under).

In next sub-section is shown the improvement of transmitted power management in case of MSE receiver.

V.2.2 MSE DETECTOR

Its main goal is to minimize frequency error between detected signal and transmitted symbol. This receiver scheme is sketched in Figure V-3.

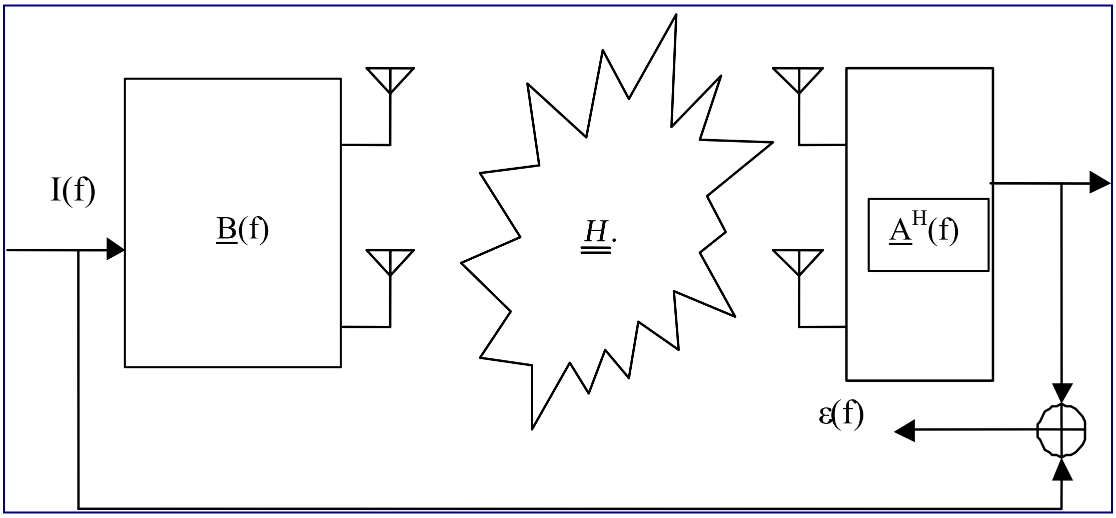


Figure V-3.- MSE receiver with reference signal and error signal (expressed in frequency).

Error expression is

$$\varepsilon(f) = I(f) - \tilde{I}(f) = \left(\underline{A}^H.\underline{\underline{H}}.\underline{B} - 1\right).I(f) + \underline{A}^H.\underline{W}(f) \qquad \text{(V.16)}$$

In order to obtain the optimal receiver expression, it's enough to derive and make the result zero. Equivalently it's also enough to force error signal and detector output signal to be independent, this is also known as orthogonality principle. The result for a receiver is (V.17) and the expression of square error is (V.18).

$$\begin{aligned} \underline{A} &= \left(\underline{\underline{R}}_0 + \underline{\underline{H}}.\underline{B}.\underline{B}^H.\underline{\underline{H}}^H\right)^{-1}.\underline{\underline{H}}.\underline{B} = \underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{B}.\left[1 - \frac{\rho}{1+\rho}\right] = \\ &= \rho(f).\underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{B} \qquad \text{with} \qquad \rho(f) = \underline{B}^H.\underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}\underline{\underline{H}}.\underline{B} \end{aligned} \qquad \text{(V.17)}$$

$$E\left(\left|\varepsilon(f)\right|^2_{MIN}\right) = \frac{1}{1+\rho(f)} \qquad \text{(V.18)}$$

Once the minimum error in every frequency is obtained, SNR is easy to calculate (V.19) if the orthogonality between error and output signal is borne in mind.

$$SNR(f) = \frac{1}{E\left(\left|\varepsilon(f)\right|^2\right)} - 1 = \rho(f) \qquad \text{(V.19)}$$

At this point transmitter design can be started. The main goal of transmitter is to maximize SNR, subject to the existence of a maximum level of transmitted energy density. Note that this design includes the idea that in these systems can exist a frequency mask M(f) which limit maximum density value. With all, the design consists of:

$$\left.\underline{B}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{H}}.\underline{B}\right|_{MAX} \qquad \underline{B}^{H}.\underline{B} \le M(f) \tag{V.20}$$

Last problem solution is, the transmitter beamformer must fit in with maximum eigenvector of channel matrix with a frequency function which square module verifies the mask. The solution will be

$$\underline{B}(f) = \underline{e}_{\max}(f).\beta^{1/2}(f) \tag{V.21}$$

Power allocation design for this problem is formulated by (V.22). Note that without mask restriction, the solution is such pathologic because advises to put all energy in the best channel, converting the system from wide band to narrow band.

$$\left.\int \beta(f).\lambda_{\max}(f).df\right|_{MAX} \qquad \int \beta(f).df = E_T \qquad \beta(f) \le M(f) \quad \forall f \subset B_T \tag{V.22}$$

The mask inclusion determines a such sophisticated method for make the energy allocation. Basically the solution consists of fill with energy up to the mask but always the first ones the best frequencies.

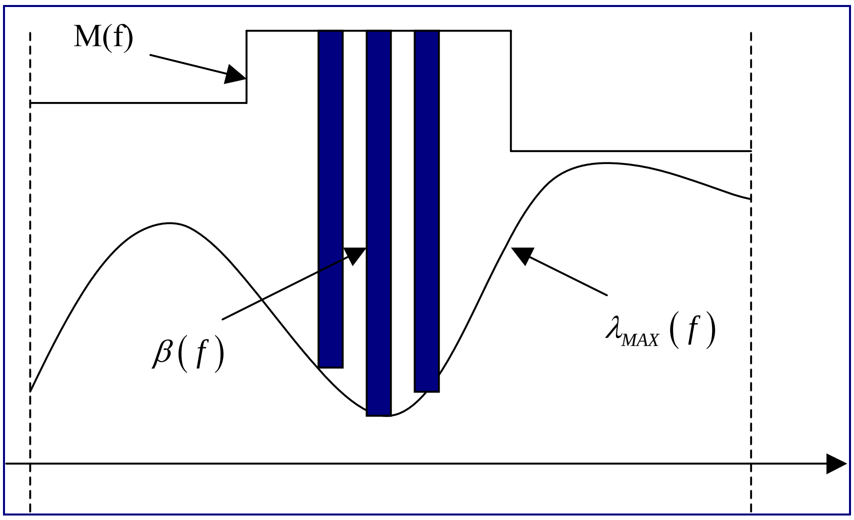


Figure V-4.- Power Allocation for MSE system.

Note the differences between this and ZF case. Whereas in ZF the first frequencies to fill were the worst ones, in MSE the beginning is opposite, that is with the best ones. Another difference is that in MSE detector, the receiver is always factible except when null values of channel in frequency are present, as in mono-channel case.

V.2.3 SEQUENCES DETECTOR

In this case the architecture is quite similar to MSE detector but in sequences detector there are two distinguished parts: The receiver processor and the desired impulsional response generation. The scheme is sketched in Figure V-5.

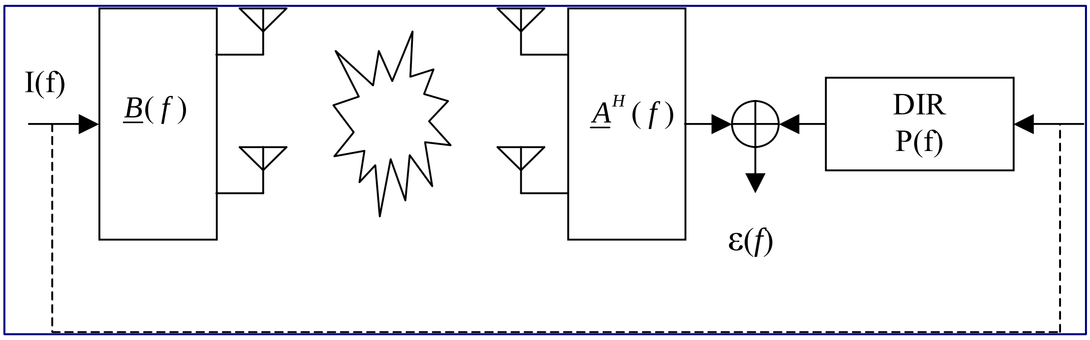


Figure V-5.- Scheme of a wide band MIMO system with sequences detector

System design is quite similar to MSE one. There are two basic differences. The first one, the DIR design takes directly to simulate signal path from transmitter to add block. Therefore DIR is equal to accumulated response of transmitter, channel and receiver.

$$P(f)=\underline{A}^H(f).\underline{\underline{H}}(f).\underline{B}(f) \tag{V.23}$$

The second difference is related to optimal character of this receiver. In order that receiver be optimal it's necessary to impose that receiver process doesn't change white character of noise in reception, that is to say it must be a frequency all-pass. The condition is,

$$\underline{A}^H(f).\underline{\underline{R}}_0(f).\underline{A}(f)=N_0 \quad \forall f \tag{V.24}$$

If receiver processor mustn't change noise, we could think that the best solution is not to include it in the design. But when interference is present, the receiver processor must remove them before decision by ML of received signal.

With these conditions SNR is given by

$$SNR=\frac{\int\left|\underline{A}^H.\underline{\underline{H}}.\underline{B}\right|^2.df}{\int\underline{A}^H.\underline{\underline{R}}_0.\underline{A}df} \tag{V.25}$$

Its maximization is identical as the other detectors but with (V.24) restriction. Proceeding in traditional way the result is the following:

$$SNR = \frac{E_T \int \left| \underline{A}^H . \underline{\underline{H}} . \underline{B} \right|^2 .df}{\int \underline{A}^H . \underline{\underline{R}}_0 . \underline{A} df} \leq \frac{E_T \int \left(\underline{A}^H . \underline{\underline{R}}_0 . \underline{A} \right) \left[\underline{B}^H \underline{\underline{H}}^H . \underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{B} \right] .df}{\int \underline{A}^H . \underline{\underline{R}}_0 . \underline{A} df} =$$
$$= \frac{E_T}{B_T} . \int \left[\underline{B}^H \underline{\underline{H}}^H . \underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{B} \right] .df \qquad (V.26)$$

The receiver to reach this SNR maximum is (V.27), where if we don't take into account the square root, the denominator is quite similar as optimal one in ZF case.

$$\underline{A}(f) = \frac{\underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{B}}{\left(\underline{B}^H . \underline{\underline{H}}^H . \underline{\underline{R}}_0^{-1} . \underline{\underline{H}} . \underline{B} \right)^{1/2}} \qquad (V.27)$$

Receiver optimization leads to the same situation as MSE case, that is to say the use of maximum eigenvector in every frequency and a power assignation which must begin by the best channel till reach the corresponding mask level.

In next section, an evaluation of these wide band detection procedures will be done, fundamentally ZF and MLSE (sequence detector).

V.3. EVALUATION AND COMPARISON

It's interesting to compare these three solutions. All of them assume perfect CSI availability because use the maximum eigenvector as beamformer and only change in their power distribution, in frequency. A comparison of ZF and MLSE will be done, regarding to UPA case, in frequency. From this comparison we will extract the great interest of UPA in practical situations.

ZF solution for power distributions and the corresponding ZF SNR distribution is shown in the following two equations.

$$\left| \beta(f) \right|_{ZF}^2 = \frac{1}{\lambda_{MAX}^{1/2}(f)} . \frac{E_T}{\int \frac{df}{\lambda_{MAX}^{1/2}(f)}}$$
$$SNR_{ZF} = \frac{E_T}{N_0} . \frac{B_T}{\left| \int \frac{df}{\lambda_{MAX}^{1/2}(f)} \right|^2} \qquad (V.28)$$

And when UPA is used the SNR is:

$$SNR_{ZF}^{UPA} = \frac{B_T}{N_0} . \frac{1}{\int \frac{df}{\left(\lambda_{MAX}(f) . \left| \underline{B} \right|^2 \right)}} = \frac{E_T}{N_0} . \frac{1}{\int \frac{df}{\lambda_{MAX}(f)}} \qquad (V.29)$$

And, if (V.30) is verified, their comparison is easy, when it's evident that is such flat channels in interesting bandwidth, UPA is a good solution, with a quality up to the optimal case.

$$\left(\int \frac{df}{\lambda_{MAX}(f)}\right).\left(\int 1.df\right) = B_T.\int \frac{df}{\lambda_{MAX}(f)} \geq \left|\int \frac{df}{\lambda_{\max}^{1/2}(f)}\right|^2 \qquad \text{(V.30)}$$
$$SNR_{ZF}^{Opt} \geq SNR_{ZF}^{UPA}$$

Evenly, the SNR for UPA solution in MSLE case is:

$$SNR_{MLSE}^{UPA} = \frac{1}{B_T}.\int \left[\underline{B}^H \underline{\underline{H}}^H .\underline{\underline{R}}_0^{-1}.\underline{\underline{H}}.\underline{B}\right].df = \frac{E_T}{B_T^2}\int \lambda_{\max}(f).df \qquad \text{(V.31)}$$

In order to compare this last SNR we will use the following inequality:

$$\left(\int \frac{df}{\lambda_{MAX}(f)}\right).\left(\int \lambda_{MAX}(f).df\right) \geq \left|\int df\right|^2 = B_T^2 \qquad \text{(V.32)}$$

From it can be concluded the UPA superiority over MLSE than over ZF, as it was logical to suppose.

$$SNR_{MLSE}^{Opt} \geq SNR_{MLSE}^{UPA} \qquad \text{(V.33)}$$

On the other hand, as (V.35) is verified, it's easy to check by means of (V.35) that UPA over MLSE is higher than the optimal power allocation over ZF.

$$\int \frac{df}{\lambda_{MAX}(f)} < \left|\int \frac{df}{\lambda_{\max}^{1/2}(f)}\right|^2 \qquad \text{(V.34)}$$

$$\frac{B_T}{\left|\int \frac{df}{\lambda_{\max}^{1/2}(f)}\right|^2} \leq \frac{B_T}{\int \frac{df}{\lambda_{\max}(f)}} =$$
$$\frac{B_T.\int \lambda_{\max}(f)df}{\int \frac{df}{\lambda_{\max}(f)}.\int \lambda_{\max}(f)df} \leq \frac{1}{B_T}\int \lambda_{\max}(f)df \qquad \text{(V.35)}$$

$$SNR_{ZF}^{Opt} \leq B_T.SNR_{MLSE}^{UPA} \qquad \text{(V.36)}$$

In next section both possibilities will be analyzed in a MISO channel environment.

V.4. FREQUENCY SELECTIVE MISO CHANNEL

The case where only transmitter has spatial diversity is detailed because of their practical interest. Assuming that frequency response of MISO channel is given by $\underline{H}^H(f)$, then the maximum eigenvector to use as a beamformer is the channel vector normalized whereas the eigenvalue associated is, directly, the square module of channel vector.

$$\underline{e}_{\max} = \frac{\underline{H}(f)}{|\underline{H}|} \quad y \quad \lambda_{\max}(f) = |\underline{H}|^2 \tag{V.37}$$

With regard to power distribution, in ZF it will be:

$$|\beta(f)|^2 = \frac{1}{|\underline{H}(f)|} \cdot \frac{E_T}{\int \frac{df}{|\underline{H}(f)|}} \tag{V.38}$$

And regarding the receiver, it is simply reduced to:

$$\underline{A}(f) = \frac{\underline{H}^H(f)\underline{B}(f)}{\left(\underline{B}^H(f).\underline{H}(f)\underline{H}^H(f)\underline{B}(f)\right)} = \frac{1}{|\underline{H}|} \tag{V.39}$$

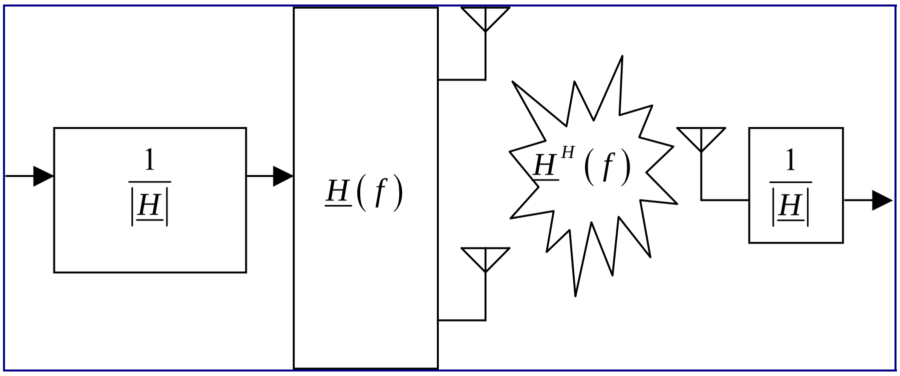


Figure V-6.- ZF design in a frequency selective MISO channel

If the channel module, denominator in (V.37), is grouped with power distribution, both terminal filters result identical. Moreover the space-time processor becomes simply an adaptive filter. Thus, in filters implementation, each FIR of every transmitter antenna consists of the antenna impulsional response, but in inverse temporal order and changing the quadrature component sign.

This scheme is exemplified in Figure V-6. Filters can be designed with linear phase using a central prediction procedure over the coefficients derived from channel response IDFT.

In next figure is presented a BPSK transmission over frequency selective MISO channel.

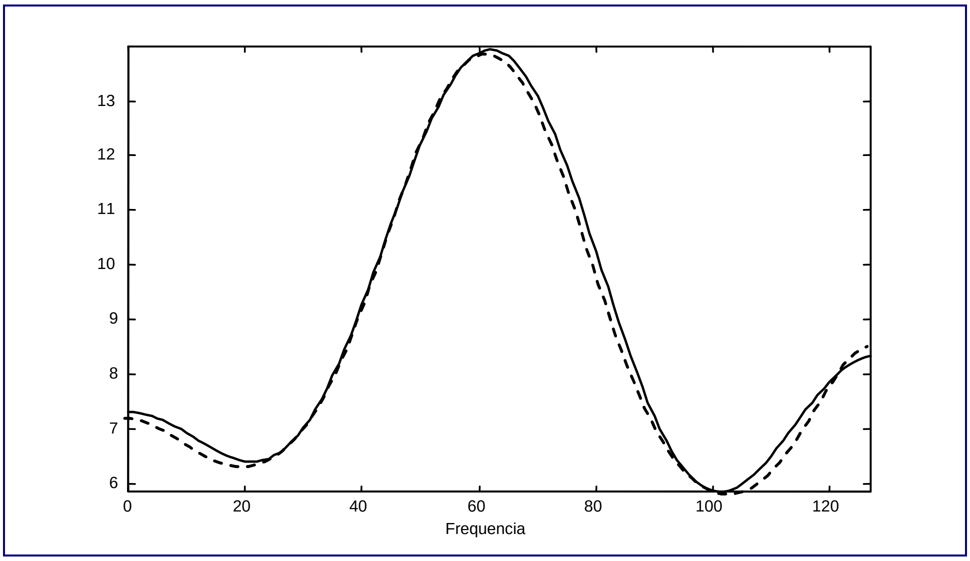


Figure V-7.- Inverse channel response (---) and 7 coeff. FIR (linear phase) approximation (—)

Next constellations transmitted by each channel antenna, the global response, ideally flat, and received constellation are presented.

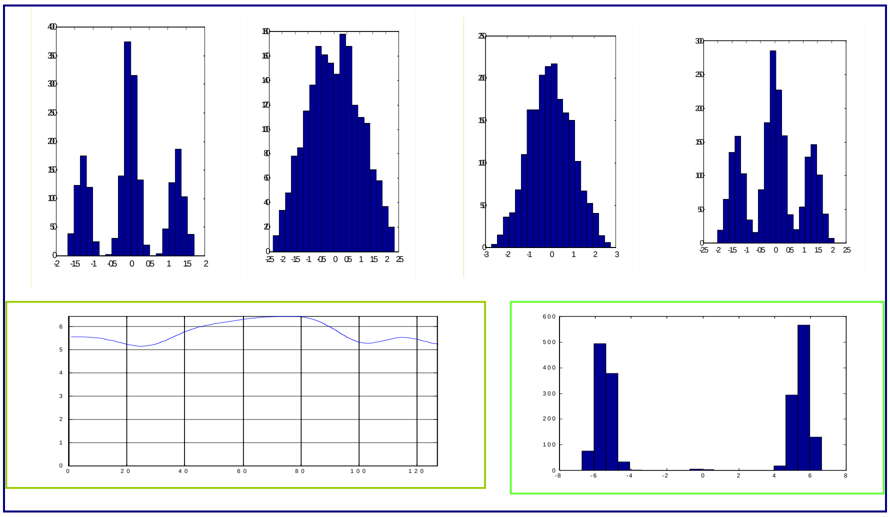


Figure V-8.- Transmitted constellations (up). Global frequency response and received constellation (down).

Such very interesting characteristic of MISO system over selective channels is its immediate implementation, only to copy every antenna response (inverse) and conjugating their coefficients. This simple structure is very robust in case of CSI errors. It's clear, very different the situation is when we must study how channel estimation errors impact on its eigenvector (MIMO case) than do it directly as in MISO case.

To emphasize the robustness, suppose than MISO channel presents the following matrix of impulsional responses of every paths from transmission antennas to receiving ones. Receiver transmits CSI as a dynamic range of 2 and then transmits normalized coefficients.

$$\underline{\underline{h}} = \begin{bmatrix} 0.5 & 1 & 0.3 & 0.1 & 0 \\ 2 & 1.5 & 0.7 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.7 & 0.6 & 0.03 \\ 0.2 & 0.3 & 0.4 & 0.4 & 0.1 \end{bmatrix} \quad \text{(V.40)}$$

Complete transmitter processor will have next figure architecture. Note that all complexity is implemented on transmitter at the expense of increase noise in receiver. It's for this that both filters are grouped in one with double power.

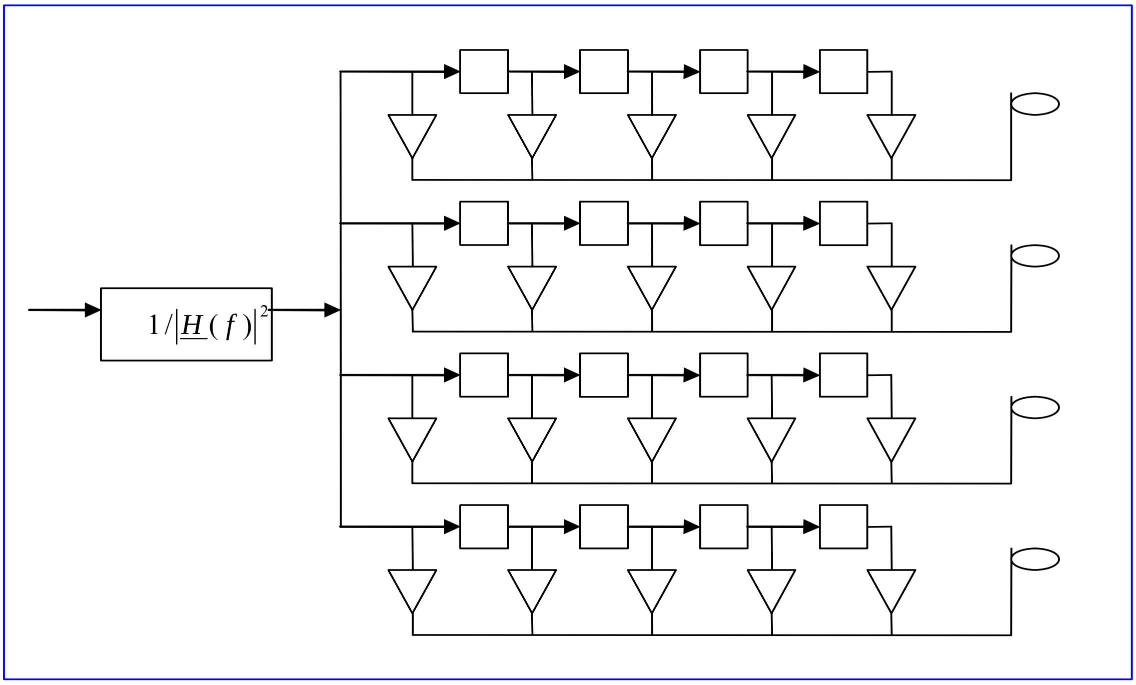


Figure V-9.- Complete processor for order 3 ISI in every channel

This channel with optimum terminal filter in transmitter was the one used in Figure V-7 and Figure V-8.

Suppose, because of robustness reasons or for reduce complexity, that only are available the two greater channel coefficients (for every channel) and only is possible assign a bit to represent its sign. The processor matrix is reduced to

$$\underline{\underline{h}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \text{(V.41)}$$

The transmitter processor has too less operations, but also its architecture has reduced considerably. Then, although this example hasn't much practical value (real coefficients), next figure shows this case, which one can be compared with previous figures ones. Differences are minimal if we take into account the huge complexity reduction.

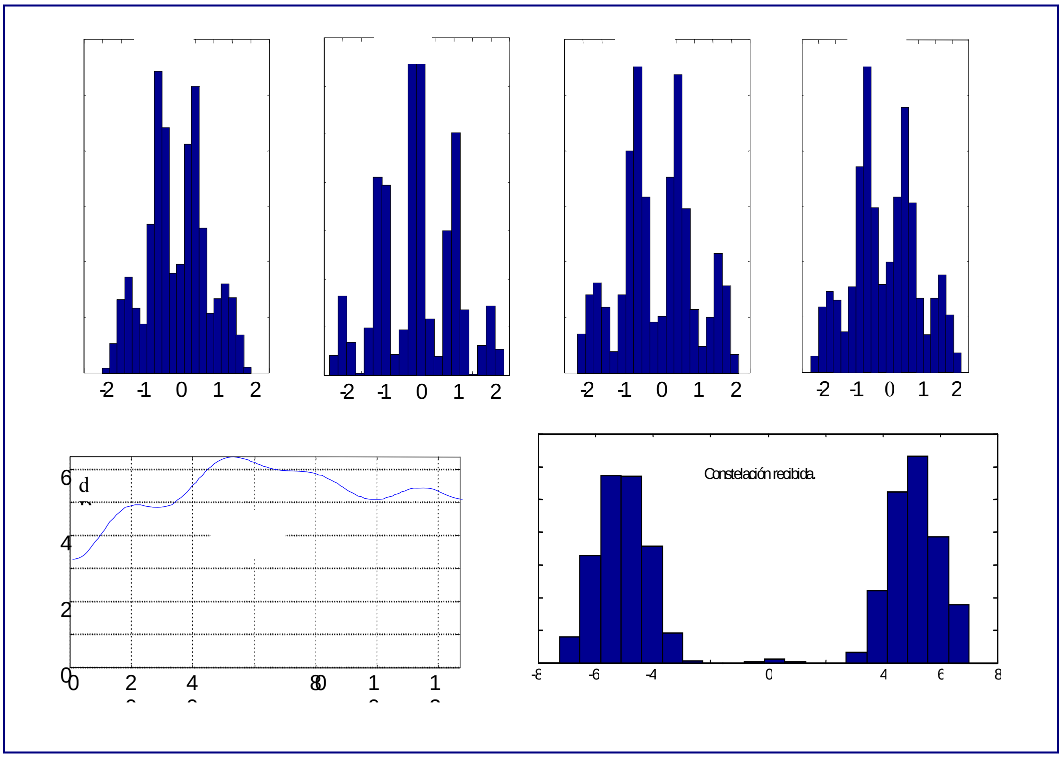


Figure V-10.- Similar results to Figure V-8 ones, after reducing complexity of coeffiecients (ruling less significatives out and reducing quantification bits)

Anyway, even though a detailed study is always necessary, MISO channel presents viable designs, with FIR structures for their frequency designs, very simple ones and with higher robustness on, for example, CSI errors. It's also easy to characterize them because maximum eigenvector is fit in with channel.

We should stress that, as all design is done in frequency domain, if we would want to traduce them to temporal level in order to implement them, the synthesis of FIR linear phase filters is needed, even in transmission than in reception. In MISO case this step is not a big problematic situation.

V.5. DESIGN WITHOUT CSI

In the design without CSI over a selective channel, matrices with complete range are needed (instead of range one election).

Basically the use of complete range in space-time processors imply the following equations for transmitted, received and detected signal:

$$\underline{\underline{X}}_{T,n} = \sum_q \underline{\underline{B}}_q . s(n-q)$$
$$\underline{\underline{X}}_{R,n} = \sum_r \underline{\underline{H}}_r . \underline{\underline{X}}_{T,n-r} = \sum_r \underline{\underline{H}}_r . \sum_q \underline{\underline{B}}_q . s(n-q-r) + \underline{\underline{W}}_n$$
$$\underline{\underline{X}}_{D,n} = \sum_p \underline{\underline{A}}_p^H . \underline{\underline{X}}_{R,n-p} + \sum_p \underline{\underline{A}}_p^H . \underline{\underline{W}}_{n-p} = \sum_p \underline{\underline{A}}_p^H . \sum_r \underline{\underline{H}}_r . \underline{\underline{X}}_{T,n-r-p} + \sum_p \underline{\underline{A}}_p^H . \underline{\underline{W}}_{n-p} =$$
$$= \sum_p \underline{\underline{A}}_p^H . \sum_r \underline{\underline{H}}_r . \sum_q \underline{\underline{B}}_q s(n-p-r-q) + \sum_p \underline{\underline{A}}_p^H . \underline{\underline{W}}_{n-p} \qquad \text{(V.42)}$$

Using the sequence detector, SNR expression, similar to the expression obtained when SNR is directly managed (for instant detection), becomes:

$$SNR = \frac{1}{N_0} . \int Trace\left[\underline{\underline{B}}^H(f) . \underline{\underline{R}}_H(f) . \underline{\underline{B}}(f) \right] . df$$
$$\text{with} \quad \underline{\underline{R}}_H(f) = \underline{\underline{H}}^H(f) . \underline{\underline{R}}_0^{-1}(f) . \underline{\underline{H}}(f) \qquad \text{(V.43)}$$

Over this expression is used again the criterion of find the minimum for the channel, and over this minimum maximize with receiver process matrix. The obtained solution is the same as in non-selective fading (but in this case for the matrix in frequency). In short, solution is:

$$\underline{\underline{B}}(f) . \underline{\underline{B}}^H(f) = \underline{\underline{I}} . \frac{E_T}{n_T . B_T} \qquad \text{(V.44)}$$

For example, next matrix response is valid for its use without CSI in selective channel.

$$\underline{\underline{B}}_0 = \frac{1}{2} \begin{pmatrix} 1 & j \\ -j & 1 \end{pmatrix} ; \underline{\underline{B}}_1 = \underline{\underline{0}} ; \underline{\underline{B}}_2 = \frac{1}{2} \begin{pmatrix} 1 & -j \\ j & 1 \end{pmatrix} \qquad \text{(V.45)}$$

In any case, the construction of matrices like this is not complicated, even if number of antennas is high. The problem is created in their use in multiuser environments. As well as non selective channels, CSI absence is covered in next chapters when space–time block codes are taken up.

V.6. SUMMARY

In this chapter, MIMO architectures for frequency selective channel have been exposed. All the design has done in frequency domain, with a beamforming scheme because CSI in transmitter is available.

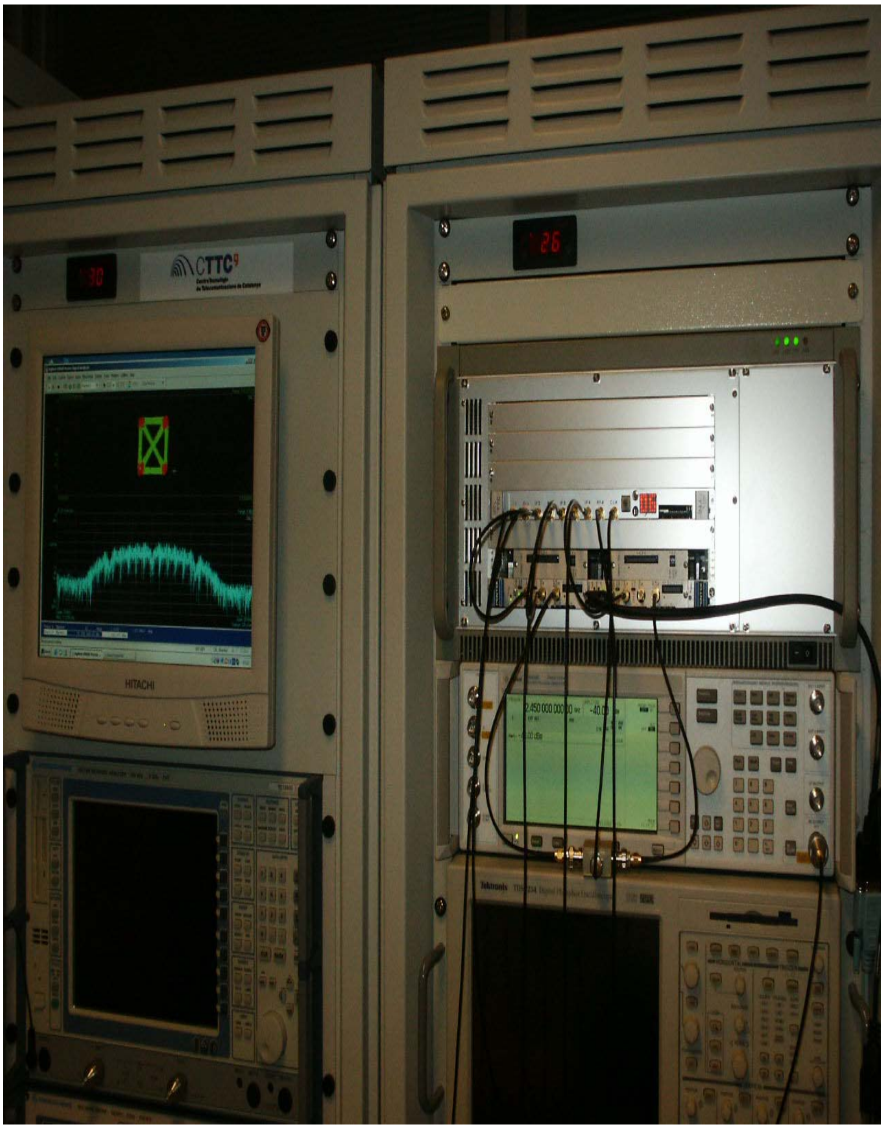
As schemes are wide band ones, without canalization by means of multicarrier systems or OFDM, designs obtained in frequency must translate to temporal domain, usually to FIR structures. This step consists on a new additional trouble.

The description has been slight and some similarities between selective channel design and not selective one have been used as much as possible in order to clarify the exposition.

It is worth mentioning than communication systems for selective channels appear nowadays in sharing-spectrum systems. Based on this perspective, mask restriction have been added in the design. This topic, not only in this aspect but in its totality, is centering a lot of actual investigation works and probably the content of this chapter must go through a lot of changes in next years. For the moment, its presence in this course must be useful as a motivation and start point for reader to this side, low studied for the moment, of MIMO systems.

VI.

VI. LOW COMPLEXITY ARCHITECTURES



Miguel Ángel Lagunas, Ana I. Pérez-Neira

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VI.1. INTRODUCTION

In last chapters it has been emphasized the interest of diagonalize MIMO channel to orthogonal channels, always assuming CSI in transmitter and receiver. It can be interesting to pose if any other decomposition (not SVD) gives interesting solutions to transmit information with multiple antennas system. This way, QR and LU decomposition will be explained.

This chapter introduces both procedures under the interest on simplify the receivers. Amazing results arise. Although this point of view (reducing complexity) is a valid justification to include this systems in these notes, there are also arguments in terms of capacity, alternatives to SVD and also very close to solution of MIMO Broadcasting problem (nowadays this is an still opened problem). Because of the chapter reference is always the complexity reducing on receivers, it will be described the interest of schemes introduced in this chapter to MIMO-MAC and MIMO-BC problems.

In a MIMO-MAC channel in which K users access to a base station with n_R antennas (and each user with n_{Tk} antennas), capacity concept for a single user can be quickly extended. In fact, the named sum-capacity increase linearly with the minimum of transmitted antennas and all receivers antennas sum.

$$C_{MIMO-MAC}^{SUM} \approx \phi_0 . \min\left[n_R , \sum_{k=1}^{K} n_{Tk} \right] \qquad \text{(VI.1)}$$

Capacity region is well known and it is composed of all combinations of user rate for every user. Also to be up to capacity in MAC case, it can be obtained with a superposition coding or if not, for a later on exposed concept, successive decoding and interference cancellation to base station. This successive decoding scheme will arise as a simple way to remove some complexity of MIMO receiver. This way, remember than decoding or successive cancellation schemes, as well as iterative receivers that in this chapter will be obtained as low complexity solutions to implement in transmitter. When these are implemented also in receiver this is the way to get capacity in a multiple access MIMO system.

For comparison, suppose a TDMA system in which only a user is active in the time slot. In the number of antennas of all users, it can be written than:

$$\frac{C_{MIMO-MAC}^{SUM}}{C_{TDMA}} \approx \min\left[\frac{n_R}{n_T} , K \right] \qquad \text{(VI.2)}$$

That is to say the improvement can be very high if number of antennas in access point is higher.

In MIMO-BC (broadcasting) case, the problem is still opened, for decide what capacity is considered and which the optimal techniques for coding and detection are. Situation changes if we consider channel degradation. Next will be shown that exists an optimal solution of coding to get capacity in this type of channels.

Firstly we will consider a SISO channel with two receivers (see Figure VI-1).

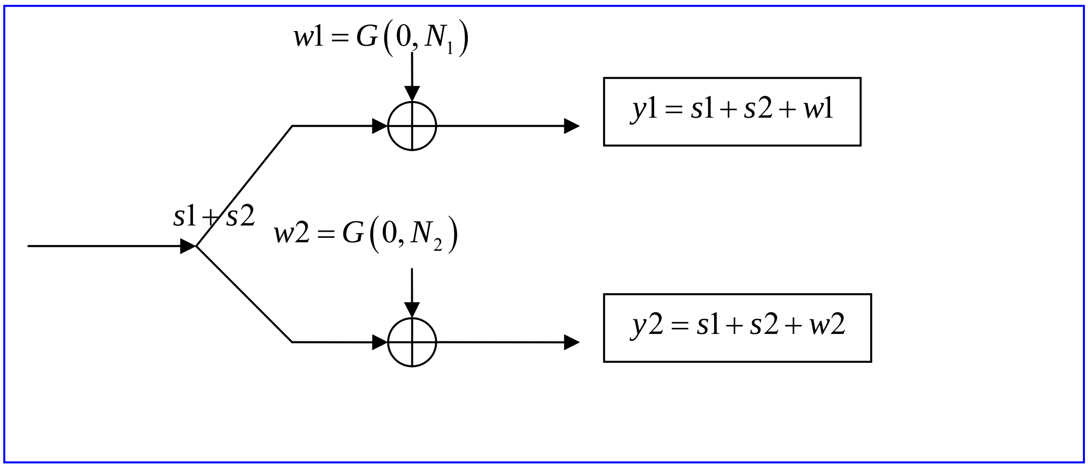

Figure VI-1.- Degraded SISO-BC channel

The solution to this problem is the successive decoding in receivers. That is to say s2 must be coded considering s1 as an interference with R2 as result. If in y1 there is less noise than in y2 this also can be decoded from y1. Subtracting s2 from y1, R1 is get as if s2 doesn't exist. Therefore, the capacity for known or unknown interferency will have the same value. In any case, the degraded channel is essential in this process.

$$R2 = \frac{1}{2}.\log\left(1 + \frac{P_2}{N_2 + P_1}\right) \qquad \text{(VI.3)}$$
$$R1 = \frac{1}{2}.\log\left[1 + \frac{P_1}{N_1}\right]$$

This concept adapted to transmitter case is known as DP (Dirty Paper Coding) because s1 is written over paper zones in which s2 have previously stained. This way, degraded channel is not necessary. In next chapters, under low complexity receivers assumption, will be explained how is the precoding system who allows to get a better quality in MIMO-BC systems.

The transformation from SISO to MIMO isn't easy. In MIMO systems the worst channel selection is not possible and therefore successive decoding is not a solution. But it's possible to extend DP concepts to MIMO. With all, calling $\underline{Q}$ to transmitted covariance matrix, capacity obtained in DP might be:

$$R1 = \frac{1}{2}\log\left[\frac{\det\left(\underline{\underline{H}}_1.\underline{\underline{Q}}_1.\underline{\underline{H}}_1^H + \underline{\underline{H}}_1.\underline{\underline{Q}}_2.\underline{\underline{H}}_1^H + \underline{\underline{\Phi}}_1\right)}{\det\left(\underline{\underline{H}}_1.\underline{\underline{Q}}_2.\underline{\underline{H}}_1^H + \underline{\underline{\Phi}}_1\right)}\right]$$
$$R2 = \frac{1}{2}\log\left[\frac{\det\left(\underline{\underline{H}}_2.\underline{\underline{Q}}_2.\underline{\underline{H}}_2^H + \underline{\underline{\Phi}}_2\right)}{\det\left(\underline{\underline{\Phi}}_2\right)}\right] \quad \text{(VI.4)}$$

It's interesting to note than sum-capacity of MIMO-BC channel is limited by capacity in a situation where receivers cooperate, hence considering $\underline{x}$ as transmitted vector for s1 next to s2, and $\underline{H}$ represents the channel for both users, then it verifies:

$$C_{MIMO-BC}^{SUM} \leq \min_{\underline{\underline{\Phi}}}\left(\max_{\underline{\underline{\Phi}}_x}\left[\log\left[\frac{\det\left(\underline{\underline{H}}.\underline{\underline{Q}}_x.\underline{\underline{H}}^H + \underline{\underline{\Phi}}\right)}{\det\left(\underline{\underline{\Phi}}\right)}\right]\right]\right) \quad \text{(VI.5)}$$

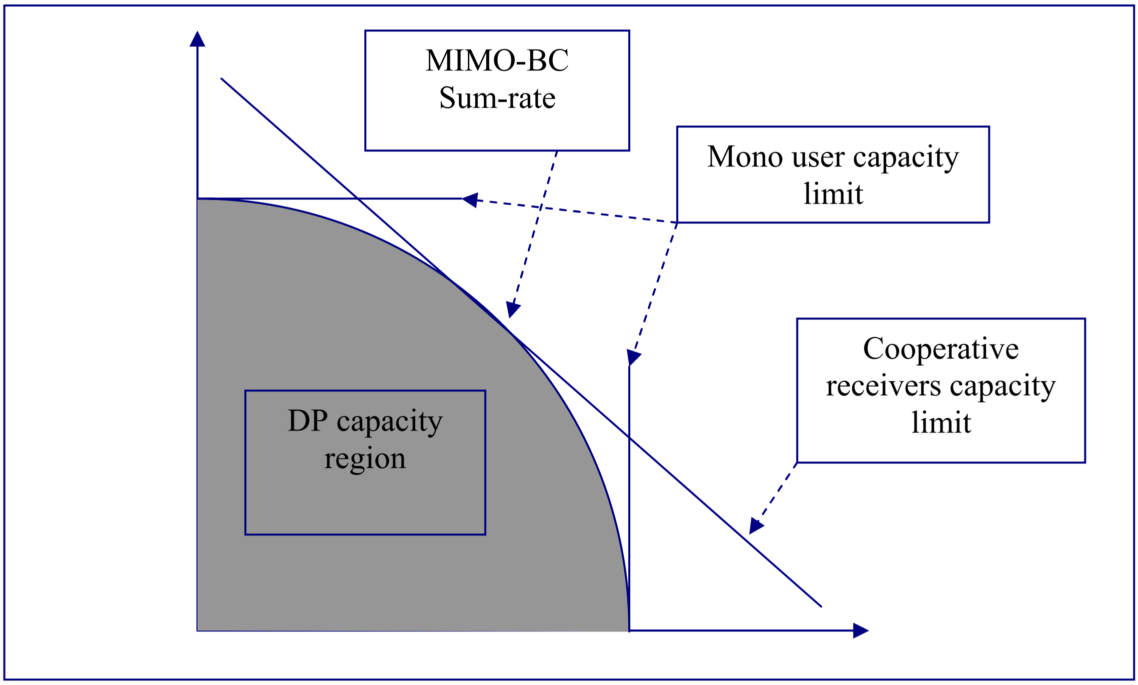

Figure VI-2.- DP capacity region for MIMO-BC and established limits.

Such interesting thing is to know that MIMO-BC capacity has an asymptotic behaviour, similar to MAC channel and it also benefits itself of add antennas in transmitter (in this case).

DP capacity region appears in Figure VI-2, and also limits are shown off. It's left to know if DP region covers completely the capacity region of MIMO-BC channel.

The following chapter describes precoding systems, extrapolable to receiver in MIMO-MAC case, which allows to bring sum-capacity point with maximum deviations of 3 bits/s/Hz in low SNR case and near to zero in SNR cases higher than 30dB.

VI.2. DIRTY-PAPER (DP) CONCEPT

First try to reduce complexity in receiver forces to make an instant detection of transmitted symbol, try that force to not use a ML detector. Second step is to forget interferent environments and think only in transmitter to receiver point to point link with the only limitation of MIMO channel and the corresponding front-end receiver noise. Finally can be concluded than maximum simplicity is obtained then design criterion is ZF.

Obviously use ZF as design criterion involves some quality loss and some limitations in system features. In order to show strong limitations of ZF scheme, we will test the case in which n_s streams are transmitted over a MIMO channel caracterized by their response, now assumed flat-fading. Having $\underline{I}$, the streams to transmit, a ZF design implies a process matrix $\underline{\underline{B}}$ in transmitter which verifies

$$\underline{\underline{B}} = \underline{\underline{H}}^H . \left(\underline{\underline{H}} . \underline{\underline{H}}^H \right)^{-1} \qquad \text{(VI.6)}$$

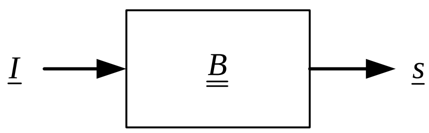

Figure VI-3.- Simplified scheme of transmitter

This way, transmitted symbols, s, will be given by

$$\underline{s} = \underline{\underline{B}} . \underline{I} \qquad \text{(VI.7)}$$

The most important problem of ZF using is, because of transmitted energy limitation, it forces to normalize receiver according to E_T. Calling α to normalization constant, this must verify

$$\alpha^{1/2} = \frac{E_T}{\beta} = \frac{E_T}{Trace\left(\underline{\underline{B}}^H . \underline{\underline{B}} \right)} = \frac{E_T}{Trace\left(\left[\underline{\underline{H}} . \underline{\underline{H}}^H \right]^{-1} \right)} \qquad \text{(VI.8)}$$

In receiver case, this will have a received signal and a SNR as indicates (VI.9), where it can be seen the crucial paper of normalization in transmitter. Note than the system is fair because presents the same SNR in reception for all channels.

$$\underline{y} = \left(\frac{E_T}{\beta} \right)^{1/2} . \underline{I} + \underline{w} \qquad SNR = \frac{E_T}{\beta . N_0} \qquad \text{(VI.9)}$$

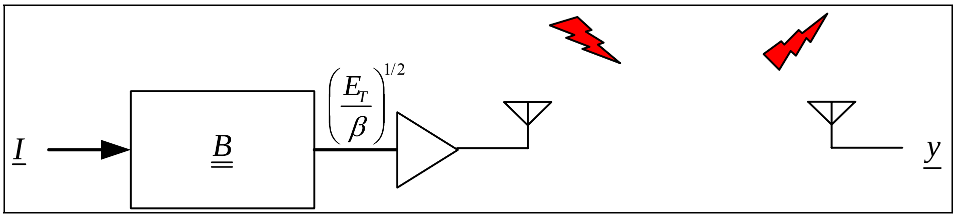

Figure VI-4.- ZF Tx-Rx scheme with transmitted energy control

Here we can appreciate first big problem in ZF schemes, because the expectancy of β tends to infinity and therefore system behaviour is unstable.

The solution to this problem is related to fairness presented for all streams, users in a multi-user environment. In fact, CSI availability shows the existence of channel eigenmodes, that is to say independent channels without ISI between them. These good channels in terms of ISI are virtual, not physically real (the beamformer designer generate them). Anyway the eigenmodes existence and the way as designer obtains them, suggest than perhaps exist another virtual channels with different properties. We should propose if exists a set of channels between the ones generated by ZF and the ones generated by SVD, with properties between both as well. This alternative must be neither channel inversion nor its eigenmodes decomposition.

Less algebraically complex as SVD, another decomposition exists. It's Q-R decomposition. This divide channel matrix in a orthonormal matrix and a lower-diagonal one. Obviously this decomposition doesn't diagonalize the channel (because this second matrix is triangular), thus its use leads to channels with ISI. The triangular structure, as we will see, indicates than there are virtual channels with ISI from zero (the first one) to the maximum one (for the last one). Now we will justify all these affirmations and also we will see which relation have this decomposition with ZF design.

Q-R decomposition is given by (VI.10), where the first matrix is orthonormal and the second one is lower-triangular.

$$\underline{\underline{H}} = \underline{\underline{R}}.\underline{\underline{Q}} \qquad \text{(VI.10)}$$

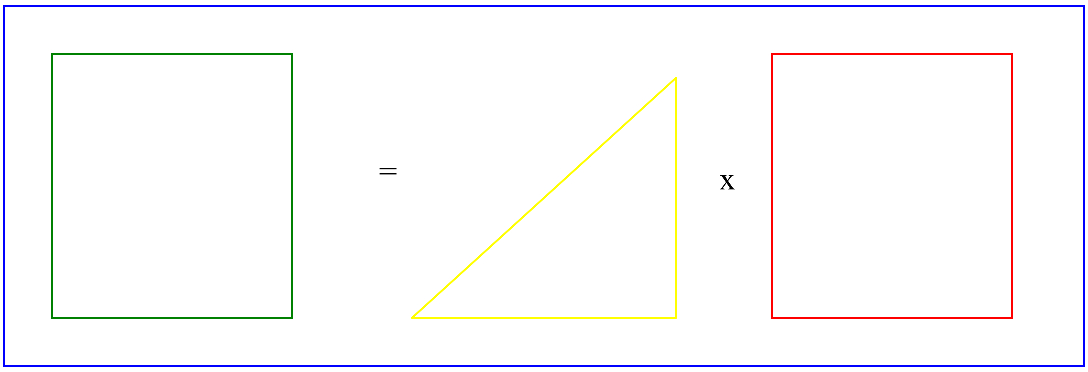

Figure VI-5.- Graphical representation of Q-R decomposition for MIMO channel decomposition

Thus, if transmitter, and specifically its space-time processor is identified with matrix $\underline{\underline{Q}}$, the channel (not diagonalized) takes on a special form which simplify receiver. At the same time, and given that second matrix is orthogonal, transmitted energy is permanently under control.

$$\underline{\underline{B}} = \underline{\underline{Q}}^H \quad and \quad \underline{\underline{B}}^H.\underline{\underline{B}} = \underline{\underline{Q}}.\underline{\underline{Q}}^H = \underline{\underline{I}} \qquad \text{(VI.11)}$$

The problem therefore is that the receiver must work against the ISI which equivalent channel leads, increasing as a result its complexity. In next section the receiver and the solution are presented as a joint work between complexity in receiver and power control in transmitter. Remember than the main goal is to prove its low complexity and it's odd that Q-R decomposition increase respect to ZF case, apparently. Later on we might see the outstandingnly of the solution, above all its objective achievement (low complexity in receiver combined with power control in transmitter).

VI.3. DIRTY-PAPER CONCEPT

Assuming the use of matrix $\underline{\underline{Q}}$ in transmitter, the equivalent channel is visualized in Figure VI-6, where it's evident that it will present ISI (this is not diagonal) and presents so much complexity in receiver.

$$\underline{u} = \underline{\underline{Q}}^H.\underline{I} \qquad \text{(VI.12)}$$

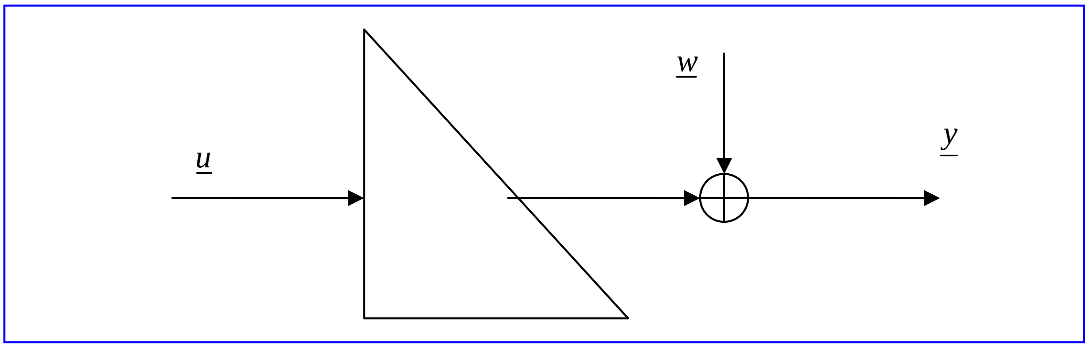

Figure VI-6.- Q-R decomposition, equivalent channel.

In order to see this channel effect over symbol vector to transmit, it will be supposed the following structure to effective channel:

$$\underline{\underline{R}} = \begin{bmatrix} d_1 & 0 & 0 & .. & 0 \\ r_{21} & d_2 & 0 & .. & 0 \\ r_{31} & r_{32} & d_3 & .. & .. \\ .. & .. & .. & .. & .. \\ r_{n_R 1} & r_{n_R 2} & r_{n_R 3} & .. & d_{n_T} \end{bmatrix} \qquad \text{(VI.13)}$$

The component structure of received vector, without noise contribution, in function of input vectors will be as indicates (VI.14). Note that maximum number of streams to transmit must be equal than channel matrix range, that is in general, equal to min(n_r, n_T).

$$\underline{y} = \begin{bmatrix} d_1 & 0 & 0 & .. & 0 \\ r_{21} & d_2 & 0 & .. & 0 \\ r_{31} & r_{32} & d_3 & .. & .. \\ .. & .. & .. & .. & .. \\ r_{n_R 1} & r_{n_R 2} & r_{n_R 3} & .. & d_{n_T} \end{bmatrix} . \begin{bmatrix} s1 \\ s2 \\ s3 \\ .. \end{bmatrix} \qquad \text{(VI.14)}$$

Or equivalently,

$$\begin{aligned} y_1 &= d_1.s1 \\ y_2 &= r_{21}.s1 + d_1.s2 \\ y_3 &= r_{31}.s1 + r_{32}.s2 + d_3.s3 \\ &.................. \end{aligned} \qquad \text{(VI.15)}$$

These equations clearly reveal the MIMO channel structure obtained. First symbol, s1 has a completely clean channel (without other channels ISI). The following symbol, s2, is written over a channel stained by s1, and so forth till arrive to worst channel which is the last symbol one. Although it contains ISI, channel hierarchy from better to worse channel is clear for the receiver, then the receiver itself or the transmitter can do something to solve the problem which this MIMO channels present (in a such easier form than the original one).

A way to tackle the problem in transmitter would be to mark s1 so that in y2 was easy to remove its contribution. This procedure is called “dirty paper”, that is, how to write over a stained paper without lose or damage new information.

In receiver there is a solution, less effective but so much simple: Detect s1 from first channel and subtract its contribution from y2. This is completely analog to Blast concept, where in its stratified or let’s say, layered version, its symbols repeat over transmitter antennas and, as a result, it is detected the first time it appears in receiver and from there it will be considered as interference. This process is sketched in next figure.

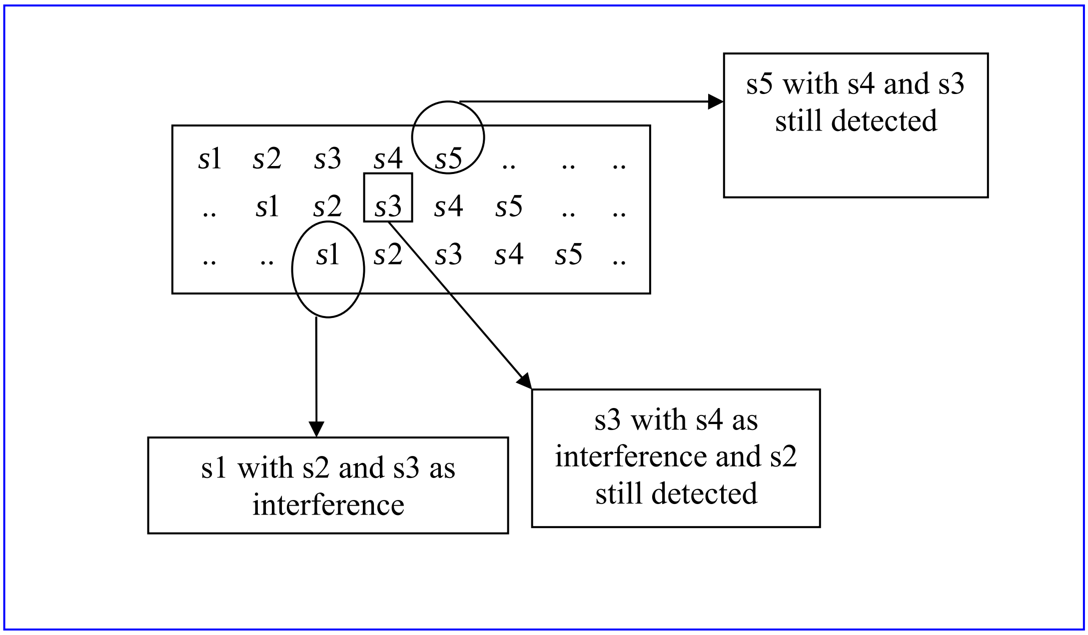

Figure VI-7.- Layered Blast architecture. How each symbol is seen in the receiver in a three antenna system.

As it can be seen, a symbol process in any column is done considering upper ones as unknown interference and lower ones as previously detected.

In capacity terms, (VI.15) allows to write capacity of each channel as indicated in (VI.16), where it can be seen how capacity decrease when the interference term increase in its denominator. The sum capacity will be the sum of whole capacities corresponding to each channel.

$$
\begin{aligned}
C_1 &= Ln\left[1+\frac{E_s}{N_0}.d_1^2\right] \\
C_2 &= Ln\left[1+\frac{E_s}{N_0}.\frac{d_2^2}{1+r_{21}^2\frac{E_s}{N_0}}\right] \\
C_3 &= Ln\left[1+\frac{E_s}{N_0}.\frac{d_3^2}{1+\left(r_{31}^2+r_{32}^2\right)\frac{E_s}{N_0}}\right]
\end{aligned}
\qquad (VI.16)
$$

....

Before follow with next section, note that for the moment in this scheme, and using Q-R decomposition, the transmitter can perfectly control its power at the expense of such more complex receiver in which interference is removed successively to previous detections. Forgetting the problems of this ISI successive cancellation, note that in a multi-user scheme in down-link, the last user would need to know the information of all other ones to give the corresponding symbol. In next section a such interesting alternative is presented.

As announced in section 1, the situation in which multiple users access an access point, this kind of scheme mounted on access point (in receiver, not in the K MAC transmitters) is a solution which get capacity in MIMO-MAC channel. The solutions that follow, situated in receiver instead of transmitter, form the base of systems which obtain capacity in MAC.

VI.4. TOMLINSON-HARASHIMA SCHEME IN MIMO

In last system explained, successive interference caused Q-R decomposition and it gat a perfect power control in transmitter but at the expense of increase receiver complexity (as we have been explaining this is not a good result in practice).

Unique solution to take up low complexity objective again is to make the successive cancellation in transmitter. This is easy to implement, it's enough to define a new diagonal matrix which contains the elements of virtual channel diagonal by means of Q-R decomposition.

$$\underline{\underline{D}} = diag\left[\underline{\underline{R}}\right] \tag{VI.17}$$

The transmitter scheme following this idea is represented in Figure VI-8.

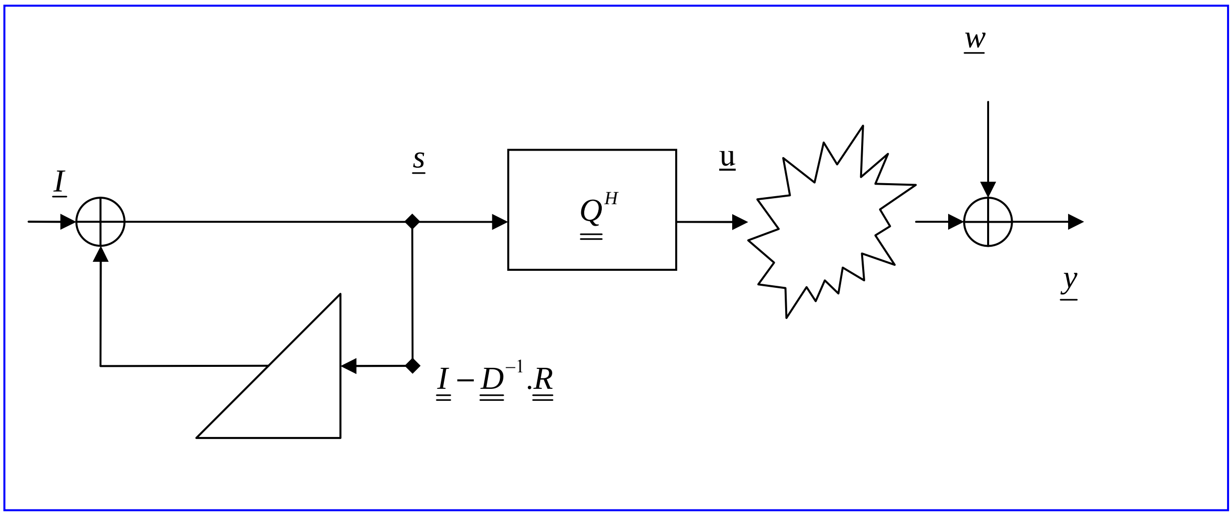

Figure VI-8.- Transmitter with ISI successive cancellation

We can see how interference cancellation is done in transmitter. The relation which link transmitted and effective symbols, after transformation, is:

$$\underline{s} = \underline{I} + \left(\underline{\underline{I}} - \underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s} \tag{VI.18}$$

This equation can be also written separating as well input from output.

$$\underline{s} = \underline{I} + \left(\underline{\underline{I}} - \underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s}$$
$$\underline{\underline{D}}^{-1}.\underline{\underline{R}}.\underline{s} = \underline{I} \quad \Rightarrow \quad \underline{\underline{R}}.\underline{s} = \underline{\underline{D}}.\underline{I} \tag{VI.19}$$

The conclusion is, as channel is reduced to the product between matrix Q and matrix R, the received signal is identical as input stream, as we wanted, and all these without any process at the receiver.

$$\underline{y} = \underline{\underline{H}}.\underline{u} = \underline{\underline{R}}.\underline{\underline{Q}}.\underline{\underline{Q}}^H \underline{s} = \underline{\underline{R}}.\underline{s}\left|\text{see } (6.13)\right| = \underline{\underline{D}}.\underline{I} \tag{VI.20}$$

For instance, when we have two streams,

$$\begin{bmatrix} s1 \\ s2 \end{bmatrix} = \begin{bmatrix} i1 \\ i2 \end{bmatrix} + \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ r_{21}/d_2 & 1 \end{pmatrix}\right].\begin{bmatrix} s1 \\ s2 \end{bmatrix}$$

and, solving,

$$s1 = u1$$
$$s2 = u2 - \frac{r_{21}}{d_2}.s1 \tag{VI.21}$$

In second expression, DP concept is used because u2 is written in an expression in which s1 has get dirty. Obviously, in the receiver:

$$y1 = d_1.i1$$
$$y2 = d_2.i2 \tag{VI.22}$$

It's then proved the way as low complexity in receiver is recovered, using complete Q-R decomposition in transmitter. The problem still not solved is transmitter power control, because (VI.19) recursion doesn't assure energy of every symbol to transmit.

This problem solution, and the global problematic of both control complexity in reception and used energy in transmission, is to turn to Tomson and Arracima precoder principle. The idea is to operate symbol vector at the output of sum (module ρ), as represent Figure VI-9.

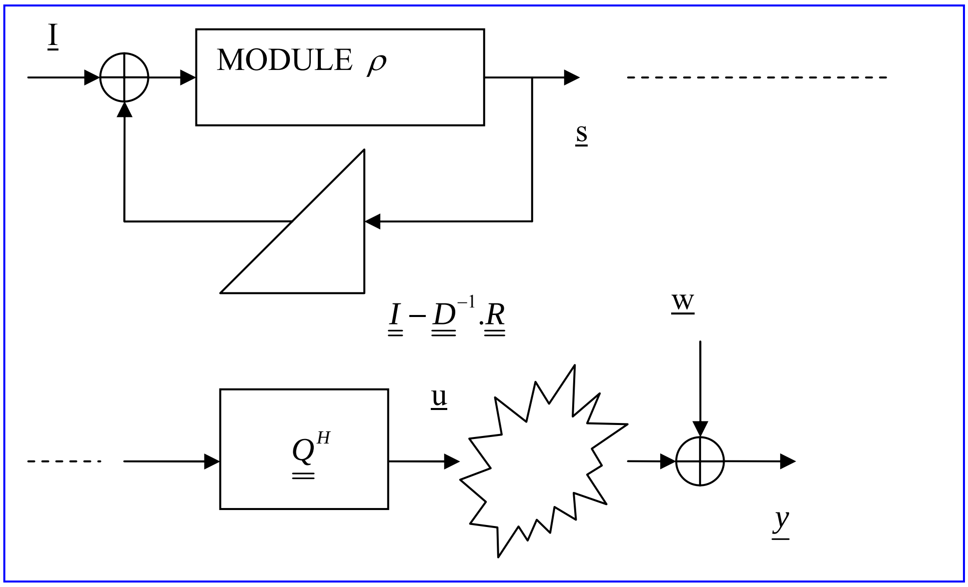

Figure VI-9.- T-H transmitter over Q-R decomposition

Remembering that Q transformation is orthonormal and assuming an uniform distribution in range [-ρ,+ρ], the transmitted energy by stream remains as (VI.23) always apart channel to use.

$$E_T = n_S.\frac{\rho^2}{12} \qquad \text{(VI.23)}$$

Now let's see how the use of T-H precoder will affect to receiver signal. Removing Q transformation from formulation, transmitted symbols are given by (VI.19), where m is a vector of integers.

$$\underline{s} = MOD_\rho\left(\underline{I}+\left(\underline{\underline{I}}-\underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s}\right) = \underline{I}+\left(\underline{\underline{I}}-\underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s}+\underline{m}.\rho \qquad \text{(VI.24)}$$

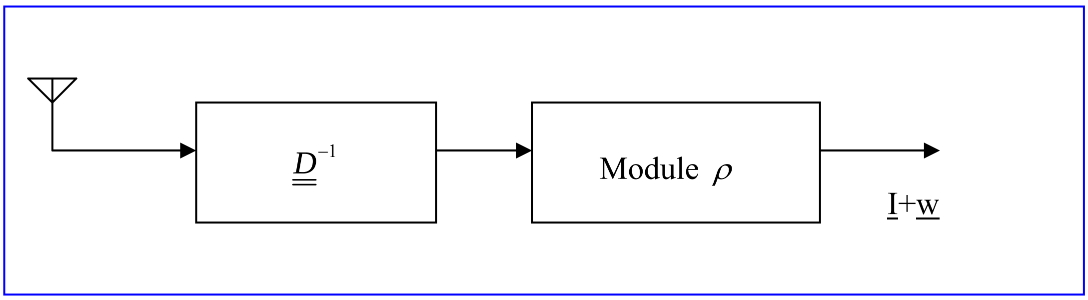

Figure VI-10.- Receiver scheme for T-H transmissions and Q-R decomposition.

Evidently in the receiver original streams are recovered and also integer vector times ρ parameter. This last one can be removed reapplying module operation. In short, the receiver is like a gain control over every received stream, followed by module operation in each of the resulting vector components. In Figure VI-10 it can be seen the outstanding simplicity of receiver, providing transmitted symbols with white Gaussian noise. Probably is difficult to think in nearer scheme to both objectives of the beginning of chapter.

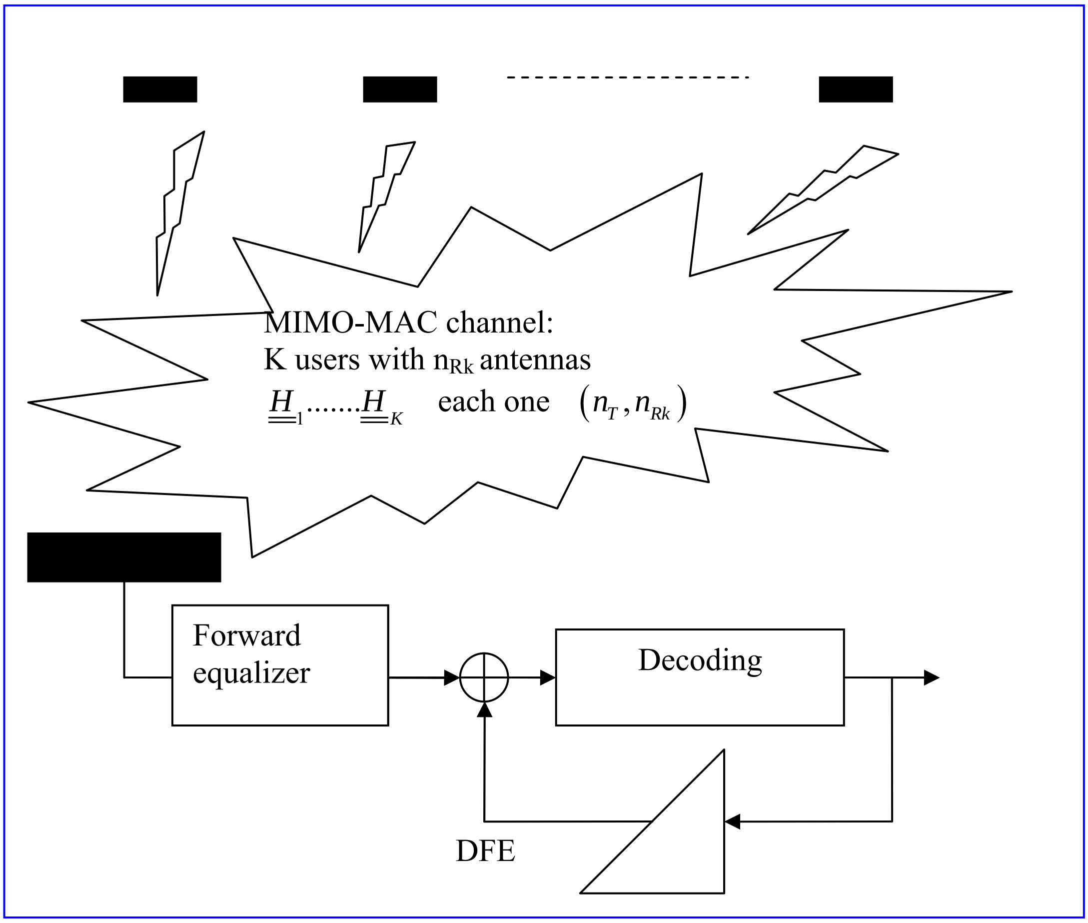

Figure VI-11.- MIMO-MAC system with successive decoding to get capacity

Recovering the MIMO-MAC case, note that if we use the system in the receiver, in a system of Q users accessing to, the scheme T-H is equivalent to the DFE (Decision Feedback) one, adapted to MIMO channel, where the successive decoding or ISI cancellation takes place, which really allows to obtain capacity in a MIMO-MAC system.

VI.5. SPHERE PRECODER

Last transmitter covers all the questions brilliantly. The unique review or clarification which can be done is the drastic way as transmitter selects power, that is to say the way as system takes directly the module without taking care of optimize this action. This is true that the module operation can't be modified or optimized but probably there be another way to things better.

In essence, the T-H processor is included in last section in order to control ZF power. The ZF energy was given by the trace of second order matrix of transmitted symbols. In other words, the Frobenius norm of ZF matrix times input streams matrix.

$$\underline{s} = \frac{1}{\sqrt{\gamma}} \underline{\underline{B}}.\underline{I} \qquad \gamma = Trace\left(\underline{\underline{B}}.\underline{I}.\underline{I}^{H}.\underline{\underline{B}}^{H}\right) = \left|\underline{\underline{B}}.\underline{I}\right|_F^2 \tag{VI.25}$$

In essence what it has been shown in last section is that, for the purposes of receiver, to transmit initial streams vector or a ρ modified one in each of its components was almost the same (see Figure VI-10). In conclusion, for the receiver it's as easy to introduce $\underline{I}$ than a module version of this.

$$\underline{I} \quad \Rightarrow \quad \underline{I} + \rho.\underline{m} \tag{VI.26}$$

The question is, now transmitted signal would be (VI.27), and the purpose it's the same. So we must search the minimum norm of transmitted vector.

$$\underline{s} = \frac{1}{\sqrt{\gamma}}.\underline{\underline{B}}.\left(\underline{I} + \rho.\underline{m}\right) \tag{VI.27}$$

In short, the design depends now on integers vector, and taking care to minimize power. Hence, the design becomes similar as ML detector and it takes the name of Sphere Encoger (SE),

$$\underline{m} = \min_{\underline{m}'} \left|\underline{\underline{B}}.\left(\underline{I} + \rho.\underline{m}'\right)\right|_F^2 \tag{VI.28}$$

The quality in terms of power control given by SE is quite better than the one given by T-H.

VI.6. MSE CRITERION

Till now all the exposition is done under no spatial ISI assumption and therefore under ZF criterion. It's interesting to note that also in MSE case, all seen until this point is perfectly extrapolable.

When MSE is used as design criterion, channel matrix will be more complex than in ZF case. In fact, the process matrix in a MSE receiver was given by

$$\underline{y} = \left[\underline{\underline{H}}.\underline{\underline{H}}^{H} + N_0.\underline{\underline{I}}\right]^{-1} \underline{\underline{H}}.\underline{x}_{\text{Received}} = \underline{\underline{\Phi}}^{-1}.\underline{\underline{H}}.\underline{x}_{\text{Received}} \tag{VI.29}$$

As well the intention is to give this process to transmitter, we will see how last matrix is in parts separated. Specifically next decomposition will be used:

$$\underline{\underline{\Phi}} = \underline{\underline{G}}.\underline{\underline{S}}.\underline{\underline{G}}^H \tag{VI.30}$$

Where $\underline{\underline{G}}$ matrix is lower triangular and $\underline{\underline{S}}$ one is diagonal.

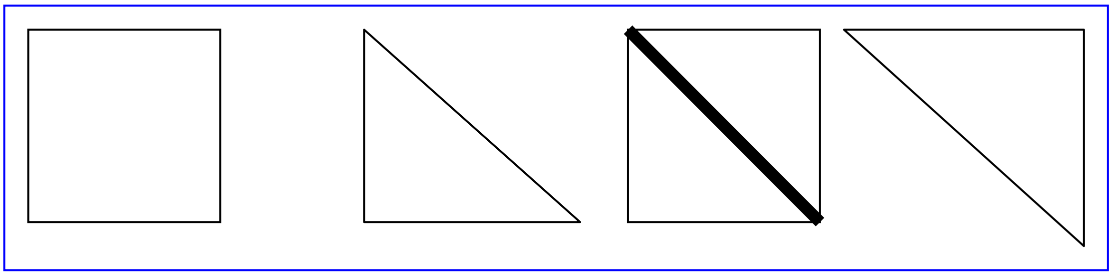

Figure VI-12.- Decomposition used for covariance of received signal over MIMO channel and white noise

Now, the transmitted vector y must contain the MMSE solution together with power normalization. Therefore, and with scheme, it will be verified that:

$$\underline{u} = \left(\frac{1}{\beta}\right)\underline{\underline{B}}_{MMSE}.\underline{s} \quad \text{, where} \quad \begin{array}{c} \underline{\underline{B}}_{MMSE} = \underline{\underline{H}}^H.\left(\underline{\underline{H}}.\underline{\underline{H}}^H + k.N_o.\underline{\underline{I}}\right)^{-1} \\ \beta = \left\lfloor \underline{\underline{B}}_{MMSE}.\underline{s} \right\rfloor^2 \end{array} \tag{VI.31}$$

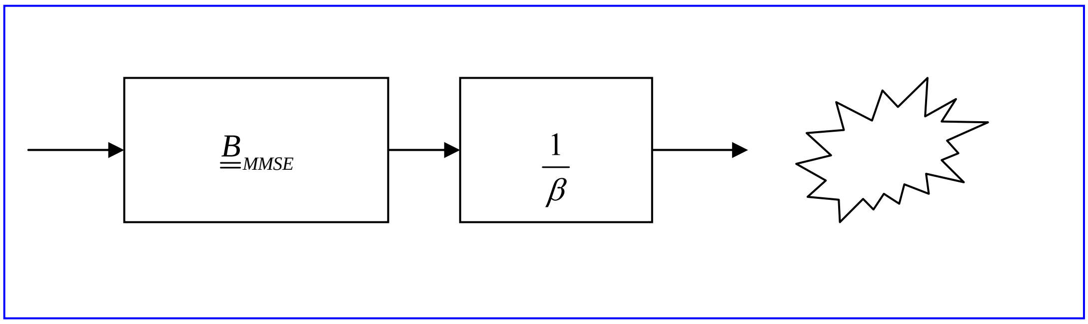

Figure VI-13.- System with MMSE precoder

The constant k is used to regularize the inverse and is optimized to reduce transmitted power.

The implementation can be done in a similar way as in a ZF. In order to do that the (VI.30) decomposition is done fragmenting this into a DFE (Decision Feedback Equalizer) scheme, followed by a forward equalizer as follows:

$$\underline{\underline{B}}_{MMSE} = \underline{\underline{H}}^H.\underline{\underline{G}}^{-1}.\underline{\underline{S}}^{-1}.\underline{\underline{G}}^{-H} = \underline{\underline{W}}_{MSE}.\underline{\underline{G}}^{-H} \tag{VI.32}$$

Implementing the inverse with a system similar to DFE (called GDFE in MIMO case) we can check the result in next figure.

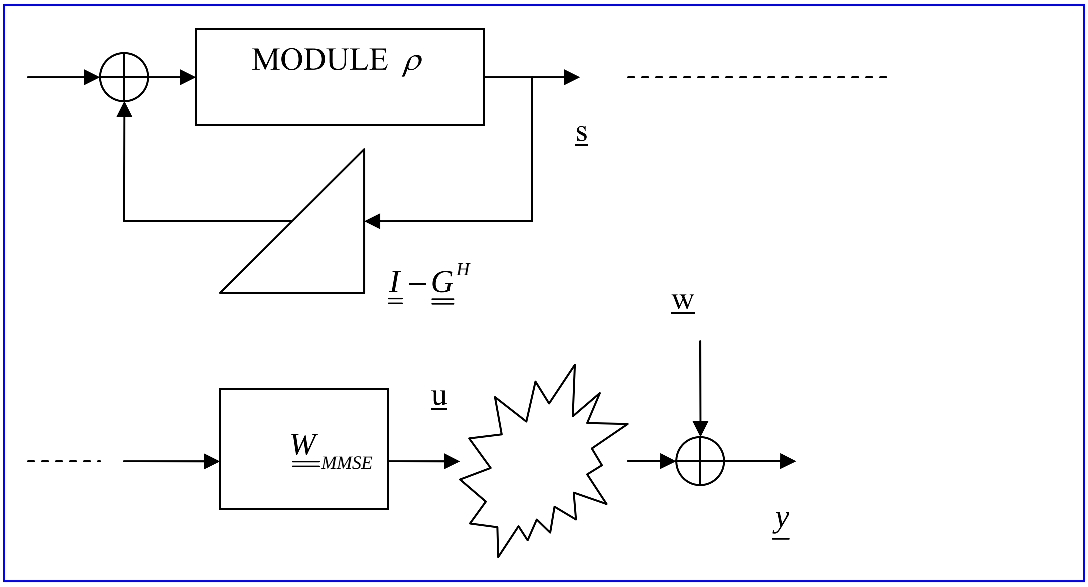

Figure VI-14.- MMSE-GDFE scheme

Obviously, and over MMSE scheme, it can be used Sphere Encoding, and whole system is called as MMSE-SE.

Both, MMSE-SE and MMSE-GDFE, present a quality in BER terms very much higher than both versions but with ZF (that is, ZF-SE and ZF-GDFE). The reason of this superiority is not clear but it's probably associated to β value (which is lower).

Some detailed analysis of received signal in every option reveals that still exists the possibility to improve more the MMSE system. Keeping as a reference the Figure VI-14, received signal can be written as follows:

$$\begin{aligned}
&\underline{y} = \underline{I} + \left[\underline{\underline{H}}.\underline{\underline{B}}_{MMSE} - \underline{\underline{I}}\right].\underline{I} + \beta_{MMSE}.\underline{w} && \textit{Direct MMSE} \\
&\underline{y} = \underline{I} + \left[\underline{\underline{H}}.\underline{\underline{B}}_{MMSE} - \underline{\underline{I}}\right].\left(\underline{I} + \rho.\underline{m}_{GDFE}\right) + \beta_{MMSE}.\underline{w} && MMSE-GDFE \quad \text{(VI.33)} \\
&\underline{y} = \underline{I} + \left[\underline{\underline{H}}.\underline{\underline{B}}_{MMSE} - \underline{\underline{I}}\right].\left(\underline{I} + \rho.\underline{m}_{SE}\right) + \beta_{SE}.\underline{w} && MMMSE-SE
\end{aligned}$$

With at the moment the best results method, which is MMSE-SE, note that integers vector $\underline{m}_{SE}$ is selected only to minimize β, that is to say receiver noise power, without taking into account that it always affect to ISI of the system represented by the second term of last expression. One possible selection of this integers vector which minimize ISI and noise power is without question the solution, nowadays, better in BER terms and in proximity to system capacity. This solution is known as MSE (Modified SE).

VI.7. SUMMARY

The most interesting architectures with multiuser environment considerations have been described. Their introduction is done in order to find solutions to MIMO channel in multiple access situations and with closed form solutions. The MIMO channel to diffusion, or MIMO-BC without a known solution has presented the alternative that nowadays is considered as the best one.

As capacity point of view belongs to multiuser environments and these will be the study objective in a next chapter, the presentation have been structured with the idea, always, to reduce complexity in receiver. Note that this criterion to reduce complexity is equivalent to pose alternatives or suggest “joint detection” receivers.

The chapter has started with the Q-R decomposition introduction as SVD alternative. This decomposition, proposed as ZF design, has allowed to show that successive decoding is the more appropriated solution and, at the same time, the optimal solution for MIMO-MAC capacities. Power control in down-link forces to explore new solutions, and in very cases ad-hoc situations without formal support. T-H and SE schemes provide an elegant solution to proposed problem of power control.

When a solution as MSE is looked for, instead of ZF, the contradiction created by searching a design in transmitter which depends on noise to its designed receiver, is solved also in a ad-hoc way. The receiver preprocessing is done by L-U decomposition of data receiver matrix and in transmitter, we use T-H and SE to control power. The design presents an additional variable which is the regularization parameter (in a classic MSE problem it represents noise power). Resulting scheme is without question the best option to obtain sum-capacity in a MIMO-BC channel.

It’s worth mentioning that presented solutions represent the linear and non linear process of maximal simplicity and following Dirty-Paper principles. Proposed schemes are simpler than DP-Coding, most of all if its excellent quality is considered.

Remember than presented ideas and architectures would be in use in multiuser systems chapter and that’s for it that, the introduction exposition was newly described in the corresponding chapter.

VII.

VII. MLSE AND STATISTICAL DESIGN

Miguel Ángel Lagunas, Ana I. Pérez-Neira

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VII.1. INTRODUCTION

This chapter consists in a revision of the topics seen in chapter II over ML detector. That chapter analyzed ML detector for a single symbol. Now, as we have could see the MIMO system facility of transmit multiple streams, the detector design in multi stream case will be analyzed.

Also a revision of some concepts on MIMO linear processing is done, before next chapter, which will contain the non linear processing or space time coding. The reader will can see the nearness of the concepts, what is viable in MIMO channel without CSI and the coding notation. The absence of CSI in transmitter force to do some processing, let's say, of the phase in transmitter. This concept, typical in SISO channels, is extended to MIMO channels in which are denominated orthogonal process matrices.

Given that no CSI availability will difficult to reach high quality levels in transmission, it will be introduced the concept of multiple accesses to the channel and also the preprocessing without CSI which these multiple channel accesses fix to quality of achieved system. These notions will be very useful when we'll want to understand as well the structural differences of space time block and convolutional codes present. Finally it will be easier to understand the advantages of complexity which involve the use of orthogonal block codes.

Related with this, the chapter also checks what's given by MIMO processing statistical design. Obviously the most part of concepts found in literature had their origin, and their success, in cable channels or in frequency diversity systems. Over these systems, the existence of instant CSI in transmitter is not such an illusion, in fact it's easy to measure with precision easily because of their less variation with time. It's obvious that radio channel presents a very different scenario. In this sense, it's too much pretentious to think full CSI or instant CSI will be present on transmitter. At the same time, adaptive modulation schemes (it also has the origin on cable and satellite systems) are presented as the unique alternative of valid CSI in transmission, in radio systems. If it must characterize adaptive modulation in a few words, it is a technique which consists on provide a partial CSI, only SNR information, without the phase. That is to say it gives a CSI with two basic limitations: Not availability of phase information and the fact of come from a measures average or old channel behaviour. With these CSI conditions, this chapter will formulate the average error rate and will provide the corresponding solutions to this type of CSI (only module, SNR) and also in no CSI case maintaining the objective in average error rate.

The content is absolutely essential to tackle the space time coding which follows in next chapter.

VII.2. THE MLSE DETECTOR

The initial scheme will be the same as used in all expositions, with transmitted process, channel and receiver process. This is reproduced in Figure VII-1.

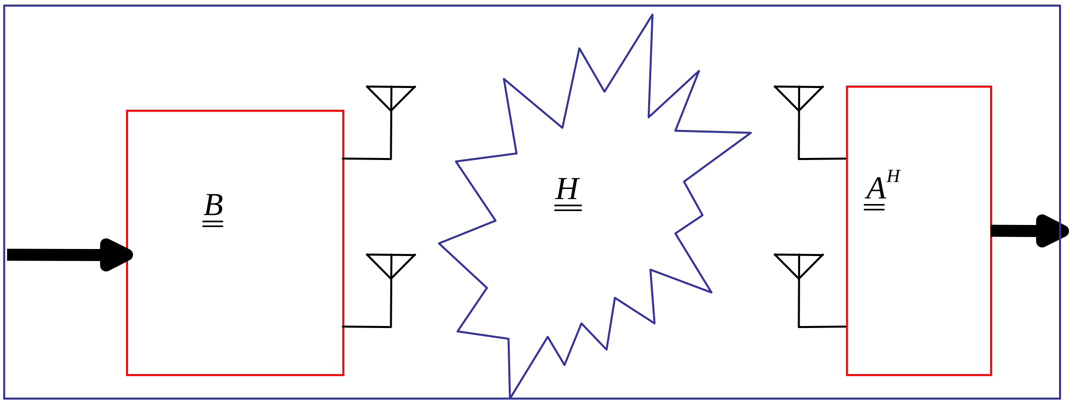

Figure VII-1.- Usual scheme in MIMO channel processing

In this architecture it is worth mentioning the three block decomposition in transmitter, as shows Figure VII-2.

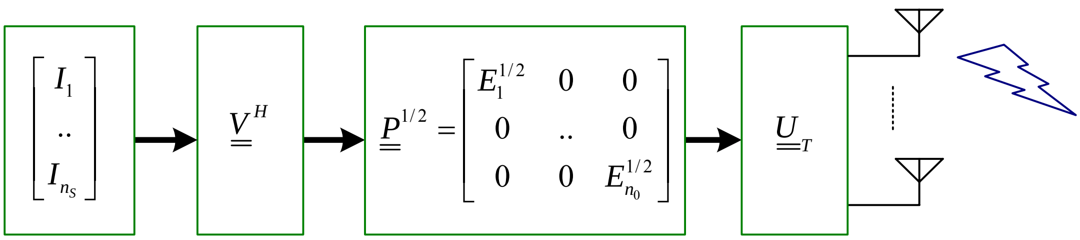

Figure VII-2.- Separation of transmitter process in three independent blocks

This way the different processes in transmitter are better understood. Right block is a set of beamformers which condition the available channel as good as they can. The best option is to use the channel matrix eigenvectors (by means of its SVD decomposition) but also has been shown that in multiuser environments these vectors come from Q matrix of QR channel decomposition. In any case it's important to think in a new channel which matrix would be given by (VII.1), where the best or the worst temporal processor is included. Note that where no instant CSI is available this matrix won't exist or, at best, it will be formed by beamformers which generate beams quite orthogonal in all interest directions, even in a pseudo-aleatory way. This last interpretation will be used when we explain multiuser techniques.

$$\underline{\underline{U}}.\underline{\underline{H}} \Rightarrow \text{new} \quad \underline{\underline{H}} \qquad \text{(VII.1)}$$

The second block (power allocation to each channel mode) is used to weight every channel mode, or to each antenna if temporal processor doesn't exist (for instance, as we have explained, if CSI is unknown). This block adopt non linear forms, so-called in last chapter as power control in transmitter, when TH or SE is used.

Finally, the most important block in this chapter will be the matrix $\underline{\underline{V}}$. This matrix only changes the n_S binary inputs into the number of channel modes or antennas, n_R, always equal or lower than n_S. This block can also be understood as the symbols constellation builder from input bits. Its paper will be crucial in all the exposition left. Note that in order not to abuse of formulation it will be considered from now that power allocation is included inside this matrix.

Formulating the ML detector, the likelihood expression is given in (VII.2). In this expression is included the option to do N channel accesses with the same information but changing the constellation, that is, the energy in every access. This cause the inclusion of the index (n) in last paragraph mentioned matrix.

$$\Im\left(\underline{I}_0\right) = -\sum_{n=1}^{N} \left| \underline{X}_n - \underline{\underline{H}}.\underline{\underline{U}}.\underline{\underline{V}}_n.\underline{I}_o.\sqrt{E_s} \right|^2 / 2.N_0 \qquad \text{(VII.2)}$$

Assuming the possible vector decided as $\underline{I}_d$, if the real vector is the one shown in (VII.2), then (VII.3) must be verified, where also the error vector is defined.

$$\begin{aligned} &\Im\left(\underline{I}_0\right) \geq \Im\left(\underline{I}_d\right) \\ &\underline{\tilde{I}}_0 = \underline{I}_0 - \underline{I}_d \end{aligned} \qquad \text{(VII.3)}$$

After some manipulation and with $\underline{W}_n$ as the received noise in every access, it can be written as

$$\begin{aligned} &Trace\left[\underline{\underline{R}}_H.\underline{\underline{A}}\right] \geq \operatorname{Re}\left\{ Trace\left[\underline{\underline{U}}^H.\underline{\underline{H}}^H.\sum_{n=1}^{N} \underline{W}_n.\underline{\tilde{I}}_0^H.\underline{\underline{V}}_n \right] \right\} \\ &\underline{\underline{R}}_H = \underline{\underline{U}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{U}} \quad and \quad \underline{\underline{A}} = \sum_{n=1}^{N} \underline{\underline{V}}_n^H \underline{\tilde{I}}_0.\underline{\tilde{I}}_0^H.\underline{\underline{V}}_n = \sum_{n=1}^{N} \underline{\phi}_n.\underline{\phi}_n^H \end{aligned} \qquad \text{(VII.4)}$$

In this expression it can be seen that, because an error doesn't exist, the right random variable is Gaussian, spatially uncorrelated, zero mean and with a variance equal to the left term in inequality, except N_0, the spectral density of noise. In short, error probability, that is when random variable is larger than threshold will be given by next equation,

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_d\right) = Q\left(\sqrt{\frac{E_s}{2.N_0}.Trace\left[\underline{\underline{R}}_H.\underline{\underline{A}}\right]} \right) \qquad \text{(VII.5)}$$

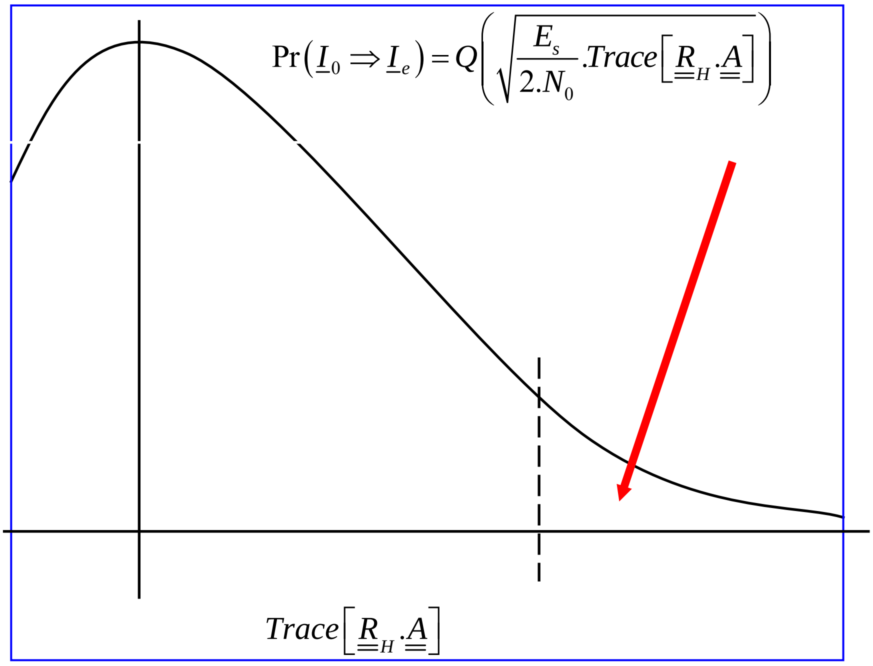

Figure VII-3.- Graphical interpretation of error probability in ML detectors

In order to keep progressing in the exposition it's necessary to consider that an appropriate system design will lead that the most probable error over a vector of n_S components be produced only in one stream. If we name the stream in which an error has appeared as 's', then only $\underline{\tilde{I}}_0(s)$ will be different from zero and it will be ± 2. Therefore only the corresponding column to this index in $\underline{\underline{V}}_n^H$ matrix will take part on probability of mentioned error.

$$\underline{\underline{V}}_n^H.\underline{\tilde{I}}_0 \rightarrow \quad stream\,\#s \rightarrow 4.\underline{v}_{s,n} \quad column\,'s'\,of \quad \underline{\underline{V}}_n^H \qquad \text{(VII.6)}$$

And minimize this error probability will be equivalent to maximize the next expression:

$$Trace\left[\underline{\underline{R}}_H.\sum_{n=1}^{N}\underline{v}_{n,s}.\underline{v}_{n,s}^H\right] \qquad \text{(VII.7)}$$

Before continuing with the design, even with or without instant CSI, the assumption of a same solution for all the streams or "fair", that is to say last expression must be the same regardless of 's' index. The following sections examine the two existing alternatives.

VII.3. MLSE WITH CSI

Reviewing the expression to maximize and imposing previously the equality relation to all the streams, it must be required to choose the constellation matrix in order to verify:

$$\underline{v}_{s,n}^{H}.\underline{\underline{R}}_{H}.\underline{v}_{s,n} = \beta_0 \ \forall n = 1,N; s = 1,n_S \tag{VII.8}$$

And if last equation is verified we can obtain a single vector as indicated as follows:

$$\underline{v}_{s,n} = \frac{\beta_0^{1/2}.\underline{a}_{s,n}}{\left(\underline{a}_{s,n}^{H}.\underline{\underline{R}}_{H}.\underline{a}_{s,n}\right)^{1/2}} \tag{VII.9}$$

With this election, total transmitted energy will be,

$$E_T = \beta_0.E_S.\sum_{s=1}^{n_S}\frac{1}{\underline{a}_{s,n}^{H}.\underline{\underline{R}}_{H}.\underline{a}_{s,n}} \tag{VII.10}$$

And finally the error probability:

$$\Pr^{CSI}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = Q\left(\sqrt{\frac{2.E_T}{N_0}.\left[\frac{N}{\sum_{s=1}^{n_S}\frac{1}{\underline{a}_{s,n}^{H}.\underline{\underline{R}}_{H}.\underline{a}_{s,n}}}\right]}\right) \tag{VII.11}$$

The minimization of error probability drives to use the channel matrix maximum eigenvector in all channel accesses. Only a constant in every access is not determined in the design. Even more, as we're analyzing full CSI situation, it's logical to assume than the channel is already diagonalized to maximum eigenvector. If it's right, the best design will consist in select the maximum eigenvector (the first one) of diagonalized matrix not including mentioned constant.

$$\underline{a}_{s,n}^{H} = \varphi(s,n).\begin{bmatrix}1 & 0 & \dots & 0\end{bmatrix} \tag{VII.12}$$

Related to last constant, there exist two apparent alternatives, even though really this is not true and the best solution is only one. The non valid alternative would be to pack all the streams inside a single n_S-QAM symbol. The problem given by big constellations in 2D (I and Q) is that scale badly its total transmitted energy. This wrong option would have an error probability as

$$\Pr_{QAM}^{CSI}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = Q\left(\sqrt{\frac{2.E_T}{N_0}.\left[\frac{3.N}{2^{n_S}-1}\right].\lambda_{\max}\left(\underline{\underline{R}}_{H}\right)}\right) \tag{VII.13}$$

For a given bandwidth, the most interesting solution is to increase the number of dimensions, that is, do each access in a different frequency. This is the case in which we use OFDM as transport. Then (VII.14) would be the constellation matrix and (7.15) would be the error probability.

$$\underline{\underline{V}}_n^H = \begin{bmatrix} ..\exp(-2\pi\, j.n.s/N).. \\ 0...0 \\ .. \\ 0...0 \end{bmatrix} \qquad \text{(VII.14)}$$

$$\Pr_{OFDM}^{CSI}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = Q\left(\sqrt{\frac{2.E_T}{N_0}.\left[\frac{N}{n_S}\right].\lambda_{\max}\left(\underline{\underline{R}}_H\right)}\right) \qquad \text{(VII.15)}$$

From comparison between (VII.13) and (VII.15) the reader can verify and understand the superiority of use simple constellation over a lot of dimensions, instead of complex constellations over I-Q diversity. Therefore, from now will be considered as a reference of instant CSI case, the expression(VII.15), where n_S/N is the system bit-rate. Moreover, if in (VII.13) channel matrix would change, in each access would be necessary to change the constellation.

VII.4. MLSE WITHOUT CSI

When no CSI is available the only option to maximize (VII.7), keeping as well the equality criterion in error, is to achieve that in all N accesses, the columns of constellation matrix together sum the identity matrix (except constant). In definitive, it must be verified:

$$Trace\left[\underline{\underline{R}}_H.\sum_{n=1}^{N}\underline{v}_{n,s}.\underline{v}_{n,s}^H\right]_{MAX} \Rightarrow \sum_{n=1}^{N}\underline{v}_{n,s}.\underline{v}_{n,s}^H = \beta_0.\underline{\underline{I}} \quad \forall s \qquad \text{(VII.16)}$$

Note the matrix channel will be constant over the N accesses to the channel. With this design, where UPA is done, the new error rate will be (VII.17) and loss because of not to have CSI is evident. We only have to compare this expression with º(VII.15) and differences are significant.

$$\Pr^{NO-CSI}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = Q\left(\sqrt{\frac{2.E_T}{N_0}.\frac{N}{n_S}.\frac{Trace\left(\underline{\underline{R}}_H\right)}{n_T}}\right)$$

$$\frac{Trace\left(\sum_{n=1}^{N}\underline{\underline{V}}_n^H.\underline{\underline{V}}_n\right)}{N}.E_s.n_s = E_T \quad \Rightarrow \quad E_T = \frac{n_S}{N}.n_T.\beta_0.E \qquad \text{(VII.17)}$$

As reader may appreciate, the difference between have or not to have CSI are the same as Chapter II expressions and reasonings. Now it's more evident the implied consequence, and it's summarized in (VII.16) propierty, that is to say the constellation matrices to be used in each access must satisfy that the sum of every column 's' of range unity matrices must regenerate the range unit arrive to identity matrix. It's obvious that number of channel accesses, because no CSI availability must be equal or larger than number of antennas in transmission.

There exist some possibilities for mount this kind of constellation matrices. The Figure VII-4 explains in a simple way, favour-based, to complete the constellation matrices columns in every case.

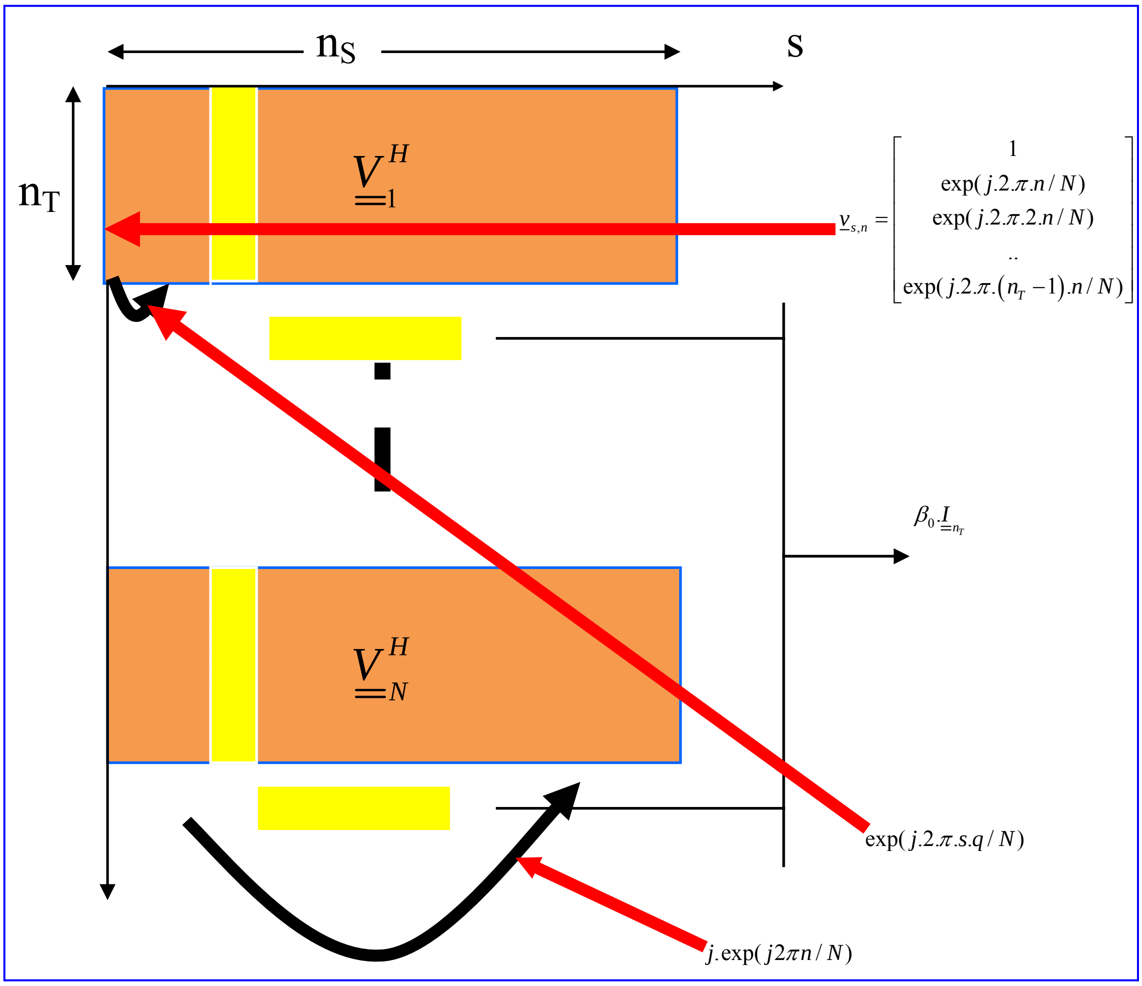

Figure VII-4.- Design of constellation matrices for N accesses, n_S streams and n_T antennas

In the following expression, two different examples of constellation matrices when there are four streams, two antennas and, obviously, two accesses to the channel

$$\underline{\underline{V}}_1^H = \begin{bmatrix} 1 & j & 1 & j \\ 1 & -j & -1 & j \end{bmatrix} \quad \underline{\underline{V}}_2^H = \begin{bmatrix} j & 1 & j & 1 \\ -j & 1 & j & -1 \end{bmatrix}$$
$$\underline{\underline{V}}_1^H = \begin{bmatrix} 1 & j & 0 & 0 \\ 0 & 0 & 1 & j \end{bmatrix} \quad \underline{\underline{V}}_2^H = \begin{bmatrix} 0 & 0 & 1 & -j \\ -1 & j & 0 & 0 \end{bmatrix} \qquad \text{(VII.18)}$$

By reasons exposed in next chapter, the second option, which is called Alamouti code, is better than the first one. This option form an OSTBC (Orthogonal Space-Time

Block Code), studied later on. In any case, for a generic case of ML detector, both have the same quality and the first one probably better to create less fatigue in transmission amplifiers and to use the most part of times at the maximum performance.

Next can be seen the selection, in case of three transmitter antennas and six receiver antennas.

$$\underline{\underline{V}}_1^H = \begin{bmatrix} 1 & 1 & 1 & j & j & j \\ 1 & \exp(j\frac{2\pi}{3}) & \exp(j\frac{4\pi}{3}) & j & j\exp(j\frac{2\pi}{3}) & j.\exp(j\frac{4\pi}{3}) \\ 1 & \exp(j\frac{4\pi}{3}) & \exp(j\frac{2\pi}{3}) & j & j\exp(j\frac{4\pi}{3}) & j.\exp(j\frac{2\pi}{3}) \end{bmatrix}$$

$$\underline{\underline{V}}_2^H = \begin{bmatrix} 1 & 1 & 1 & j\exp(j\frac{2\pi}{3}) & j.\exp(j\frac{2\pi}{3}) & j\exp(j\frac{2\pi}{3}) \\ \exp(j\frac{2\pi}{3}) & \exp(j\frac{4\pi}{3}) & 1 & j\exp(j\frac{4\pi}{3}) & j & j\exp(j\frac{2\pi}{3}) \\ \exp(j\frac{4\pi}{3}) & \exp(j\frac{2\pi}{3}) & 1 & j & j\exp(j\frac{4\pi}{3}) & j\exp(j\frac{2\pi}{3}) \end{bmatrix}$$

$$\underline{\underline{V}}_2^H = \begin{bmatrix} 1 & 1 & 1 & j\exp(j\frac{4\pi}{3}) & j\exp(j\frac{4\pi}{3}) & j\exp(j\frac{4\pi}{3}) \\ \exp(j\frac{4\pi}{3}) & 1 & \exp(j\frac{2\pi}{3}) & j\exp(j\frac{2\pi}{3}) & j\exp(j\frac{4\pi}{3}) & j \\ \exp(j\frac{2\pi}{3}) & 1 & \exp(j\frac{4\pi}{3}) & j & j\exp(j\frac{4\pi}{3}) & j\exp(j\frac{2\pi}{3}) \end{bmatrix} \quad \text{(VII.19)}$$

Before go to the following section, it's important to remark the interest of this kind of matrices, probably they seems to be the only alternative to this situation, even if, as in next sections happen, the transmitter design criterion is changed. Next will be described the design modification when we have partial or non instantaneous channel knowledge.

VII.5. STATISTICAL MIMO DESIGN WITH CSI

Returning to error expression which provides the receiver in every block of transmitted streams, (VII.20), expressed in function of n_R columns and $\underline{\underline{H}}$ matrix,

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_0.\exp\left[-\frac{2E_S}{N_0}.\sum_{p=1}^{n_R} \underline{h}_p^H.\underline{\underline{A}}.\underline{h}_p\right] \quad \text{(VII.20)}$$

When error rate is expressed this way, let's say instantaneous way, a more pragmatic design criterion can be used instead of minimize its instantaneous value, such complex operation. Maintaining this objective, different channel realizations must be meant and this way we will obtain an average of error rate. When the average operation is done, we will assume Gaussian distribution and, at the same time, we will assume that correlation in receiver doesn't exist. This last detail is important and is solved in the habitual case where receiver uses a single antenna or uses the best available antenna. In the long run it will be observed that is very useful and pragmatic to use antenna selection in MIMO channel receiver end, unless it can be assured no correlation in this side of transmission system.

$$E_{\underline{\underline{H}}}\left[\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right)\right] \approx k_0.\iiint \det\left(\underline{\underline{\Sigma}}_p^{-1}\right).\exp\left(\underline{h}_p.\left(\underline{\underline{\Sigma}}_p^{-1} + \frac{2E_S}{N_0}.\underline{\underline{A}}\right).\underline{h}_p\right) \quad \text{(VII.21)}$$

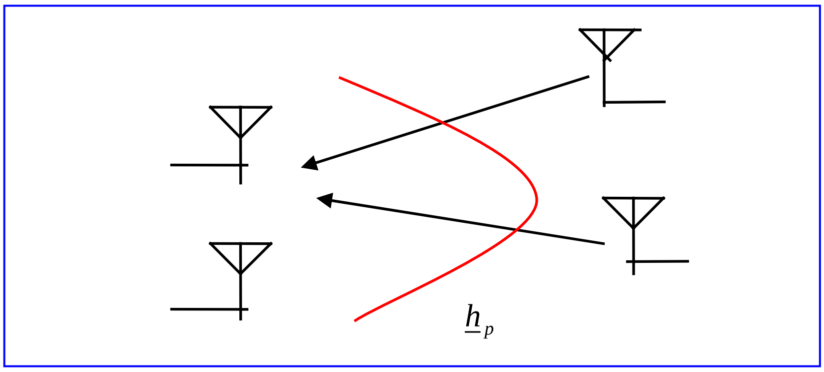

Figure VII-5.- Observed channels by each transmitter antenna opening

Therefore, and considering uncorrelated columns and a Gaussian distribution of its components, the average error rate can be written as (VII.21). This average has a simple closed expression in which we only should multiply and divide by the matrix in the exponent, and remember that pdf integrate is the unity. With this, we finally arrive to the following expression, which expresses the average error rate.

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1.\prod_{p=1}^{n_R} \frac{1}{\det\left[\underline{\underline{I}}_{n_T} + \frac{2E_S}{N_0}.\underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]} \quad \text{(VII.22)}$$

This expression reveals that constellation matrix design is still not done and, at the same time, that its impact won't be exponential but it will be a determinant. Moreover, if we allow CSI it's clear that the transmitter has the information corresponding to covariance matrices. If correlation in transmitter doesn't exist, its diagonal values are the SNR that each receiver antenna receive from every transmitter antenna. Is for that reason that the following can be considered as an extension of adaptive modulation SISO systems to MIMO case.

It's assumed that covariance matrices are diagonal due to low correlation in transmitter. The constellation matrix design, concretely the matrix A which minimize error rate will be also diagonal.

$$\underline{\underline{\Sigma}}_p = diag\left(\sigma_{pq}\right) \quad y \quad \underline{\underline{A}} = diag\left(\beta_q\right) \tag{VII.23}$$

The design of diagonals of this matrix will be done as follows, in any case, it's interesting to note that if we want this matrix diagonal, we must RECURRIR another time to orthogonal matrices, that is to say the constellation matrices in this case would be given by (VII.24), which uses the matrices that, in other context and goal, were used in case of no CSI availability.

$$\underline{\underline{V}}_n^H \rightarrow \left(diag\left[\beta_q\right]\right)^{1/2} .\underline{\underline{V}}_n^H \tag{VII.24}$$

Using these matrices inside average error rate we obtain,

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = k_1 . \prod_{q=1}^{n_T} \prod_{p=1}^{n_R} \left(1 + \frac{2E_S}{N_0}\beta_q .\sigma_{p,q}\right)^{-1} \tag{VII.25}$$

In this expression, two different approaches can be used. The first one, the most recommendable, consists in select the receiver antennas with better or similar SNR, thus next approach is quite correct.

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1 . \prod_{q=1}^{n_T} \left(1 + \frac{2E_S}{N_0}\beta_q .\gamma_q\right)^{-n_R} \quad with \quad \gamma_q = \left(\prod_{p=1}^{n_R} \sigma_{p,q}\right)^{1/n_R} \tag{VII.26}$$

We can see effectively that channel sign in the objective with the geometric mean, situation in which only good receiver antennas will be admitted. But if no antenna selection is done, the approach must be quite pessimist and culminate in an arithmetic mean, losing the receiver diversity in exponent, as indicated as follows:

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1 . \prod_{q=1}^{n_T} \left(1 + \frac{2E_S .n_R}{N_0}\beta_q .\gamma_q\right) \quad with \quad \gamma_q = \frac{1}{n_R}\sum_{p=1}^{n_R} \sigma_{p,q} \tag{VII.27}$$

It's clear that the most recommendable thing is to use (VII.26) for the good antennas and take this value as an inferior COTA (the worst case) and perform the design over that. Remember that it's easy to perform antenna selection in receiver because SNR information in each receiver antenna is easy to obtain.

Using (VII.26) and the total transmitted energy restriction, we arrive to (VII.28). This has been called statistical water-filling, given its similarity with frequently used procedure to reach capacity on instant CSI.

$$\beta_q = \left\lfloor \mu - \frac{1}{\gamma_q}.\left(\frac{N_0}{2E_S}\right)\right\rfloor^+ = \left| \begin{array}{l} \text{Only good} \\ \text{receiver antennas} \rightarrow \\ \text{All gains activated} \end{array} \right| = \mu - \frac{1}{\gamma_q}\left(\frac{N_0}{2E_S}\right) \tag{VII.28}$$

Furthermore it's interesting to note some duality between capacity and quality. In capacity with CSI, the solution is water-filling and the maximization of ergodic (average) capacity is UPA. In average error rate the solution is statistical water-filling and, in instant without CSI, is UPA.

Recovering the average error rate expression, case with CSI, the expression that results is

$$\bar{P}_e^{CSI} = \left(\frac{2E_T.\gamma_0}{N_0}\right)^{-n_T.n_R} \cdot \left[\frac{N}{n_s.n_T} + \left(\frac{E_S}{E_T}\right).\frac{1}{n_T}\sum_{q=1}^{n_T}\frac{1}{\gamma_q}\right]^{-n_T.n_R}$$
$$con \quad \gamma_0 = \left(\prod_{q=1}^{n_T}\gamma_q\right)^{1/n_T} \qquad \text{(VII.29)}$$

Such interesting is the following expression, directly derived from the last one.

$$\bar{P}_e^{CSI} = \left[\frac{2E_T.\gamma_0}{N_0}\frac{N}{n_s.n_T} + \frac{2.E_S}{N_0.n_T}\frac{\gamma_{0,GEO}}{\gamma_{0,HAR}}\right]^{-n_T.n_R} \qquad \text{(VII.30)}$$

Maybe we anticipate information, but it can be seen that this equation is the quotient between geometric mean and harmonic mean, that is the way as CSI makes the goal better. When this quotient isn't significative, for high number of antennas in transmitter or low EsNo rates, the solution will be the same as in no CSI case, that is, UPA (as we might see later).

VII.6. STATISTICAL MIMO DESIGN WITHOUT CSI

In case of no CSI availability, the only available design for matrix $\underline{\underline{A}}$ is another time an identity matrix, except constant.

$$\underline{\underline{A}} = \beta_0.\underline{\underline{I}} \qquad \text{(VII.31)}$$

With this design the average error rate, as we have commented, in case of selected antennas in transmitter becomes

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = \left|\begin{matrix}\text{Good}\\ \text{SNR}\\ \text{case}\end{matrix}\right| \approx k_1.\prod_{q=1}^{n_T}\left(\frac{2E_S}{N_0}\beta_0.\gamma_q\right)^{n_R} \qquad \text{(VII.32)}$$

Finally when total transmitted energy is included, the expression (VII.33) is obtained. Seeing this result, it can be understood what is lost in the objective due to CSI absence. These channel losses can be insignificant, even if we consider the difference of complexity between to have and not to have CSI available in transmitter.

$$\overline{P}_e^{NO-CSI} = \left(\frac{2E_T.\gamma_0}{N_0}\right)^{-n_T.n_R} . \left[\frac{N}{n_s.n_T}\right]^{-n_T.n_R} \qquad \text{(VII.33)}$$

becuse $\frac{Trace\underline{\underline{A}}}{N}.n_S.E_S = E_T$

It's worth mentioning that penalisation due to a larger number of antennas might no appear in articles that cover this topic. Here appears only in order to consider that's not right to increase total transmitted energy with the number of antennas because formally, the MIMO use is done, with so much other things, to increase efficiency and link quality without increase spectral contamination.

VII.7. IMPACT OF L.O.S. PRESENCE

Till nowadays there is a lot of work done about the topics described, but any of them which give a correct explanation to the capacity and the quality obtained in called colloquially position of good radio coverage. Moreover, al lot of works talk about correlation in transmitter and receiver when this hypothesis can't be accomplished from an electromagnetic or electronic point of view of the corresponding installation. In addition, it must be talked about the wrong use of a statistical notation when we assume a Rayleigh channel when terminals are very near to access point. It's obvious that not to include a measure in the statistical channel model doesn't allow to explain some behaviours and confusions, for example, that exist correlation between two entries of channel matrix, when this is simply the presence of a non null mean in both variables.

This section is going to analyze the impact into transmission channel of L.O.S. (Line Of Sight) presence, even with or without CSI.

For simplicity reasons in whole presentation will be supposed that only one receiver antenna works, without limit the partial or global conclusions extracted in what follows.

Basically the channel distribution seen by the only receiver antenna will have now a mean different from zero.

$$\Pr\left(\underline{h}\right) = \det\left(\underline{\underline{\Sigma}}^{-1}\right).\exp\left\{-\left(\underline{h}-\underline{h}_0\right)^H.\underline{\underline{\Sigma}}^{-1}.\left(\underline{h}-\underline{h}_0\right)\right\} \qquad \text{(VII.34)}$$

Using the following inequality:

$$\begin{aligned}&\left(\underline{h}^H.\underline{\underline{A}}.\underline{h}\right)\left(\underline{h}-\underline{h}_0\right)^H.\underline{\underline{\Sigma}}^{-1}.\left(\underline{h}-\underline{h}_0\right) = \left(\underline{h}-\underline{\underline{\Phi}}\underline{h}_0\right)^H.\underline{\underline{\Sigma}}^{-1}.\left(\underline{h}-\underline{\underline{\Phi}}\underline{h}_0\right)\\&+\underline{h}_0^H.\underline{\underline{\Sigma}}^{-1}.\underline{h}_0 - \underline{h}_0^H.\underline{\underline{\Phi}}^H\left(\underline{\underline{\Sigma}}^{-1}+\underline{\underline{A}}\right).\underline{\underline{\Phi}}.\underline{h}_0\end{aligned} \qquad \text{(VII.35)}$$

And the fact that:

$$\underline{h}_0^H.\underline{\underline{\Sigma}}^{-1}.\underline{h}_0 - \underline{h}_0^H.\underline{\underline{\Phi}}^H\left(\underline{\underline{\Sigma}}^{-1}+\underline{\underline{A}}\right).\underline{\underline{\Phi}}.\underline{h}_0 = \left| \begin{array}{l} con \\ \underline{\underline{\Phi}} = \left(\underline{\underline{\Sigma}}^{-1}+\underline{\underline{A}}\right)^{-1}.\underline{\underline{\Sigma}}^{-1} \end{array} \right| = \tag{VII.36}$$

$$\underline{\underline{\Sigma}}^{-1} - \underline{\underline{\Sigma}}^{-1}\left(\underline{\underline{\Sigma}}^{-1}+\underline{\underline{A}}\right)^{-1}.\underline{\underline{\Sigma}}^{-1} = \left(\underline{\underline{\Sigma}}+\underline{\underline{A}}^{-1}\right)^{-1}$$

The new error rate is obtained when L.O.S. exists.

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = k_1 \frac{\exp\left\{-\left(\underline{h}_0^H.\left[\underline{\underline{I}}_{n_T} + \underline{\underline{\Sigma}}.\underline{\underline{A}}\left(\frac{2E_S}{N_0}\right)\right]^{-1}.\underline{\underline{A}}\left(\frac{2E_S}{N_0}\right).\underline{h}_0\right)\right\}}{\det\left[\underline{\underline{I}}_{n_T} + \underline{\underline{\Sigma}}.\underline{\underline{A}}\left(\frac{2E_S}{N_0}\right)\right]} \tag{VII.37}$$

This expression suggest how many gain a system can have when the receiver has the mean available, because the system investment in power or diversity is rewarded in a exponential way instead of less determinant of denominator. Also the expression reveals that, on the contrary as so many authors have said and following our intuition, the variance is not good and take advantage of their development and use is so many difficult as do the following with the mean. This observation has the origin on the commented benefits of last expression numerator term versus denominator, but also it's so much amazing to note that variance is not zero, and to increase transmitted power saturate exponential term and therefore limits system quality by increasing transmitted energy. This saturation doesn't exist if variance is so low. Then, to give up L.O.S. presence is to give up the best thing a radio channel can provide.

If we proceed to design without CSI, newly the obtained solution is UPA and the matrix A can be written as

$$\underline{\underline{A}} = \alpha.\underline{\underline{I}} = \left(\frac{2E_T}{N_0}.\frac{N}{n_S.n_T}\right).\underline{\underline{I}} \tag{VII.38}$$

And its application into error rate expression generates (VII.39), where the spectacular contribution of L.O.S. presence in the numerator can be appreciated, most of all compared with the case in which this doesn't exist and only the denominator contributes to quality.

$$\Pr_{NO-CSI}^{LOS}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = k_1.\frac{\exp\left\{-\sum_{a=1}^{n_T}\left|h(q)\right|^2.\frac{\alpha}{1+\alpha.\sigma_q^2}\right\}}{\prod_{q=1}^{n_T}\left(1+\alpha.\sigma_q^2\right)} \tag{VII.39}$$

In case of no CSI availability, the formulation of the problem as (VII.40) reveals that this can be easily solved together with constant transmitted energy restriction.

$$Ln\left(P_e\right) = k_2 - \sum_{q=1}^{n_T} \rho_q . \frac{\beta_q^0}{1+\beta_q^0.\sigma_q^2} - \sum_{q=1}^{n_T} Ln\left(1+\beta_q^0.\sigma_q^2\right)$$
$$with \quad \rho_q = \left|h(q)\right|^2 \quad ; \quad \underline{\underline{A}} = diag\left(\beta_q\right) \quad and \quad \beta_q^0 = \beta_q . \frac{2E_S}{N_0} \qquad \text{(VII.40)}$$

The solution consisting on force the Lagrangian value to zero gives two different solutions, in which only one of them presents a minimum, which correspond to positive sign in

$$1+\sigma_q^2.\beta_q^0 = \mu.\sigma_q^2\left(1 \pm \sqrt{1+\left(\frac{2\rho_q}{\mu.\sigma_q^2}\right)}\right) \qquad \text{(VII.41)}$$

This solution can be also written as

$$\frac{2E_S}{N_0}.\beta_q = -\frac{1}{\sigma_q^2} + \mu + \mu\sqrt{1+\left(\frac{2\rho_q}{\mu.\sigma_q^2}\right)} \qquad \text{(VII.42)}$$

And for low L.O.S. it drives another time to statistical water-filling, and for moderate or high L.O.S. drives approximately to

$$\frac{2E_S}{N_0}.\beta_q = -\frac{1}{\sigma_{0q}^2} + \mu, \text{ where } \quad \sigma_{0q}^2 = \frac{\sigma_q^2}{1-\rho_q^{1/2}} \qquad \text{(VII.43)}$$

As in other situations, the μ parameter is adjusted in order to verify the total transmitted energy; the thing that could happen may be no all matrix entries were different from zero. Assuming that transmitted energy is high enough to activate all modes then the μ parameter is given by

$$\mu = \frac{E_T.N}{E_S.n_S.n_T} + \frac{1}{n_T}\sum_{q=1}^{n_T}\sigma_{0q}^{-2} \qquad \text{(VII.44)}$$

Average error rate is then (VII.45) where another time can be appreciated the presence of numerator term, a consequence of L.O.S. presence.

$$\Pr_{CSI}^{LOS}\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) = k_1 . \frac{\exp\left\{-\sum_{a=1}^{n_T}\left|h(q)\right|^2 . \frac{\left(2E_S/N_0\right)\left(\mu-\sigma_{0q}^{-2}\right)}{1+\left(2E_S/N_0\right)\left(\mu-\sigma_{0q}^{-2}\right).\sigma_q^2}\right\}}{\prod_{q=1}^{n_T}\left(1+\left(2E_S/N_0\right)\left(\mu-\sigma_{0q}^{-2}\right).\sigma_q^2\right)} \qquad \text{(VII.45)}$$

In case of CSI existence or absence, the comparison between (VII.45), average error rate with CSI, and (VII.40), average error rate without CSI, shows that the α parameter of the expression without CSI improves as (7.44) due to CSI availability.

$$\alpha = \frac{2E_T}{N_0}\frac{N}{n_S.n_T} \Rightarrow \left(2E_S / N_0\right)\left(\mu - \sigma_{0q}^{-2}\right) \approx \alpha + \left(\frac{2E_S}{N_0.n_T}\right)\sum_{q=1}^{n_T}\frac{\rho_q}{\sigma_q^2} \qquad \text{(VII.46)}$$

It's interesting to note that conclusions have always a big similarity. CSI use is interesting in case of high EsNo and/or low transmitter antennas. The CSI contribution is traduced in the quotient of channel L.O.S. energy and its corresponding variance, even if we keep in mind the validity of the approximation done and that improve this for variances different from zero and changes its form when is zero.

VII.8. SUMMARY

This chapter has described ML detector in MIMO systems, or concretely the quality measures which characterize it. Even in an instantaneous level or in an average level (also called ergodic regime) has been described the corresponding error rate and also the optimal transmitter design even with or without CSI. Both measures and analysis and design will have a great utility for understanding next chapter, in space time coding. Even more, the unitary structure of constellation matrix, in one or multiple channel accesses, will reveal as a standard in channel pre-processing in transmitter as in space time coding.

Finally it's worth mentioning the analysis of MIMO channel with L.O.S., not only as what happen in radio channel, but at the same time as an amazing opportunity to reach high quality and capacity levels, even if CSI in transmitter is available or not.

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VIII.

VIII. SPACE TIME CODES

Miguel Ángel Lagunas, Ana I. Pérez-Neira

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VIII.1. INTRODUCTION

Every time the distance which separate linear and non linear (normally known as coding systems) processing in communication systems is smaller. In this chapter the existing similarities will be developed, in order to describe its formal design, using and features, beginning with linear processing until now explained.

The utility of orthonormal matrices, already described (the last time, in last chapter), dedicated to MSE detection will be converted in the most used tool in code design.

Any codification system, up to a point, doesn't need channel information to work. For this reason, the reader will find formal similarities of what has been exposed in systems seen until this point, to the case where transmitter doesn't know channel state. In fact, the average error rate notion exposed in last chapter will be converted in basic quality evaluation parameter of a space-time coder.

The exposition begins with the simplest system, the called OSTBC (Orthogonal Space-Time Block Codes). Its peculiarity is that OSTBC allow instant detection of received symbols with a minimum complexity in receiver. Two different formulations will be exposed. One specify way and another which connect OSTBC directly to block codes and convolutional.

The exposition will continue with the MIMO version of convolutional codes, also known as "trellis codes", without question the best alternative when quality is the problem and we might assume complexity in decoding. Differential systems will be eventually described, furthermore because they are the only efficient alternative when CSI is neither available in receiver.

VIII.2. OSTBC CODES

Recovering transmission system of a single symbol s1(n), over MIMO channel and the corresponding design for CSI absence, the reader should remember that process matrix must verify orthogonality (VIII.1), that is, an uniform power distribution in all diversity axes available.

$$\underline{\underline{B}}.\underline{\underline{B}}^H = \underline{\underline{I}}_{n_T} \qquad \text{(VIII.1)}$$

With this transmitter process, ML receiver will be directly the trace operator over the process adapted to transmitter. That is to say if (VIII.2) was transmitted signal, then (VIII.3) was the transmitted symbol expression adopted by ML detector.

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s1(n) \tag{VIII.2}$$

$$\begin{aligned} &\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{X}}_{T,n} + \underline{\underline{W}}_n \\ &\widehat{s1} = \operatorname{sgn}\left(Trace\left[\underline{\underline{B}}^H \underline{\underline{H}}^H .\underline{\underline{X}}_{R,n}\right]\right) \end{aligned} \tag{VIII.3}$$

It's important to emphasize that the estimation of transmitted symbol directly from trace is only valid in BPSK or QPSK symbols. In other cases this detector is not optimal. In spite of so many authors take this for granted, we will insist another time on it because, as the reader will remember, in chapter II this topic was clarified proving that ML detector only coincide with (VIII.3) for these constellations. Moreover it must be remembered that if detector isn't the optimal one, it doesn't mean their use in a quality detriment.

Even if any constellation is used, it's clear that the statistic of any detector must be the trace appeared in second term of (VIII.3) and in this one now we will center our attention.

Remembering the condition in (VIII.1), it's quite obvious that so many matrices which verify this condition exist. In fact, in last chapter it was made clear that the use of phasors as multidimensional matrices input could archieve this property, even if the number of channel accesses were larger or equal than transmitter antennas. Without loss of generality we will concentrate our presentation in codes with lower redundancy possible, that is to say, number of channel accesses equal to transmitter antennas. Because of that, the matrix (VIII.2) will be a square matrix.

Following with design possibilities of transmitter matrix, with the only condition of the orthogonality, there are multiple options. The most interesting ones in this section will be those who have zero, one or minus one in their input, and then a linear processing system will be converted in a code. For instance in (VIII.3) are the available options which accomplish (VIII.1), with two transmitter antennas.

$$\begin{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & \cdots \end{matrix} \tag{VIII.4}$$

The thing is, if all these matrices were valid, we could use more of one simultaneously. The motivation of this argument is that, when two channel accesses are done for a single real symbol, the system will has a very low speed. The possibility of use more than one matrix will allow increasing the transmission system speed.

If two real symbols are transmitted, s1 and s2, with some different matrices, the transmitted signal would be

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 + \underline{\underline{B}}_2.s2 \tag{VIII.5}$$

This signal, after MIMO channel and when detection of one of both symbols (let’s say, s1) is done, it will generate a desired component and an interference of the other symbol, as well as receiver noise.

$$\hat{s}1 = \underbrace{Traza\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{B}}_1\right].s1}_{desired} + \underbrace{Trace\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{B}}_2\right].s2}_{ISI} \qquad \text{(VIII.6)}$$

It’s clear that criterion of non distortion for desired signal means the orthonormality of both matrices used, that is:

$$\underline{\underline{B}}_1.\underline{\underline{B}}_1^H = \underline{\underline{B}}_2.\underline{\underline{B}}_2^H = \underline{\underline{I}}_{n_T} \qquad \text{(VIII.7)}$$

However, in order to remove ISI, the condition appears in principle so much complicated. Clearly, and using the circularity symmetric trace property, the solution for no ISI would be the orthogonality of two matrices.

$$Trace\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{B}}_2\right] = Trace\left[\underline{\underline{R}}_H.\underline{\underline{B}}_2.\underline{\underline{B}}_1^H\right] = 0 \quad \Rightarrow \quad \underline{\underline{B}}_2.\underline{\underline{B}}_1^H = \underline{\underline{0}}\,? \qquad \text{(VIII.8)}$$

This condition for a square matrix isn’t possible. Then it seems the only possibility is to send a single symbol. The solution comes if we develop a little bit more the problem, as we’ll see.

Due the ISI term will be, in general, complex, it will be written as

$$Trace\left[\underline{\underline{R}}_H.\underline{\underline{B}}_2.\underline{\underline{B}}_1^H\right] = \alpha + j.\beta \qquad \text{(VIII.9)}$$

As channel matrix is hermetic, the conjugate expression of the last one will be

$$Trace\left[\underline{\underline{R}}_H.\underline{\underline{B}}_1.\underline{\underline{B}}_2^H\right] = \alpha - j.\beta \qquad \text{(VIII.10)}$$

If we want the ISI real part be zero in order to detect symbol s1 without ISI, it’s necessary that (VIII.9) and (VIII.10) be equal (but with the sign inverted). With this condition, the elected matrices require verifying a so much facile or relaxed condition, written in (VIII.11).

$$\underline{\underline{B}}_1.\underline{\underline{B}}_2^H = -\underline{\underline{B}}_2.\underline{\underline{B}}_1^H \qquad \text{(VIII.11)}$$

If this condition is verified, the obtaining of two transmitted symbols would be:

$$\begin{aligned} \hat{s}1 &= \mathrm{Re}\left(Trace\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right) \\ \hat{s}2 &= \mathrm{Re}\left(Trace\left[\underline{\underline{B}}_2^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right) \end{aligned} \qquad \text{(VIII.12)}$$

The condition in (VIII.11) has a special name, friendly matrices, that means these matrix establish their link in a more "friendly" way that orthogonality, quite impossible condition. For instance, in (VIII.13) there is a possible two friendly matrices selection for simultaneous transmission and reception without ISI of two real symbols.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 + \underline{\underline{B}}_2.s2 = \begin{pmatrix} s1 & -s2 \\ s2 & s1 \end{pmatrix} \tag{VIII.13}$$

The question now is if the multiplexed symbols can be higher, keeping the detector as optimal. In concrete the detector will also kept if there are QPSK symbols in use, and then there are the possibility of include two more streams but now over imaginary part of transmitted signal.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 \quad \underline{\underline{B}}_4$$
$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 + \underline{\underline{B}}_2.s2 + j.\underline{\underline{B}}_3.s3 + j.\underline{\underline{B}}_4.s4 \tag{VIII.14}$$

These two symbols will be detected taking the imaginary part of corresponding trace detector. In order to not to interfere with last ones, there is a condition for real part, which i

$$\mathrm{Re}\left(Trace\left[\underline{\underline{R}}_H \cdot j.\underline{\underline{B}}_3.\underline{\underline{B}}_1^H\right]\right) = \mathrm{Im}\left(Trace\left[\underline{\underline{R}}_H.\underline{\underline{B}}_3.\underline{\underline{B}}_1^H\right]\right) = 0 \tag{VIII.15}$$

To accomplish this condition is only necessary the matrices to be friendly with without sign changing.

$$\underline{\underline{B}}_1.\underline{\underline{B}}_3^H = \underline{\underline{B}}_3.\underline{\underline{B}}_1^H$$
$$\underline{\underline{B}}_2.\underline{\underline{B}}_4^H = \underline{\underline{B}}_4.\underline{\underline{B}}_2^H \tag{VIII.16}$$

On the other hand, and because there don't be interference between them, both new symbols will also require their matrices were friendly with minus sign. Therefore, equation (VIII.17) has all the possibilities of four matrices.

$$\underline{\underline{B}}_1.\underline{\underline{B}}_1^H = \underline{\underline{B}}_2.\underline{\underline{B}}_2^H = \underline{\underline{B}}_3.\underline{\underline{B}}_3^H = \underline{\underline{B}}_4.\underline{\underline{B}}_4^H = \underline{\underline{I}}_2$$
$$\underline{\underline{B}}_1.\underline{\underline{B}}_2^H = -\underline{\underline{B}}_2.\underline{\underline{B}}_1^H \quad \underline{\underline{B}}_4.\underline{\underline{B}}_3^H = -\underline{\underline{B}}_3.\underline{\underline{B}}_4^H$$
$$\underline{\underline{B}}_1.\underline{\underline{B}}_3^H = \underline{\underline{B}}_3.\underline{\underline{B}}_1^H \quad \underline{\underline{B}}_2.\underline{\underline{B}}_4^H = \underline{\underline{B}}_4.\underline{\underline{B}}_2^H \tag{VIII.17}$$

The four streams will be recovered over real/imaginary part of detector output to each symbol.

$$\begin{aligned}
\hat{s}1 &= \mathrm{Re}\left(Trace\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{X}}_R\right]\right) \\
\hat{s}2 &= \mathrm{Re}\left(Trace\left[\underline{\underline{B}}_2^H . \underline{\underline{R}}_H . \underline{\underline{X}}_R\right]\right) \\
\hat{s}3 &= \mathrm{Im}\left(Trace\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{X}}_R\right]\right) \\
\hat{s}4 &= \mathrm{Im}\left(Trace\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{X}}_R\right]\right)
\end{aligned} \tag{VIII.18}$$

An example of matrices selection for an scheme with two transmitter antennas is the following one.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \underline{\underline{B}}_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 .s1 + \underline{\underline{B}}_2 .s2 + \underline{\underline{B}}_3 .j.s3 + \underline{\underline{B}}_4 .j.s4 = \begin{pmatrix} s1 + j.s3 & -s2 + js4 \\ s2 + j.s4 & s1 - j.s3 \end{pmatrix} = \begin{pmatrix} z1 & -z2^* \\ z2 & z1^* \end{pmatrix} \tag{VIII.19}$$

This OSTBC for two antennas is popularly known as Alamouti code, and in last chapter it appeared with a different formulation in ML detection context. The thing which characterizes an OSTBC is its easier ML detection because it doesn't require a search over the constellation. In fact, it's worth mentioning that even if detector is only optimum for QPSK symbols, the simplicity of detector makes it to be used even if M-QAM constellation is used.

The interest of OSTBC is the simultaneous symbol transmission in a number of channel accesses equal to transmitter antenna number with a simple detector and being fair because all symbols are detected with an equal SNR, and a gain value, over channel without diversity, which is equal to channel matrix trace and, without require CSI in transmitter.

If we have an OSTBC which transmits two complex symbols in two accesses all its effects are good for radio channel. The problem is that this can't be translated to systems with more than two antennas and full rate (transmits simultaneously n_T complex symbols in the same number of accesses), because friendly matrices for this case doesn't exist. But for non full-rate systems there exists friendly matrices in order to form the code. For example (VIII.20) shows an OSTBC in a system with four antennas with rate ¾.

$$\begin{bmatrix} s1 & 0 & s2 & -s3 \\ 0 & s1 & s3^* & s2^* \\ -s2^* & -s3 & s1^* & 0 \\ s3^* & -s2 & 0 & s1^* \end{bmatrix} \tag{VIII.20}$$

Checking this equation we might have the temptation of include a fourth symbol in empty spaces of this matrix. As an exercise for the reader, it's possible to check that then system will have ISI and it must renounce to simplicity of trace detector.

VIII.3. CONVOLUTIONAL CODING: TRELLIS CODES

As we have seen in last chapter and also in this one, in case of OSTBC the streams to be transmitted access the channel in so many different times. Concretely, in an OSTBC each symbol accesses the channel a number of times not lower than number of transmitted antennas. A signal processing approach will identify all as a FIR system in which every sample keeps in filter during an interval of its length. Obviously the alternative would be to consider the IIR case, that is to say include a recursion which determinates the accesses that every symbol makes to the channel. In case that this recursion was done over a finite arithmetic we would be in front of a code, badly called convolutional. Next it will be seen as with this perspective can be generated some convolutional codes in case of space-time diversity.

The coder will have, as in a IIR filter, two different parts. The first one corresponding to input symbols and the other corresponding to system memory. If considered coder admits R symbols a(1), a(2), …, a(R), all of them grouped in vector $\underline{a}$, the coder produces an output with a symbol vector x which will depends even on input symbols as its memory, called state and denoted as vector $\underline{b}$. The formulation is similar to equation of measure of Gauss model.

$$\underline{x} = \left\lfloor \underline{\underline{G}}_1 . \underline{a} + \underline{\underline{G}}_2 . \underline{b} \right\rfloor^{\square} \qquad \text{(VIII.21)}$$

Where $\lfloor . \rfloor^{\square}$ denotes that the result is translated (in every component of vector $\underline{x}$) to integers between 0 and 2^L-1. That is to say L bits are produced in each vector component and these determinate the constellation point to be transmitted by the corresponding antenna. For instance, in a 2bits/Hz system the constellation will be QPSK and the components of $\underline{x}$ will have the value 0,1,2 or 3. Depending on this value the corresponding antenna will transmit 1, j, -1 or –j. This way the rate of space-time coder will be

$$Rate = \frac{R}{L} \quad , n_T \text{ antennas} \qquad \text{(VIII.22)}$$

The equation which completes the coder is the one which capture the state evolution. This equation, also similar to the state equation of a Gauss model, takes the following form:

$$\underline{b} = \left\lfloor \underline{\underline{G}}_3 . \underline{a} + \underline{\underline{G}}_4 . \underline{b} \right\rfloor^{\oplus} \qquad \text{(VIII.23)}$$

Where $\lfloor . \rfloor^{\oplus}$ denotes that operation is done in 2^k levels. The size of component numbers of state vector M determines to a large extent the detector complexity because it fixes the Trellis size a ML detector must follow.

In short, in addition to matrices used in two equations of coder, it can be seen that parameters which characterize them.

- Number of antennas, of design, n_T.
- Bits/Hz or size of constellation used (L bits).
- Complexity or number of states $2^{K \cdot M}$.
- Rate, quotient between input symbols and L.

This way, for instance, a code, st2bh2est4rate1, with 2 bits/Hz or QPSK constellation for each antenna, has four possible states and admits all input bits. Without loss of generality, in what follows, the presentation will be limited to binary case, that is, K will be the unity.

The Figure VIII-1 summarizes the scheme of a coder as we have described until now.

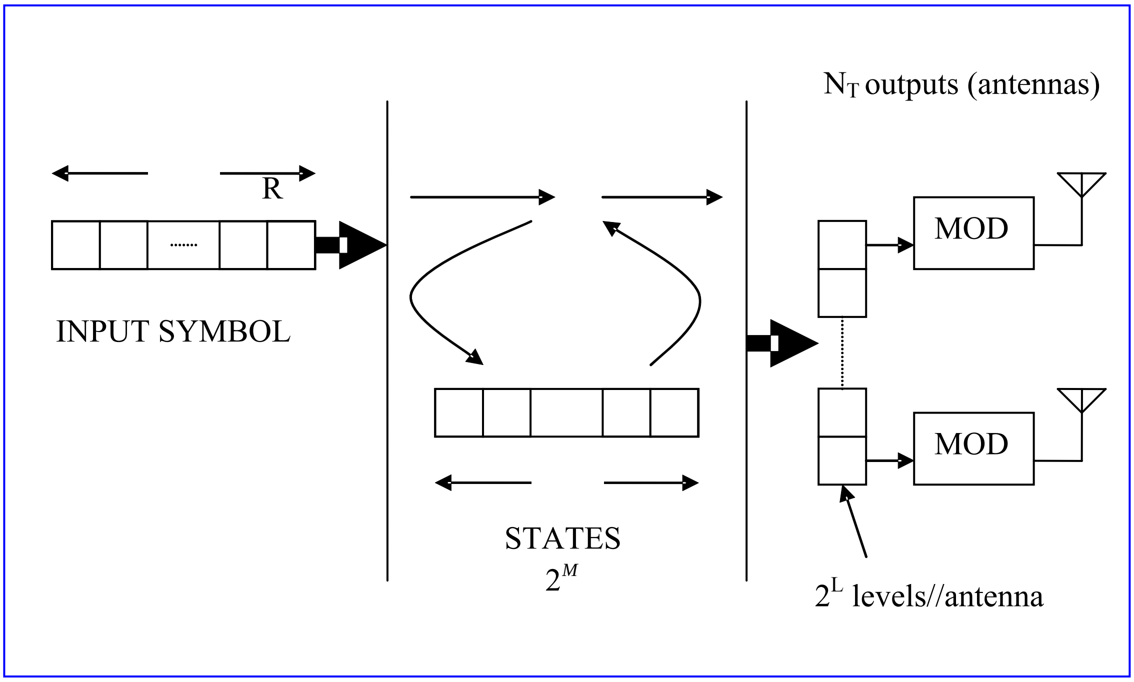

Figure VIII-1.- Trellis Coder scheme with its parameter most relevant.

Away from general formulation, which presents a great number of degrees of freedom, it exists a more compact and simple way to implement a Trellis coder. Fundamentally, relating to measure equation, it is done in a compact way grouping input and state in a single vector.

$$\underline{c} = \begin{bmatrix} a(1) & \ldots & a(R) & b(1) & \ldots & b(M) \end{bmatrix} \qquad \text{(VIII.24)}$$

And the vector which contains the index which determine the constellation signal to use, is given by (VIII.25), where $\lfloor . \rfloor^{\oplus}$ indicates that the result (operation) is done in L bits arithmetic.

$$\underline{x} = \left\lfloor \underline{\underline{G}}.\underline{c} \right\rfloor^{\oplus} \qquad \text{(VIII.25)}$$

The state vector operation is done by means of scrolling the vector c, M positions to the right. With this, coder implementation complexity in transmitter can be reduced.

For instance, the coder st2bh2est2rate1 is done as indicate (VIII.26), with x1 and x2 the indexes which determinate QPSK signals to transmit through antennas 1 and 2 respectively. All operations are done module 4.

$$\begin{bmatrix} a(1) & a(2) & b(1) & b(2) \end{bmatrix} . \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x1 & x2 \end{bmatrix} \qquad \text{(VIII.26)}$$

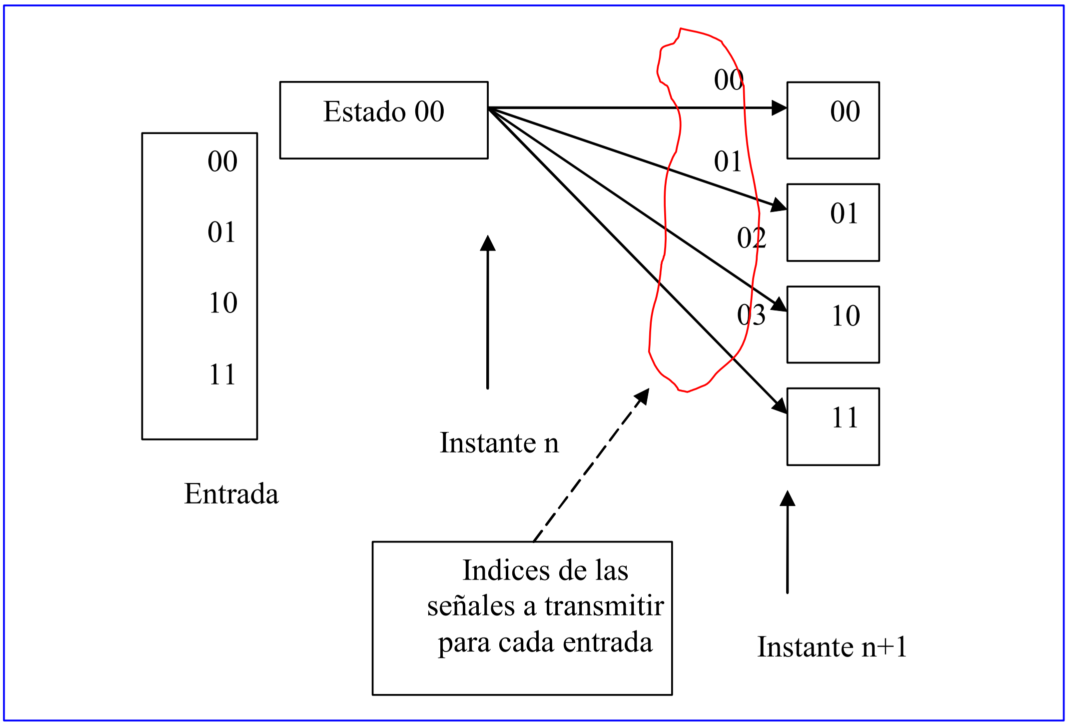

Figure VIII-2.- Input, initial state, transition with transmitted signals and new state (coder st2bh2est2rate1)

The best way to describe the coder is by means of its lattice scheme, let's say, Trellis scheme. This diagram contents in every instant and in vertical arrangement, the

2^M nodes which correspond to the state in the instant n. Depending on input, this system jumps to a different node or state for n+1 instant, and over each transition branch are indicated the signal indexes which has been transmitted in instant n. The diagram in lattice for last code would be the one sketched in Figure VIII-2. In a more summarized way and as it is used to appear in literature, the Figure VIII-3 summarizes the code used in example.

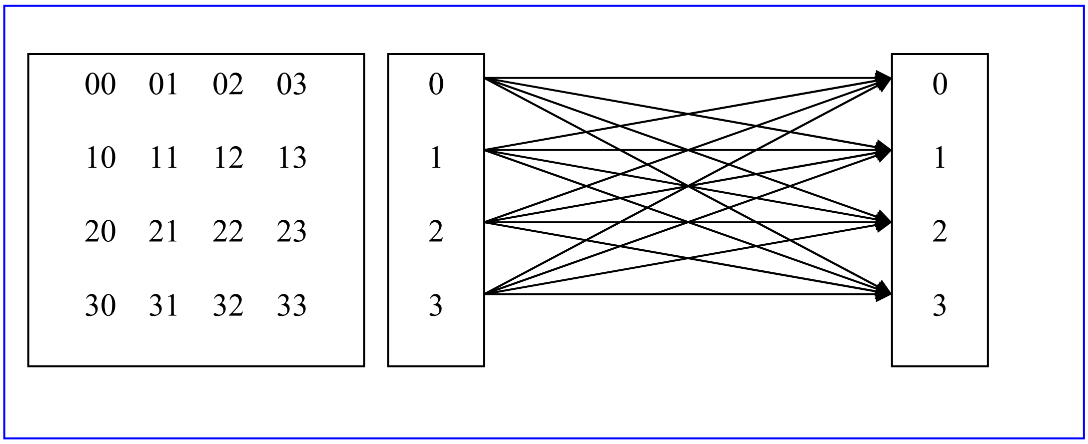

Figure VIII-3.- Lattice diagram. The left part represents indexes of transmitter signal for every node to the inputs in order

Relative to pass from indexes to transmitted signal, the Figure VIII-4 clarifies that with an example.

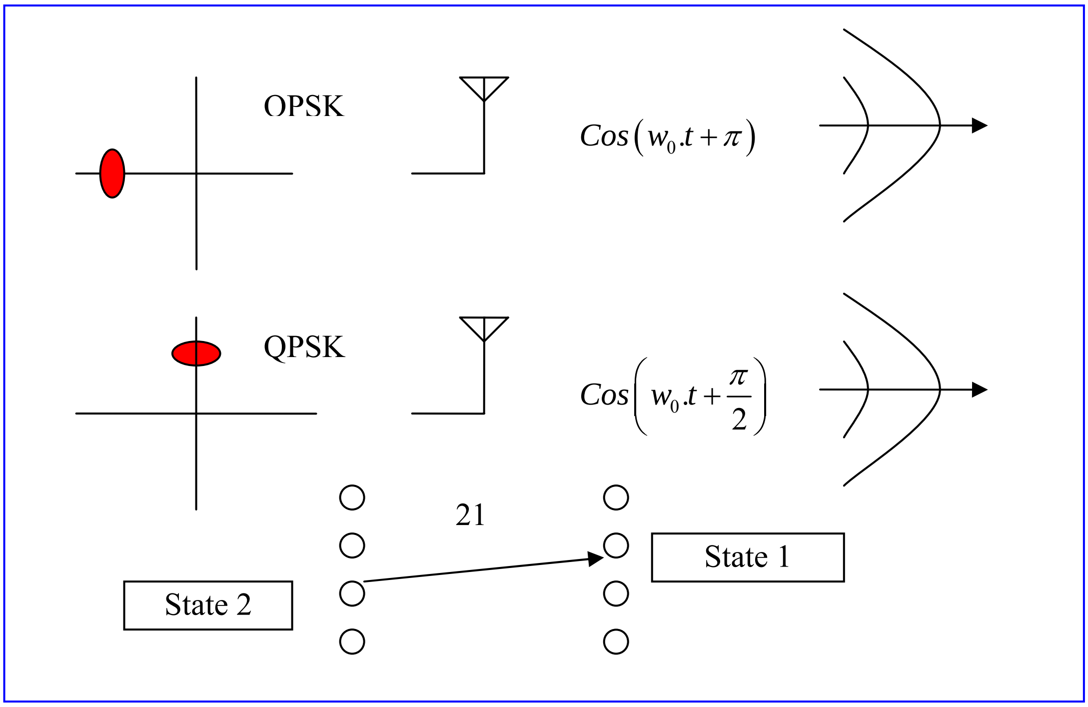

Figure VIII-4.- Transition example, from state 2 to state 1. The transition indexes are 2 for the antenna1 and 1 for the antenna2. It's indicated the waveform transmitted for these indexes.

As transmitter only emits directly the signals, the analysis of how receiver works will be simple. Essentially the receiver must compute the likelihood of all possible paths between a start point and a finish point. This last one can be determinated from decoding delay. The receiver is suboptimal but the complexity remains delimited. Imagine that delay decoding is set in four access, for instance. The detector must compute all possible paths which from start point, arrives four accesses later. This case is represented in Figure VIII-5. Imagine that transmitted sequence follows the path marked with thick trace; if other path, for instance the discontinuous one, presents a greater likelihood, an error will be produced.

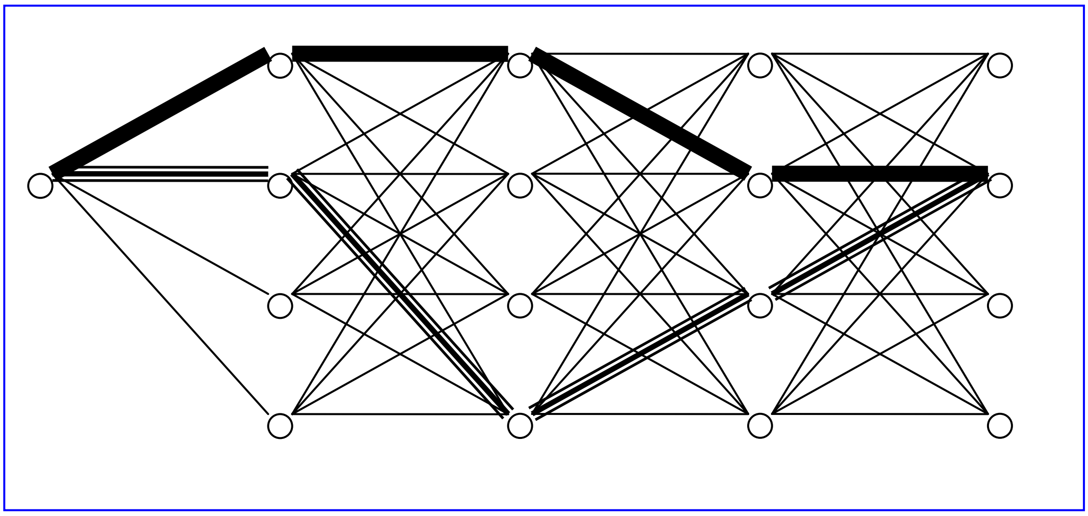

Figure VIII-5.- Evolution of correct and one erroneous paths over the lattice in a decoder with a 4 accesses delay.

If we denominate $\underline{s}_n$ the wave forms transmitted correctly, and $\underline{b}_n$ to the ones which have to be transmitted if detector doesn't be in error, it's clear that error probability will have the expression (VIII.27), as it can be seen in last chapter. In this expression, N is the number of accesses over decision is based.

$$\Pr\left(\underline{s}_n \Rightarrow \underline{b}_n; n=1,N\right) = Q\left(\sqrt{\left(\frac{E_s}{2.N_0}\right).Traza\left(\underline{\underline{R}}_H . \underline{\underline{A}}\right)}\right) \qquad \text{(VIII.27)}$$

At the same time, matrix $\underline{\underline{A}}$, as it doesn't exist a process directly before transmission, will be directly (VIII.28). Last chapter can be checked again.

$$\underline{\underline{A}} = \sum_{n=1}^{N} \left(\underline{s}_n - \underline{b}_n\right).\left(\underline{s}_n - \underline{b}_n\right)^H \qquad \text{(VIII.28)}$$

Obviously, over this expression can be used the same reasoning and proceed forms applied or used in last chapter.

Even less formal, the way as this codes has been designed is based in average error probability. Concretely, from expression of mentioned error rate is done the good

SNR approximation, and the original formula is reduced to a more simple expression, as indicated as follows:

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1 . \prod_{p=1}^{n_R} \frac{1}{\det\left[\underline{\underline{I}}_{n_T} + \frac{2E_S}{N_0} . \underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]} \approx \left|\begin{matrix}\text{For}\\ \text{high}\\ \text{SNR}\end{matrix}\right| \approx$$

$$\approx k_1 . \left(\frac{2E_S}{N_0}\right)^{-n_R.n_T} . \prod_{p=1}^{n_R} \frac{1}{\det\left[.\underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]} \qquad \text{(VIII.29)}$$

Even more, considering a unitary channel, the error probability is only function of the determinant of error matrix, as indicates the following equation:

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1 . \left(\frac{2E_S}{N_0}\right)^{-n_R.n_T} . \prod_{p=1}^{n_R} \frac{1}{\det\left[.\underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]} \approx k_2 . \left(\frac{2E_S}{N_0}\right)^{-n_R.n_T} . \left(\det\left(\underline{\underline{A}}\right)\right)^{-n_R} \qquad \text{(VIII.30)}$$

This criterion, called “determinant criterion”, and due to Calderbank, reduces the design of code matrix to those which maximize the worst determinant, let’s say, over all possible paths, the one which is searched is the one with lower determinant and then it is maximized over code matrix in (VIII.25).

Obviously this exhaustive search is unworkable and it can be reduced to a large extent if the code is uniform, that is to say the design can be done taking as base any sequence. It’s interesting to note that, although the condition seems mathematically complex , it’s enough to observe the lattice diagram of code and verify that changing the node or state order from larger to smaller, instead of its habitual from smaller to larger, this finally won’t change. If the code is non uniform, the sequence can be whatever and it’s keep, for simplicity, the derivate of an all zeros entry, that is, the upper branches.

Once reference sequence is taken, it must be searched the worst in terms of determinant. This above-mentioned search is simple because it’s enough to find the sequence that works with a lower determinant, beginning from the same state (0) and arriving from a different path another time to state 0. Once the worst determinant is found, the coding gain will be given by (VIII.31), which is equal to geometric mean of error matrix eigenvalues different from zero. It’s clear that in order to maximize this gain, before that we must get the complete range, that is, equal to number of transmitter antennas.

$$Gain = \left[\det\left(\underline{\underline{A}}\right)_{bigger}\right]^{-r} \qquad \text{(VIII.31)}$$

An example of code st2bh1est2rate0.5 is given. The optimal code, found by means of direct search, has the matrix (VIII.32) and its implementation appeared in Figure VIII-6.

$$\underline{\underline{G}} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{VIII.32}$$

Its Trellis diagram is simple and appears in the same figure

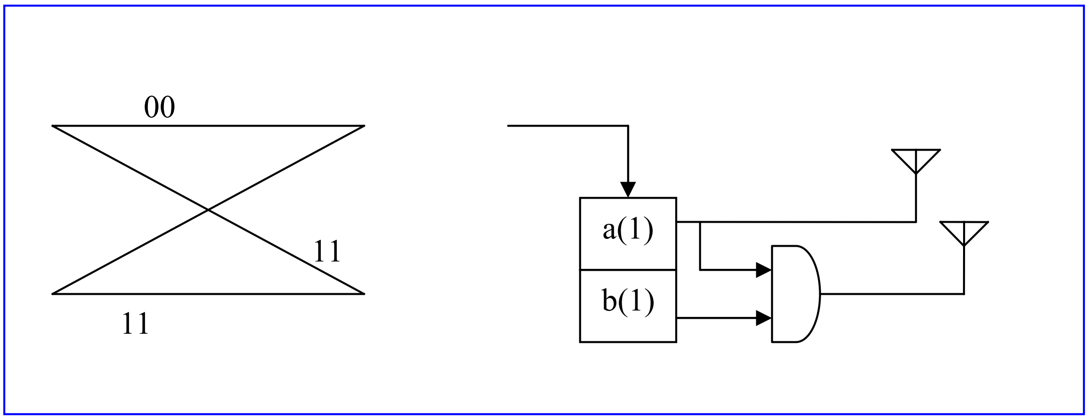

Figure VIII-6.- Lattice and realization of code st2bh1est2rate0.5.

The search of nearer sequence (minimum determinant) that starts with state 0 and returns to this is a two accesses one, and is refered to the signals with index 11/01 over BPSK (because it's a 1 bit/Hz/seg code). The Figure VIII-7 visualizes the reference sequence signalized with 00/00 and the nearer 11/01.

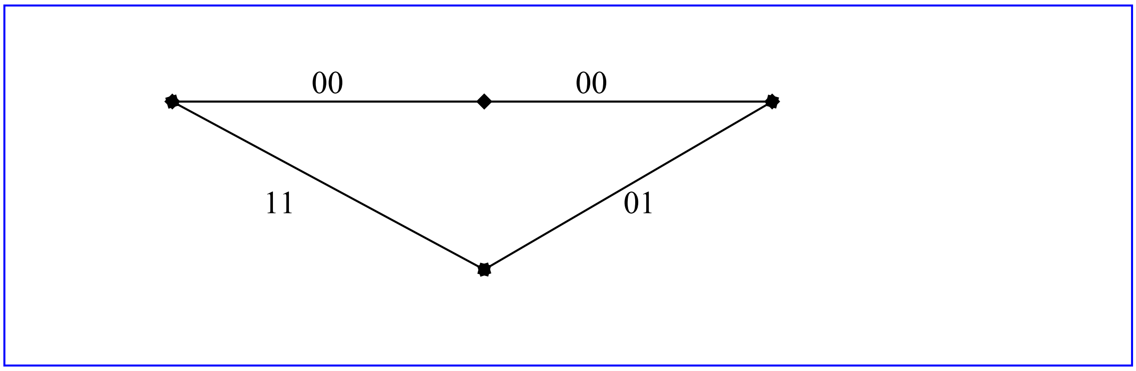

Figure VIII-7.- Correct sequence and "nearer" one, over trellis diagram of last figure code.

In order to calculate the determinant, must be remembered that in BPSK, 0 refers to +1 and 1 refers to –1 (the result won't change if this assignation is changed). This way, error vectors in both accesses are:

$$\underline{s}_1 - \underline{b}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad y \quad \underline{s}_2 - \underline{b}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{VIII.33}$$

Thus, the worst error matrix and code gain will be:

$$\underline{\underline{A}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.\begin{bmatrix} 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}.\begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 8 \end{pmatrix} \qquad \text{(VIII.34)}$$

$$\det\left(\underline{\underline{A}}\right) = 16 \qquad Gain = \sqrt{16} = 4$$

Related to this expression, as the origin of determinant criterion isn't formal and in last chapter are found better procedures, there exist another reviews or objections. The first one, the important thing is not only the worst path, but in so much complicated codes, the error probability shown by the code differs also depending on the probability of this selected path (in gain calculus) be very high or very low. A bad gain which comes from a little probable path won't impact seriously the system quality with a moderate gain, which is very probable.

As commented, a more formal study of these codes would have to be done in last chapter suggested lines, even for instantaneous design as for statistical design.

The most used Trellis codes are in the following list. These are only based in determinant criterion (and also take this as code gain), raised to $-n_T$.

St2bh2est8 Gain 2 $\quad \underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$

St2bh2est16 Gain $\sqrt{32}$ $\quad \underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 2 & 0 & 2 \end{bmatrix}$

St2bh1est2 Gain 4 $\quad \underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

St2bh1est4 Gain $\sqrt{48}$ $\quad \underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$

St2bh1est8 Gain $\sqrt{80}$ $\quad \underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

St2bh1est16 Gain $\sqrt{128}$ $\quad \underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

St3bh1est8 Gain $\sqrt[3]{256}$ $\quad \underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Next section, unlike the previous ones, tackles the problem of system design when the receiver hasn't CSI either. An habitual situation is done when radio channel presents a large variability making unfeasible any channel estimation technique by receiver, and of course also useless in transmitter.

VIII.4. CODES TO RECEIVE WITHOUT CSI

In case the receiver hasn't CSI available, the transmission using space time codes is still possible. Suppose that code matrices alphabet is designed under (VIII.35), hence $\log_2 M$ bits are coded for every N accesses (that will be supposed equal to n_T, without limit the general presentation form).

$$\underline{\underline{C}}_m \quad m = 1, M \tag{VIII.35}$$

Imposing the orthogonality condition so that any codeword must verify

$$\underline{\underline{C}}_m . \underline{\underline{C}}_m^H = \underline{\underline{I}}_{n_T} \qquad \forall m = 1, M \tag{VIII.36}$$

Received signal will be

$$\underline{\underline{Y}}_R = \left(\frac{2E_s}{N_0} \right) . \underline{\underline{H}} . \underline{\underline{C}}_0 + \underline{\underline{W}} = \rho . \underline{\underline{H}} . \underline{\underline{C}}_0 + \underline{\underline{W}} \tag{VIII.37}$$

Under unknown channel condition, the received matrix has a Gaussian distribution with zero mean and covariance matrix as (VIII.38). Note that covariance formulation, transposed times direct instead of the contrary, will adopt the channel expectancy value (the channel is supposed Rayleigh).

$$\underline{\underline{\Sigma}} = E\left[\underline{\underline{Y}}_R^H . \underline{\underline{Y}}_R \right] = \underline{\underline{I}} + \underline{\underline{C}}_0^H . E\left[\underline{\underline{H}}^H . \underline{\underline{H}} \right] . \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho . \underline{\underline{C}}_0^H . \underline{\underline{R}}_{HA} . \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho \left| H_0 \right|^2 . \underline{\underline{C}}_0^H . \underline{\underline{C}}_0 \tag{VIII.38}$$

Moreover is logical to suppose this channel matrix will present a diagonal structure in all its terms, and therefore when defining a new ρ value, its formulation will be more compact and will essentially depend of codeword. The probability of received signal is given by (VIII.39) and, after to use the inverse lemma over covariance, and keeping in mind the orthogonality condition of codewords (see (VIII.36)), this allows writing the optimal detector.

$$\begin{aligned} \Pr\left(\underline{\underline{Y}}_R / \underline{\underline{C}}_0 \right) &= k_0 . \exp - \left[Trace\left(\underline{\underline{Y}} . \underline{\underline{\Sigma}}^{-1} . \underline{\underline{Y}}^H \right) \right] = \\ &= k_0 . \exp - \left[Trace\left(\underline{\underline{Y}} . \left(\underline{\underline{I}} + \rho' . \underline{\underline{C}}_0^H . \underline{\underline{C}}_0 \right) . \underline{\underline{Y}}^H \right) \right] \end{aligned} \tag{VIII.39}$$

$$\hat{m} = \underset{\underline{\underline{C}}_m, m=1,M}{\arg\max} \left[Trace\left(\underline{\underline{Y}}_R . \underline{\underline{C}}_m^H . \underline{\underline{C}}_m . \underline{\underline{Y}}_R^H \right) \right] \tag{VIII.40}$$

The (VIII.40) detector is the optimal receiver when no CSI in receiver is present. Now it will be seen how is the quality of this detector and how this is degraded comparing to the case when CSI is available.

If the transmitted codeword is zero, it's clear than its likelihood, in order to not to produce an error with the codeword one, must be larger than this second. In detector terms, it must be verified:

$$Trace\left(\underline{\underline{Y}}_R.\underline{\underline{C}}_0^H.\underline{\underline{C}}_0.\underline{\underline{Y}}_R^H\right) > Trace\left(\underline{\underline{Y}}_R.\underline{\underline{C}}_1^H.\underline{\underline{C}}_1.\underline{\underline{Y}}_R^H\right) \qquad \text{(VIII.41)}$$

If we apply in this expression the correct received signal expression, we arrive to the following inequality, in which the square error term has been omitted.

$$\rho.Trace\left[\underline{\underline{H}}.\left(\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H.\underline{\underline{C}}_1.\underline{\underline{C}}_0^H\right).\underline{\underline{H}}^H\right] > 2\rho^{1/2}.\mathrm{Re}\left\{Trace\left[\underline{\underline{W}}.\left(\underline{\underline{C}}_0^H.\underline{\underline{C}}_0-\underline{\underline{C}}_1^H.\underline{\underline{C}}_1\right).\underline{\underline{C}}_0^H.\underline{\underline{H}}^H\right]\right\} \qquad \text{(VIII.42)}$$

In this expression, the left term is a random variable and the right one is a threshold. When random variable doesn't verify the inequality an error is made. Reasoning in the same way in last chapter, the following expressions can be written: The error probability and also the averaged one respect to channel statistic.

$$\begin{aligned}
&\Pr\left(\underline{\underline{C}}_0 \rightarrow \underline{\underline{C}}_1\right) \approx Q\left(\sqrt{\left(\frac{2E_s}{N_0}\right).Trace\left(\underline{\underline{H}}.\underline{\underline{A}}_{NOCSI}.\underline{\underline{H}}^H\right)}\right) \approx \\
&\approx k_1.\exp\left[-\left(\left(\frac{2E_s}{N_0}\right).Trace\left(\underline{\underline{H}}.\underline{\underline{A}}_{NOCSI}.\underline{\underline{H}}^H\right)\right)\right] \qquad \text{(VIII.43)} \\
&\Pr^{AVE} \approx k_2.\prod_{p=1}^{n_R}\frac{1}{\det\left[\underline{\underline{I}}-\left(\frac{E_s}{2.N_0}\right)\underline{\underline{A}}_{NOCSI}.\underline{\underline{\Sigma}}_p\right]}
\end{aligned}$$

Note than transmitted energy, with N accesses to the channel is given by

$$E_S = \left(\frac{N}{n_T}\right).E_T \qquad \text{(VIII.44)}$$

The fundamental difference is, now the code gain isn't give by error matrix between both matrices, but from their product (subtracted to identity). In (VIII.45) the reader can see the differences between both gains.

$$\begin{aligned}
&\underline{\underline{A}}_{CSI} = \left(\underline{\underline{C}}_0-\underline{\underline{C}}_1\right)\left(\underline{\underline{C}}_0-\underline{\underline{C}}_1\right)^H \\
&\underline{\underline{A}}_{NOCSI} = \underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H.\underline{\underline{C}}_1.\underline{\underline{C}}_0^H
\end{aligned} \qquad \text{(VIII.45)}$$

In order to compare both expressions, note the following:

$$\left|\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\right|^2=\underline{\underline{I}}+\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\underline{\underline{C}}_1.\underline{\underline{C}}_0^H-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H-\underline{\underline{C}}_1.\underline{\underline{C}}_0^H=$$
$$2.\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H-\underline{\underline{C}}_1.\underline{\underline{C}}_0^H-\underline{\underline{A}}_{NOCSI}=\underline{\underline{A}}_{CSI}-\underline{\underline{A}}_{NOCSI}$$
$$Asi \quad pues,$$
$$\underline{\underline{A}}_{CSI}=\underline{\underline{A}}_{NOCSI}+\left|\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\right|^2 \qquad \text{(VIII.46)}$$

That is to say, CSI loss in receiver reduce code gain which is measured by the determinant in (VIII.45), but it was clear (see (VIII.46)) that this will be this way.

A solution to stabilize this decrease with a low complexity receiver is given by differential space-time codes, these codes are not just an extrapolation to diversity case of differential modulations of any basic communications course.

VIII.5. DIFFERENTIAL S-T CODES

Before to analyze the differential problem and in order to make easier the explanation it's interesting to remember how the detector is, for a MIMO system in which codewords are matrices. These M matrices, equal to 2^L, allow accommodating L streams in N accesses, that is to say codewords will have dimension (n_T, N). The codewords alphabet, as in last section, will be represented:

$$\underline{\underline{C}}_m \quad m=1,M \qquad \text{(VIII.47)}$$

If these matrices are orthonormal, as in previous section, the ML detector is reduced to (VIII.48), considering for the moment that detector has channel information (CSI).

$$\left|\underline{\underline{X}}_R-E_s^{1/2}.\underline{\underline{H}}.\underline{\underline{C}}_m\right|_F \Rightarrow \hat{\underline{\underline{C}}}=\underset{\underline{\underline{C}}_m;m=1,M}{Max}\left[\mathrm{Re}\left(Traza\left(\underline{\underline{H}}.\underline{\underline{C}}_m.\underline{\underline{X}}_R\right)\right)\right] \qquad \text{(VIII.48)}$$

An analysis similar to the one previously made, will give the expressions in (VIII.49): error probability, exponential level and average. Always in the habitual case, low antenna correlation in receiver and Rayleigh channel.

$$Pe=\Pr\left(\underline{\underline{C}}_m\to\underline{\underline{C}}_n;\tilde{\underline{\underline{C}}}\equiv\underline{\underline{C}}_m-\underline{\underline{C}}_n\right)=k_0.Q\left(\sqrt{\frac{E_s}{2.N_0}.Traza\left(\underline{\underline{R}}_H.\tilde{\underline{\underline{C}}}.\tilde{\underline{\underline{C}}}^H\right)}\right)$$
$$Pe\approx k_1.\exp\left[-\frac{E_s}{4.N_0}.Traza\left(\underline{\underline{R}}_H.\tilde{\underline{\underline{C}}}.\tilde{\underline{\underline{C}}}^H\right)\right]=k_1.\exp\left[-\frac{E_s}{4.N_0}.\sum_{p=1}^{n_R}\underline{h}_p^H.\tilde{\underline{\underline{C}}}.\tilde{\underline{\underline{C}}}^H.\underline{h}_p\right] \qquad \text{(VIII.49)}$$
$$Pe^{aver}=k_2.\prod_{p=1}^{n_R}\frac{1}{\det\left[\underline{\underline{I}}+\frac{E_s}{4.N_0}.\tilde{\underline{\underline{C}}}.\tilde{\underline{\underline{C}}}^H.\underline{\underline{\Sigma}}_j\right]}$$

Recovering the reason of this section, if receiver doesn't have CSI either, the problem to implement (VIII.48) is that transmitter must make two accesses. In the first one, it transmits the identity matrix in order to allow receiver to estimate the channel, and in the second one, it sends a codeword. In other words, the transmitter would work as represented in Figure VIII-8, using two matrix channel accesses, from those only one contains information.

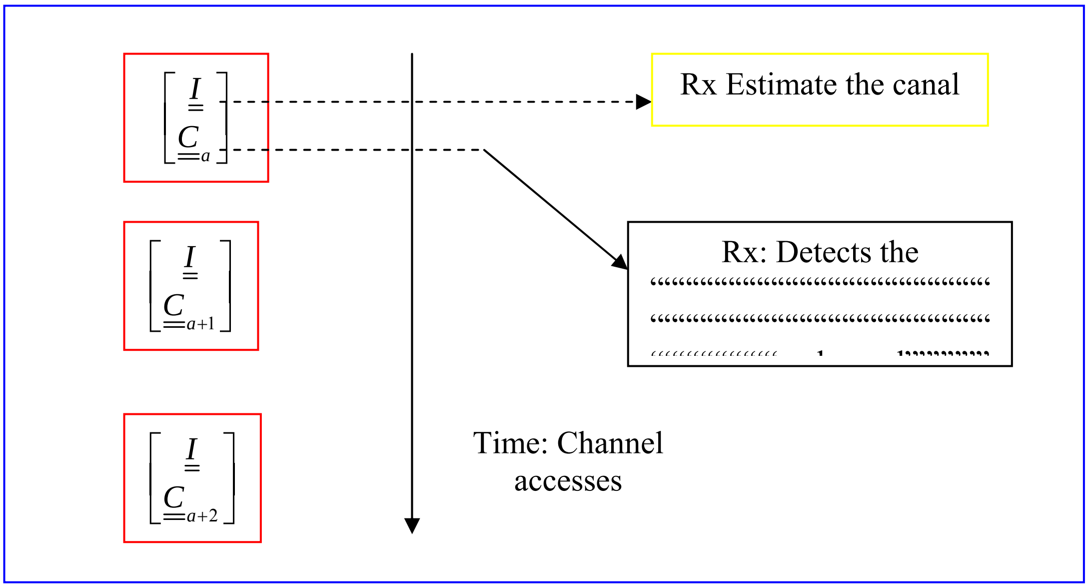

Figure VIII-8.- System with CSI in Rx for high variability channel

Another more interesting possibility, based on the previous one, consists on overlap the packets of two accesses. This overlapping would be done multiplying the matrices. The received words are represented on the following figure.

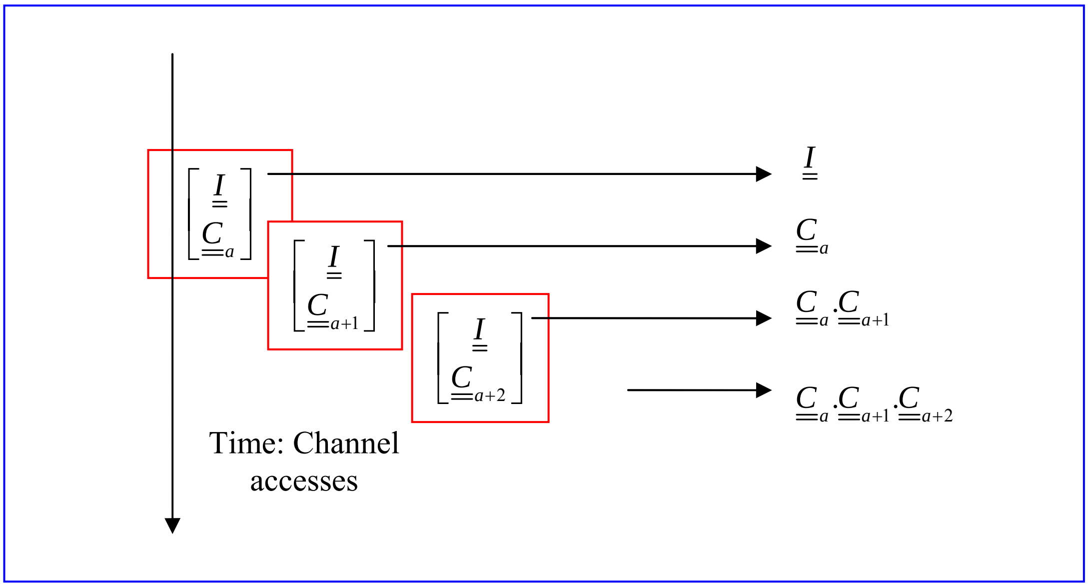

Figure VIII-9.- Differential coding principle

As it can be seen, in every access all codeword product is accumulated, and there are only a new one in every access (except the first one). Apparently, the system has suppressed needed redundancy to channel estimation. This part is the favorable one, one codeword instead of two, but the receiver keep on needing two received matrices.

The product matrix will be grouped in a new labeled one.

$$\underline{\underline{Z}}_{n-1} = \prod_{a=1} \underline{\underline{C}}_{n-a} \tag{VIII.50}$$

If the received signal in instant 'n-1' is the following:

$$\underline{\underline{X}}_{R,n-1} = \underline{\underline{H}}.\underline{\underline{Z}}_{n-1} + \underline{\underline{W}}_{n-1} \tag{VIII.51}$$

While the received one in instant 'n' can be written as

$$\begin{aligned} \underline{\underline{X}}_{R,n} &= \underline{\underline{H}}.\underline{\underline{Z}}_{n-1}.\underline{\underline{C}}_{n} + \underline{\underline{W}}_{n} = \left(\underline{\underline{X}}_{R,n-1} - \underline{\underline{W}}_{n-1}\right).\underline{\underline{C}}_{n} + \underline{\underline{W}}_{n} \\ \underline{\underline{X}}_{R,n} &= \underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n} + \left(\underline{\underline{W}}_{n} - \underline{\underline{W}}_{n-1}.\underline{\underline{C}}_{n}\right) = \underline{\underline{H}}_{nuevo}.\underline{\underline{C}}_{n} + \underline{\underline{W}}_{nuevo} \end{aligned} \tag{VIII.52}$$

As we can see in the second part of this formula, the receiver gets the transmitted codeword with a new channel and with a new noise. The new channel is the matrix received in previous case. The new noise, due to code matrices orthonormality, is still white but with the power doubled (these are the loss of all differential systems, that is, 3dB). Then, the optimal receiver will be:

$$\left|\underline{\underline{X}}_{R,n} - \underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n}\right|_F \Rightarrow \quad \hat{\underline{\underline{C}}} = \underset{\underline{\underline{C}}_n;n=1,M}{Max}\left[\operatorname{Re}\left(Trace\left(\underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n}.\underline{\underline{X}}^{H}_{R,n}\right)\right)\right] \tag{VIII.53}$$

This result is correct. But not the analysis (this is not suitable for system evaluating). In order to analyze with detail the error probability of this kind of coder , we should suppose that transmission begins with a initial word, $\underline{D}$, orthogonal as the codewords are. Moreover, both codewords and this initial matrix will be chosen so that any transmitted matrix verifies the orthogonality, that is to say it verifies

$$\begin{aligned} &\text{if} \quad \underline{\underline{Z}}_{k-1}.\underline{\underline{Z}}^{H}_{k-1} = \underline{\underline{I}} \quad \text{then} \quad \forall \underline{\underline{C}}_{m}; m = 1, M \\ &\text{it's verified that, if} \quad \underline{\underline{Z}}_{k} = \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{m} \quad \text{then also} \quad \underline{\underline{Z}}_{k}.\underline{\underline{Z}}^{H}_{k} = \underline{\underline{I}} \end{aligned} \tag{VIII.54}$$

The optimum detector for this transmitter's system would be take all received matrices and maximize the likelihood of that sequence over the possible sequences of codewords. A detector like this is too much complex. A suboptimal receiver but so much simple is to consider two consecutive received symbols $\left[\underline{\underline{X}}_{R,k-1} \quad \underline{\underline{X}}_{R,k}\right]$ as one. This, considered as a single symbol contains the information of a new codeword which consists of:

$$\left[\underline{\underline{Z}}_{k-1} \quad \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{k}\right] \equiv \overline{\underline{\underline{C}}}_{k} \qquad \text{(VIII.55)}$$

This way we have a code for no CSI in the receiver as we analyzed in last section. Note that, because of (VIII.56) is verified, the square detector is reduced to classical differential detector as we had announced. This is newly reproduced in (VIII.57).

$$\overline{\underline{\underline{C}}}_{k}^{H}.\overline{\underline{\underline{C}}}_{k} = \begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_{k} \\ \underline{\underline{C}}_{k}^{H} & \underline{\underline{I}} \end{pmatrix} \quad y \quad \overline{\underline{\underline{C}}}_{k}.\overline{\underline{\underline{C}}}_{k}^{H} = 2.\underline{\underline{I}} \qquad \text{(VIII.56)}$$

$$Trace\left[\begin{pmatrix} \underline{\underline{X}}_{R,k-1} & \underline{\underline{X}}_{R,k} \end{pmatrix}\begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_{k} \\ \underline{\underline{C}}_{k}^{H} & \underline{\underline{I}} \end{pmatrix}\begin{pmatrix} \underline{\underline{X}}_{R,k-1}^{H} \\ \underline{\underline{X}}_{R,k} \end{pmatrix}\right] = Trace\left[\underline{\underline{X}}_{R,k}\underline{\underline{C}}_{k}^{H}\underline{\underline{X}}_{R,k-1}^{H}\right] \qquad \text{(VIII.57)}$$

With respect to error probability, the matrix A of square detector becomes:

$$\begin{aligned}
&2.\underline{\underline{I}} - \frac{1}{2}.\overline{\underline{\underline{C}}}_{ko}.\overline{\underline{\underline{C}}}_{k1}^{H}.\overline{\underline{\underline{C}}}_{k1}.\overline{\underline{\underline{C}}}_{k0}^{H} = \\
&= 2\underline{\underline{I}} - \frac{1}{2}.\left(\underline{\underline{I}} + \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{0}.\underline{\underline{C}}_{1}^{H}.\underline{\underline{Z}}_{k-1}^{H} + \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{1}.\underline{\underline{C}}_{0}^{H}.\underline{\underline{Z}}_{k-1}^{H} + \underline{\underline{I}}\right) = \\
&= \underline{\underline{I}} - \frac{1}{2}.\left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{0}.\underline{\underline{C}}_{1}^{H}.\underline{\underline{Z}}_{k-1}^{H} + \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{1}.\underline{\underline{C}}_{0}^{H}.\underline{\underline{Z}}_{k-1}^{H}\right) = \qquad \text{(VIII.58)} \\
&= \frac{1}{2}\left[\left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{0} - \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{1}\right).\left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{0} - \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{1}\right)^{H}\right] = \\
&= \frac{1}{2}.\left[\underline{\underline{Z}}_{k-1}.\left(\underline{\underline{C}}_{0} - \underline{\underline{C}}_{1}\right).\left(\underline{\underline{C}}_{0} - \underline{\underline{C}}_{1}\right)^{H}.\underline{\underline{Z}}_{k-1}^{H}\right] = \underline{\underline{A}}_{DIF}
\end{aligned}$$

Moreover, and keeping in mind that the determinant of the product allows to commute the matrices and the orthogonality of $\underline{\underline{Z}}$, then it can be seen that code gain in differential case is identical to the one presented by perfect CSI in receiver case, except the factor ½ which are the habitual 3dB of loss in a differential and sub-optimal system.

VIII.6. EXAMPLE OF S-T DIFFERENTIAL CODES

The tremendous interest of codes for differential systems is its adaptation to radio environments. The way as these codes obtain diversity (or complete range in matrix A) and maximize its determinant, is the same as when the system had CSI, that's for it that matrices to use are the same as explained, only with the exception that now they must group, that is to say the multiplication of two codewords must continue been a codeword, respecting this way the whole previous explanation. Apparently this give way to a different matrices from the ones we saw, but the reader quickly will appreciate this is not this way.

Beginning with the code with lower $\left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1\right).\left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1\right)^H$ rate, and always for two antennas, the code will transmit a bit by codeword, therefore only two matrices are required. These matrices form Φ:

$$\Phi = \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{(VIII.59)}$$

With a matrix $\underline{\underline{D}}$ equal to (VIII.60), the transmission would be started.

$$\underline{\underline{D}} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad \text{(VIII.60)}$$

The code transmits a bit with two accesses, and then the rate will be 0.5. The constellation needed to antennas is BPSK, that is, 1 bit/s/Hz. The determinant of the minimum distance over two valid words, as some authors have defined as code gain, would be 8.

A code of a rate 1 implies to have four matrices at one's disposal, that over the initial matrix will form a system with also 1 bit/s/Hz, using BPSK as constellation. Its gain is 4.

$$\Phi = \left\{ \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\} \quad \text{(VIII.61)}$$

In case of a rate of 1.5 with two accesses, 8 matrices will be required. This group is denominated quaternion, and the matrices are the following.

$$\Phi = \left\{ \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm\begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix}, \pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \pm\begin{pmatrix} 0 & j \\ j & 0 \end{pmatrix} \right\} \quad \text{(VIII.62)}$$

The system works over QPSK (2 bits/s/Hz), it have also a gain of 4, but with a rate higher. This is essentially the Alamouti code.

For higher rates, the corresponding matrices can be easily generated with next generator system.

$$w_Q = \exp\left(j2\pi / Q\right)$$

$$\Phi = \left\{ \begin{pmatrix} 0 & w_Q \\ 1 & 0 \end{pmatrix}^m ; m = 0, 2Q-1 \right\} \quad \text{(VIII.63)}$$

Thus, for instance, the code with a rate of 2 would be formed with Q=8, and the one with a rate of 2.5, with Q=16. Their corresponding gains would be 1.531 and 0.7804 respectively.

VIII.7. SUMMARY

In the present chapter, the different types of space time codes have been explained. The first ones, the OSTBC, imply most of all the simplicity on receiver. On the contrary, they have a limited rate (under 1 for openings with two or more antennas). If we would exceed this number, we would change to a traditional ML detector or to some suboptimal detectors with ISI cancellation.

With higher gain than OSTBC, but with ML detection, we have seen the space time version of convolutional codes, also called Trellis or lattice. Its presentation has been very short and also it has been noted the youth of this topic. A lot of concepts exposed are nowadays in discussion, as for instance the quality measures of code and its optimal design, in consequence with the questions which support the exposed criterions of quality. These codes are more adapted to variability of radio channel but they present problems when assembling to upper layers of communication system.

Thus, detector for no CSI availability in the receiver has been exposed. The optimal detector results to be the square one, and the consequence of not to have CSI (respecting to have it) in receiver, is to pass from an error matrix to a product matrix, with a considerably lower gain. The interesting thing of this situation is the simplicity or low complexity of this detector associated to quality loss in error rate.

The last sections have been dedicated to the best solution in radio environments, the differential codes. After an interpretation in form of covered CSI, and recovering what we knew about square detector, a suboptimal detector has been presented. This one process two symbols at the same time and allows detecting with the same quality as perfect CSI, with the same error form and only with 3dB of loss, in exchange for an amazing transmitter and receiver simplicity. The square detector applied to a two symbol system over differential coding allows to reach great qualities with no dependence on CSI, even in Tx and Rx.

Additional restriction over the codewords possible, doesn't represents a bigger problem that the one which reader could have in case of block codes, most of all, in alphabet election. The last section of this chapter collects some examples of space-time differential codes in case of two antennas systems.

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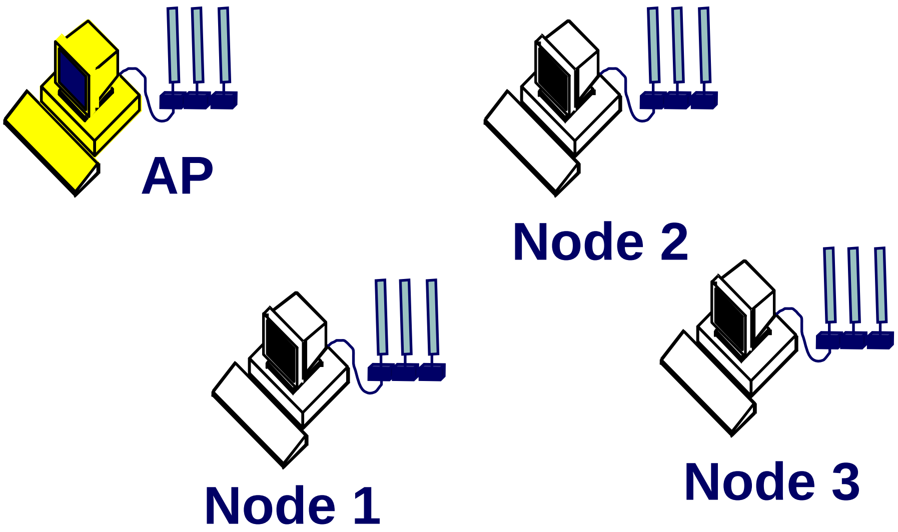
IX.

IX. MIMO MULTIUSER COMMUNICATIONS



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IX.1. INTRODUCTION

Multiuser MIMO communications from the information theory point of view

Along the present section we have studied several specific multiuser multiantenna designs, but a natural question is: what are the "optimal" multiple access schemes? Information theory can be generalized from the point-to-point scenario, considered in chapter III, to the multiuser ones, providing limits to multiuser communications and suggesting optimal multiple access strategies.

In chapter III we have seen that in a Gaussian vector channel $\mathbf{y}=\mathbf{Hx}+\mathbf{z}$. When **cooperation** is possible both among the transmit terminals and among the receive terminals, the capacity of the vector channel under a power constraint is the solution to the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{R_x}} \frac{1}{2}\log\frac{\left|\mathbf{HR_xH^T}+\mathbf{R}_z\right|}{\left|\mathbf{R_z}\right|} = \max_{\mathbf{R_x}} \frac{1}{2}\log\left|\mathbf{H^T}\mathbf{R}_z^{-1}\mathbf{HR_x}+\mathbf{I}\right| \\ &s.t. \quad tr(\mathbf{R}_x)\le P \qquad\qquad \text{(IX.1)} \\ &\qquad \mathbf{R}_x \ge 0 \end{aligned}$$

This leads to the well-known water-filling solution based on the singular-value decomposition of the equivalent channel correlation matrix $\mathbf{H}^T\mathbf{R}_z^{-1}\mathbf{H}$, as it was shown in chapter III. As the eigenvalues majorize the diagonal of a matrix, in order to maximize the determinant, the transmitter correlation matrix diagonalizes the equivalent channel correlation. Assume that **R**z=**I**, then the optimum **R**x must have its eigenvectors equal to the right singular vectors of **H** and its eigenvalues obeying the water-filling power allocation on the singular values of **H**. Further, the receive matrix can be chosen to match the left singular vectors of **H**, so that the vector Gaussian channel is diagonalized into a series of independent scalar channels onto which single-user codes can be used to collectively achieve the vector channel capacity. But this solution is only possible in a cooperative scheme.

When coordination is possible only among the receive terminals, but not among the transmit terminals, the vector channel becomes a Gaussian multiple-access channel or **MAC channel**. Although the sum capacity of a multiple-access channel is still a maximum mutual information , the transmit terminals of the multiple-access channel must be uncorrelated. Thus, the water-filling covariance, which is optimum for a coordinated vector channel, can no longer necessarily be synthesized. The optimum covariance matrix for the multiple-access channel must be found by solving an optimization problem that restricts the off-diagonal entries of the covariance matrix to zero

$$\max \frac{1}{2}\log\frac{\left|\mathbf{H}\mathbf{R}_x\mathbf{H}^T+\mathbf{R}_z\right|}{\left|\mathbf{R}_z\right|}$$
$$s.t.\quad tr(\mathbf{R}_x)\le P$$
$$\mathbf{R}_x(i,j)=0 \quad \forall(i,j)\ uncoordinated$$
$$\mathbf{R}_x\ge 0 \qquad \text{(IX.2)}$$

Thus, in terms of capacity, the value of cooperation at the transmitter side lies in the ability for the transmitters to send correlated signals. In addition, the lack of transmitter coordination makes the diagonalization of the vector channel impossible. Instead, the vector channel can only be triangularized . Such a triangularization decomposes a vector channel into a series of single-user subchannels each interfering with only subsequent subchannels. This enables a coding method based on the superposition of single-user codes and a decoding method based on successive decision feedback to be implemented. The optimal form of triangularization is a GDFE (General Decision Feedback Equalizer). If decisions on previous subchannels are assumed correct, GDFE achieves the sum capacity of a Gaussian vector

multiple-access channel. From an algebraic point of view, when only the transmitter or the receiver can face the superuser channel (i.e. BC or MAC channel), the channels is better viewed under a two matrix decomposition, as for instance the **QR** or **RQ**. Then either the transmitter or the receiver can perform $\mathbf{Q}^H$, thus, just leaving a triangular interference.

When coordination is possible only among the transmit terminals, but not among the receive terminals, the vector channel becomes a Gaussian vector broadcast channel or **BC channel**. We will see in this chapter that the sum capacity of a Gaussian vector broadcast channel is the saddle-point of a max-min problem

$$\max_{R_x}\min_{R_z} \frac{1}{2}\log\frac{\left|\mathbf{H}\mathbf{R}_x\mathbf{H}^T+\mathbf{R}_z\right|}{\left|\mathbf{R}_z\right|}$$
$$s.t.\quad tr(\mathbf{R}_x)\le P \qquad \text{(IX.3)}$$
$$\mathbf{R}_{z_i}\ known$$
$$\mathbf{R}_x,\mathbf{R}_z\ge 0$$

Although the actual noise distribution may not have the same joint distribution as the least favourable noise, because the marginal distributions $\mathbf{R}_{z_i}$ are the same (usually $\mathbf{R}_{z_i}=\sigma_i^2\mathbf{I}$), in a broadcast channel a transmitter designed for the least favourable noise performs as well as with the actual noise. The key point is that because of the lack of coordination, the receivers can no longer distinguish between different noise correlations and the capacity is as if “nature” has chosen a least favourable noise correlation. In other words, the capacity in BC cannot be better than in any cooperative situation. Thus, from a capacity point of view, the value of cooperation at the receiver lies in the ability for the receivers to recognize and to take advantage of the true correlation among the noise and received signals.

Further, in the next section we will see that the structure of the sum-capacity achieving coding strategy for the Gaussian vector broadcast channel is a decision-feedback equalizer. The optimal coding strategy again decomposes the vector channel into independent scalar subchannels each interfering into subsequent subchannels, with the interference pre-subtracted using “writing on dirty paper” coding. When full coordination is not possible, GDFE has emerged as a unifying structure that is capable of achieving the sum capacities of both the multiple-access channel and the broadcast channel sum capacity.

Next figure plots the broadcast and mac channel structures and summarizes their relationship both, between them and among the point to point MIMO channel.

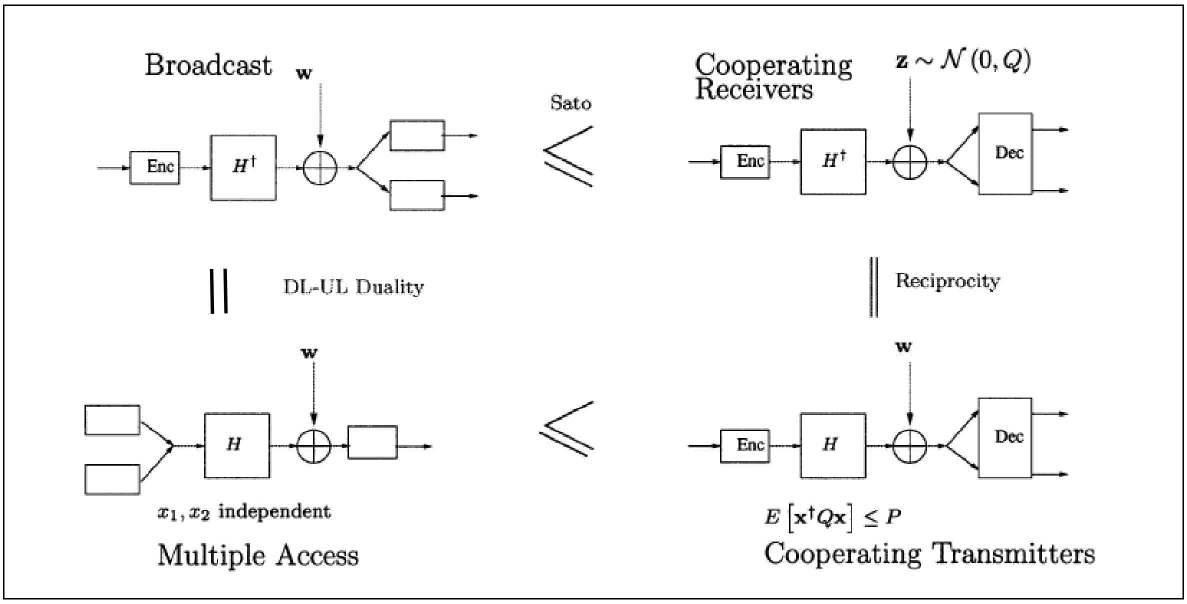

Figure IX-1.- The four channels, multiple access, broadcast, and their corresponding point-to-point channels, depicted along with the relationship between their capacities.

The study of the multiuser channels is a topic which is not closed in the literature. This chapter differs from many of the existing works because it aims at presenting the basics of the topic under a perspective as much related as possible with signal processing or filtering. Thus, making the capacity results amenable to practical coding schemes, such as those presented in past Chapter VI. First the MAC channel is addressed, second the BC channel and finally considerations on practical multiuser schemes or schedulers are presented.

Along the chapter there are two main concepts that are widely used:

A.- Assume that y =H x + w then R=I(X,Y)

$$R = I(X,Y) = H(Y) - H(Y/X) = \log\frac{|R_y|}{|R_{y/x}|} = \log\frac{|R_y|}{|R_z|} \qquad \text{(IX.4)}$$

Then y is filtered to obtain an estimation of x, x' such that

x'=Ay

If x =x' + e with E{x'e}=0 then R=I(X,X')

$$R = I(X,Y) = H(X) - H(X/Y) = \log\frac{|R_x|}{|R_{x/y}|} = \log\frac{|R_x|}{|R_e|} \qquad \text{(IX.5)}$$

And filter A is **capacity lossless**. For instance the MMSE is capacity lossless.

Observe also that there are two possible expressions for I(X,Y). When computed as $I(X,Y) = H(Y) - H(Y/X) = \log\left|\mathbf{I} + \mathbf{R}_z^{1/2}\mathbf{H}\mathbf{R}_x\mathbf{H}^H\mathbf{R}_z^{1/2}\right|$

When computed as $I(X,Y) = H(X) - H(X/Y) = \log\left|\mathbf{I} + \mathbf{R}_x\mathbf{H}^H\mathbf{H}\right|$ if the MMSE is considered, where $\mathbf{R}_e = \left(\mathbf{H}^H\mathbf{H} + \mathbf{R}_x^{-1}\right)^{-1}$

B.- If $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $|R_e| = |R_{e1}||R_{e2}|$ (i.e. the Hadamard's inequality fulfils with equality) then there is maximum rate transfer from x1 to x1' and from x2 to x2'

$$R = I(X,Y) = \log\frac{|R_x|}{|R_e|} \geq \log\frac{|R_{x_1}|}{|R_{e_1}|} + \log\frac{|R_{x_2}|}{|R_{e_2}|} \qquad \text{(IX.6)}$$

We note that equality is fulfilled when Re is diagonal, that means that E{e1.e2}=0, which is not equivalent to saying that E{x1'.x2'}=0.

IX.2. MIMO MULTIUSER MAC CHANNEL.

IX.2.1 MAC CAPACITY REGION

In the point to point case, the capacity of a channel provides the performance limit: reliable communication can be attained at any rate R<C; reliable communication is impossible at rates R>C. In the multiuser case, we should extend this concept to a capacity region C: this is the set of all pairs (R1,R2) (i.e. in the 2 user case) such that simultaneously user1 and user2 can reliably communicate at rate R1 and R2, respectively. Because signalling dimensions can be allocated to different users in an infinite number of different ways, multiuser channel capacity is defined by a rate region rather than a single number. This region describes all user rates that can be simultaneously supported by the channel with arbitrarily small error probability. From this capacity region, one can derive other scalar performance measures of interest.

For example, the symmetric capacity

$$C_{sym} = \max_{(R1,R2)\in C} R \qquad \text{(IX.7)}$$

Is the maximum common rate at which both the users can simultaneously reliably communicate.

The sum capacity

$$C_{sum} = \max_{(R1,R2)\in C} R_1 + R_2 \qquad \text{(IX.8)}$$

Is the maximum total throughput that can be achieved.

With a single receive antenna at the Base station or access point, the capacity region of the two-user MAC or uplink channel is defined by the following equations

$$\begin{aligned} R_1 &< \log\left(1+\frac{P_1}{N_o}\right) = I(x_1;y|x_2) \\ R_2 &< \log\left(1+\frac{P_2}{N_o}\right) = I(x_2;y|x_1) \\ R_1+R_2 &< \log\left(1+\frac{P_1+P_2}{N_o}\right) = I(x_2,x_1;y) = I(x_1;y)+I(x_2;y|x_1) \neq I(x_1;y|x_2)+I(x_2;y|x_1) \end{aligned} \qquad \text{(IX.9)}$$

Where P1 and P2 are the average power constraints on users 1 and 2 respectively. The individual rate constraints correspond to the maximum rate that each user can get if it has the entire channel to itself; the sum rate constraint is the total rate of a point-to-point channel with the two users acting as two transmit antennas of a single user, but sending independent signals. The three constraints define the pentagon of the figure, where, for instance, point C is obtained after matched filtering for each of the users.

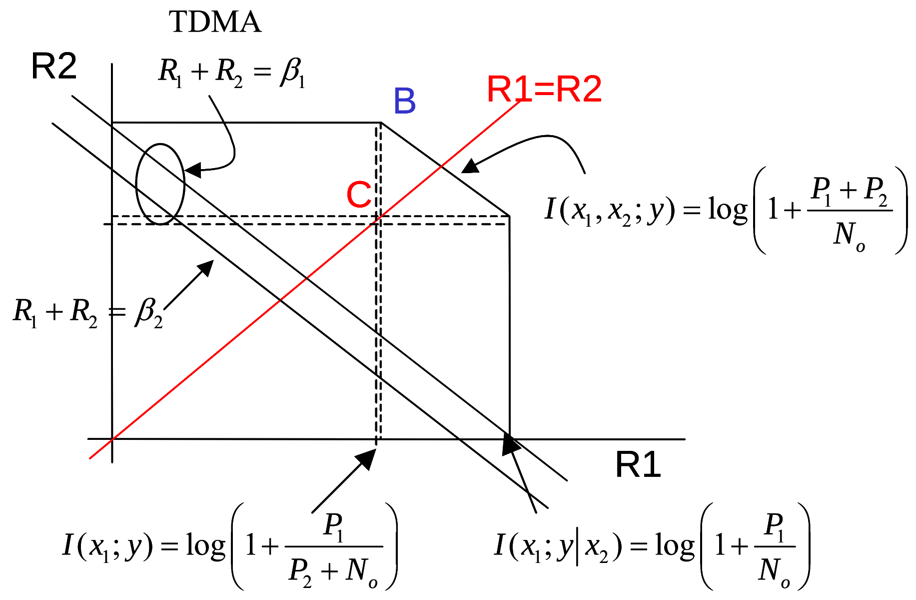

Figure IX-2.- MAC capacity region for the single tx. antenna. For 2 users is the so-called Cover-Wyner pentagon

The capacity region of the multiple access channel is the convex hull[1] of the union of these pentagons over all possible independent input distributions subject to the appropriate individual average cost constraints, i.e.,

$$C = convex\,hull\ of\ \left(\bigcup_{P_{x1},P_{x2}} C(P_{x1}, P_{x2})\right) \qquad \text{(IX.10)}$$

The convex hull operation means that we not only include points such C, but also all their convex combinations: $\alpha R_1 + (1-\alpha) R_2 \quad 0 \le \alpha \le 1$ (i.e. the diagonal lines in the figure). This is natural since the convex combinations can be achieved by time-sharing. The diagonal lines in the figure correspond to time-sharing access. For the MAC channel with single transmit antennas, the capacity region is a pentagon, because there is a unique set of input distributions that simultaneously maximizes the different constraints for R1, R2 and R1+R2. With a single transmit antenna at each user, the transmitter architecture simplifies considerably: there is only one data stream and the entire power is allocated to it. By varying the power allocations (in the case of full CSIT), the users can communicate at rate pairs in the union of the pentagons, which is itself a pentagon. For example, 2 users SDMA with multiple antenna just at the base station is a natural extension of the single antenna case

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[1] The convex hull of a set C is the set of all convex combinations of points in C:

$$conv\,C = \left\{\theta_1 x_1 + ... + \theta_k x_k \,\middle|\, x_i \in C, \theta_i \ge 0, i = 1,...,k, \theta_1 + ... + \theta_k = 1\right\}$$

$$
\begin{aligned}
&R_1 < \log\left(1+\frac{\|\mathbf{h}_1\|^2 P_1}{N_o}\right) = I(x_1;y|x_2) \\
&R_2 < \log\left(1+\frac{\|\mathbf{h}_2\|^2 P_2}{N_o}\right) = I(x_2;y|x_1) \\
&R_1 + R_2 < \log\det\left(\mathbf{I}_{nr} + \frac{1}{N_o}\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathbf{H}}\right) \qquad \mathbf{R}_x = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}
\end{aligned} \tag{IX.11}
$$

Figure IX-3.- MAC capacity region for single tx antenna. For 2 users is the so-called Cover-Wyner pentagon

The right hand side of the third inequality is the sum capacity and it is the total rate achieved in a point-to-point channel with the two users acting as two transmit antennas of one user with independent inputs at the antennas. Note that

$$
\log\left|\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H\right| = \log\left|\mathbf{I} + \sum_i p_i \mathbf{h}_i \mathbf{h}_i^H\right| \tag{IX.12}
$$

and to get some insight this expression can be further work out and results that there is a gain in rate when using multiple antennas at reception if the channels of the users are not orthogonal

$$
\begin{aligned}
&\log\left|\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H\right| = \log\left|\mathbf{I} + \sum_i p_i \mathbf{h}_i \mathbf{h}_i^H\right| = \log\left(1 + \frac{P_1|h_1|^2 + P_2|h_2|^2}{N_o} + \frac{P_1P_2}{N_o}\left|\mathbf{H}\mathbf{H}^H\right|\right) \\
&\left|\mathbf{H}\mathbf{H}^H\right| = |\mathbf{h}_1|^2|\mathbf{h}_2|^2 - \left|\mathbf{h}_1^H\mathbf{h}_2\right|^2
\end{aligned} \tag{IX.13}
$$

To define the capacity region in the general case of Ntot! users, and MIMO multiuser access, we can extend the previous region to

$$
\begin{aligned}
&R_k \le \log\det\left(\mathbf{I}_{nt} + \frac{1}{N_o}\mathbf{H}\mathbf{R}_k\mathbf{H}^H\right) \quad k = 1 \ldots Ntot \\
&\sum_{k=1}^{Ntot} R_k \le \log\det\left(\mathbf{I}_{nt} + \frac{1}{N_o}\sum_{k=1}^{Ntot}\mathbf{H}\mathbf{R}_k\mathbf{H}^H\right)
\end{aligned} \tag{IX.14}
$$

with constraints on each users power $tr\left[\mathbf{R}_k\right]=P_k$. The last inequality is upperbounded by the superuser capacity, which is a generalization of that obtained for the single transmitting antenna case.

This inequalities that define the MAC capacity region can be summarized in a single inequality and the MAC capacity region can be defined in the following more compact way

$$C_{MAC}=\bigcup_{\{Tr(\mathbf{R}_i)\leq P_i\ \forall i\}}\left\{\left(R_1...R_K\right):\sum_{i\in S}R_i\leq\frac{1}{2}\log\left|I+\sum_{i\in S}\mathbf{H}_i\mathbf{R}_i\mathbf{H}_i^H\right|\quad\forall S\subseteq\{1...Ntot\}\right\}\tag{IX.15}$$

In the convex hull process, however, if there are multiple transmit antenna at the users, no single pentagon may dominate over the other pentagons (as it is shown in next figure).

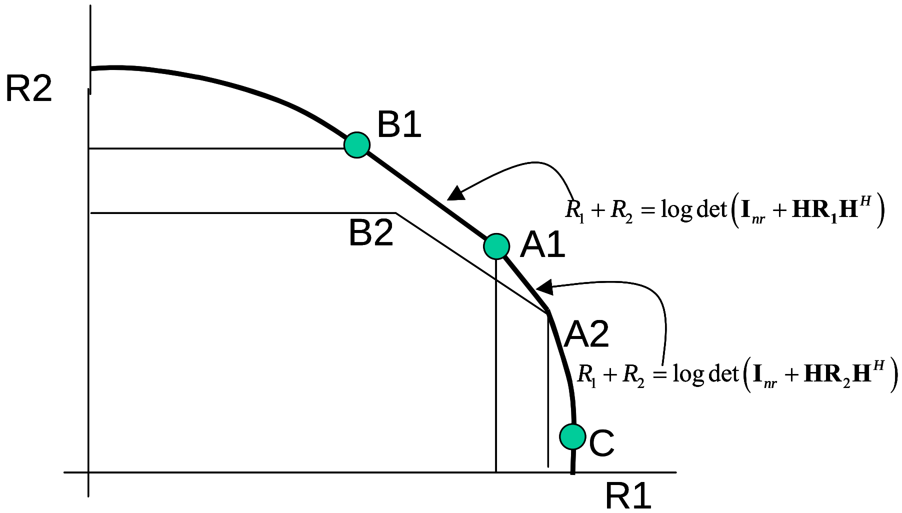

Figure IX-4.- The achievable rate regions (pentagons) corresponding to two different input distributions may not fully overlap with respect to one another. Multiple transmitting and receiving antennas.

In that case, there is no single choice of covariance matrix that simultaneously maximize the constraints: the capacity region is the convex hull of the union of the pentagons created by all the possible covariance matrices (subject to the power constraints on the users). The global capacity region is generated by the union of such polyhedrons, each one corresponding to a specific power allocation satisfying the power constraints. The resulting boundary of the global capacity region is curved, except at the sum rate point, where the boundary is a straight line, and it is generated by the union of well-selected vertices. At point C, user1 is decoded last and achieves his single-user capacity by choosing R1 as a waterfill of the channel H1 (independent of H2 or R2). User 2 is decoded first, in the presence of interference from user 1, so R2 is chosen as a waterfill of the channel H2 and the interference from user1. The sum-rate corner points A1 and B1 are the two corner points of the pentagon corresponding to the sum-rate optimal covariance matrices R_1^{sum},R_2^{sum}. We will see in IX.2.2. that at point A1 user 1 is decoded last, whereas at point B1 user 2 is decoded last. Thus, points A1 and B1 are achieved using the same covariance matrices but different decoding orders.

With multiple transmit antennas, we have a choice of power splits among the data streams and also the choice of the rotation **U** before sending the data streams out of the transmit antennas. We recall that in the time-invariant point-to-point MIMO channel, the rotation matrix **U** was chosen to correspond to the right rotation in the singular value decomposition of the channel and the powers allocated to the data streams correspond to the waterfilling allocations over the squared singular values of the channel matrix. In the MAC MIMO, in general, different choices of power splits and rotations lead to different pentagons and the capacity region in general is not a pentagon. This is because, unlike the single transmit antenna case, there are no covariance matrices **R**k, that simultaneously maximize the right hand of the all the inequalities. Depending on how one wants to trade off the performance of the two users, one would use different input strategies. In any case, note that the sum capacity is concave on **R**i, thus, in general there is no closed-form solution to the optimization problem considering sum rate, but efficient algorithms that arrive at numerical solutions exist. The obtained sum-rate maximizing covariance matrix of any user in the system should be the single-user water-filling covariance matrix of its won channel with noise equal to the actual noise plus the interference from the other K-1 transmitters [45].

The question to answer is what is the optimal receiver architecture that achieves sum capacity. How can corner points A1 and B1 be reached? Next section presents the Decision Feedback structure as the one that answers these questions.

IX.2.2 DECISION FEEDBACK EQUALIZER AND MAC SUM CAPACITY

The DFE structure achieves the entire capacity region of the multiple-access channel.

The DFE structure

Let us have

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w} \tag{IX.16}$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \end{bmatrix} \quad \mathbf{R}_x = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix} \tag{IX.17}$$

where **x**i (i=1,2) are vectors and they are statistically independent, therefore, Rx is a block diagonal matrix (in contrast to the BC channel as we will see later on).

Next we show that the DFE decomposes the vector channel into two subchannels that can be independently encoded and decoded because of the error diagonalization that the DFE performs. The achievable rates of the two subchannels are R1=I(X1',X1) and R2=I(X2',X2), being xi' the output that estimates each subchannel xi at the output of the DFE, and the sum rate is R1+R2=I(X;Y). Thus, the DFE is capacity lossless). In order to prove it 2 key ideas are involved: 1) the MMSE filter is capacity lossless, in terms of sum capacity, 2) block Cholesky factorization of the minimum MMSE noise matrix.

Next consider the following figure, where **A** is the MMSE filter

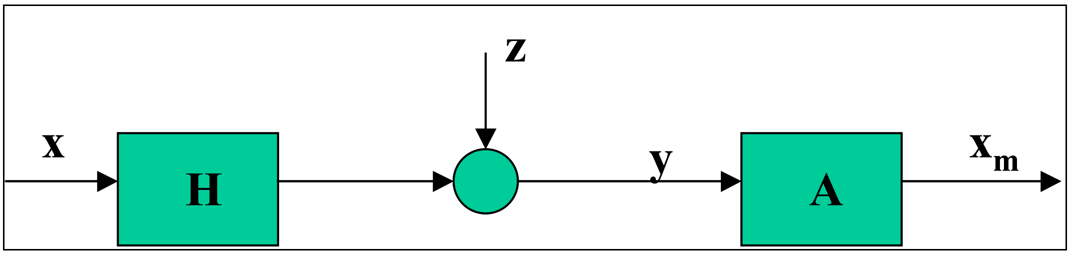

Figure IX-5.- DFE system diagram.

1.- The MMSE filter is capacity lossless: I(X;Xm)=I(X;Y)

Let **x=xm+e**, where E{x_m,e}=0 thus

$$I(X;Xm) = I(Xm;X) = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_e|} \qquad \text{(IX.18)}$$

As

$$I(X;Y) = I(Y;X) = H(X) - H(X/Y) = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_{x/y}|} = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_e|} \qquad \text{(IX.19)}$$

However if we define $\mathbf{e} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 - \mathbf{x}_{1m} \\ \mathbf{x}_2 - \mathbf{x}_{2m} \end{bmatrix}$, e1 and e2 are not necessarily uncorrelated.

$\mathbf{x}_m = \mathbf{AHx} + \mathbf{Az}$ with **e=Az.** Taking into account that the MMSE filter

$$\mathbf{A} = \mathbf{R}_{xy}\mathbf{R}_y^{-1} = \mathbf{R}_x\mathbf{H}^T\left(\mathbf{H}^T\mathbf{S}_x\mathbf{H} + \mathbf{I}\right)^{-1} = \left(\mathbf{H}^T\mathbf{H} + \mathbf{S}_x^{-1}\right)^{-1}\mathbf{H}^T \qquad \text{(IX.20)}$$

the correlation matrix of the error is

$$\mathbf{R}_e = E\left\{\mathbf{e}\mathbf{e}^T\right\} = E\left\{\left(\mathbf{x} - \mathbf{x}_m\right)\left(\mathbf{x} - \mathbf{x}_m\right)^T\right\} = \mathbf{R}_x - \mathbf{R}_x\mathbf{H}^T(\mathbf{H}\mathbf{R}_x\mathbf{H}^T + \mathbf{I})^{-1}\mathbf{H}\mathbf{R}_x = \left(\mathbf{H}^T\mathbf{H} + \mathbf{S}_x^{-1}\right)^{-1} \qquad \text{(IX.21)}$$

where $\left(\mathbf{H}^T\mathbf{H} + \mathbf{S}_x^{-1}\right)^{-1} = \mathbf{R}_x - \mathbf{R}_x\mathbf{H}^T(\mathbf{H}\mathbf{R}_x\mathbf{H}^T + \mathbf{I})^{-1}\mathbf{H}\mathbf{R}_x$ is obtained by way of the matrix inversion lema.

So, by Hadamard's inequality, $|\mathbf{R}_e| \le |\mathbf{R}_{e_1}||\mathbf{R}_{e_2}|$. This implies

$$\frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_e|} \geq \frac{1}{2}\log\frac{|\mathbf{R}_{x1}|}{|\mathbf{R}_{e1}|} + \frac{1}{2}\log\frac{|\mathbf{R}_{x2}|}{|\mathbf{R}_{e2}|} \qquad \text{(IX.22)}$$

That the independent decoding of x1 based on x1m and decoding of x2 based on x2m are capacity-lossy.

The goal of the DFE is to use a decision-feedback structure to enable the independent decoding of x1 and x2. This is accomplished by a diagonalization of the MMSE error e, while preserving the "information" in xm.

2) The diagonalization of the MMSE error can be done via a Block Cholesky factorization as follows

$$\mathbf{R}_e = \mathbf{G}^{-1}\boldsymbol{\Delta}^{-1}\mathbf{G}^{-T} \qquad \mathbf{G} = \begin{bmatrix} \mathbf{I} & \mathbf{G}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \qquad \boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\Delta}_{11} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta}_{22} \end{bmatrix} \qquad \text{(IX.23)}$$

then

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\mathbf{y} + \mathbf{e} = \\ &= \mathbf{G}^{-1}\boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{H}^{T}\mathbf{y} + \mathbf{e} \end{aligned} \qquad \text{(IX.24)}$$

In order to decouple the error

$$\mathbf{G}\mathbf{x} = \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{w} + \mathbf{G}\mathbf{e} = \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{w} + \mathbf{G}\mathbf{e} \qquad (*)$$

$$\mathbf{e}' = \mathbf{G}\mathbf{e} = \begin{pmatrix} \mathbf{I} & \mathbf{G}_{22} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}\begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{pmatrix} \rightarrow \mathbf{R}_{e'} = \boldsymbol{\Delta}^{-1} \qquad \text{(IX.25)}$$

$$|\mathbf{R}_{e'}| = |\Delta_{11}^{-1}||\Delta_{22}^{-1}|$$

Thus **e**' is uncorrelated. From equation (*) we get

$$\mathbf{x} = \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{w} + (\mathbf{I} - \mathbf{G})\mathbf{x} + \mathbf{e}' = \mathbf{x}' + \mathbf{e}' \qquad \text{(IX.26)}$$

Which gives the DFE structure of the new receiver shown in the figure, where the feedback filtering part can be implemented a successive interference cancellation due to the triangular structure of **G**. Note that in case **R**e were factorized following the SVD decomposition, then the successive interference cancellation interpretation is lost.

The achievable rates of the two subchannels are

$$
\begin{aligned}
R_1 &= I(X_1^{'};X_1) = \frac{1}{2}\log\frac{|\mathbf{R}_1|}{|\mathbf{R}_{e'1}|} \\
R_2 &= I(X_2^{'};X_2) = \frac{1}{2}\log\frac{|\mathbf{R}_2|}{|\mathbf{R}_{e'2}|} \\
R_1 + R_2 &= I(X_1^{'};X_1) + I(X_2^{'};X_2) = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_e|} = I(X;Y)
\end{aligned}
\qquad \text{(IX.27)}
$$

Thus, proving that the GDFE is capacity lossless. Note that now **the independent decoding of x1 based on x1' and decoding of x2 based on x2' are capacity-lossless.** Therefore, trying to obtain a diagonal error matrix is useful both, in terms of BER, because there is no interference among branches, as seen in past chapters [tesis Daniel Palomar], and in terms of capacity.

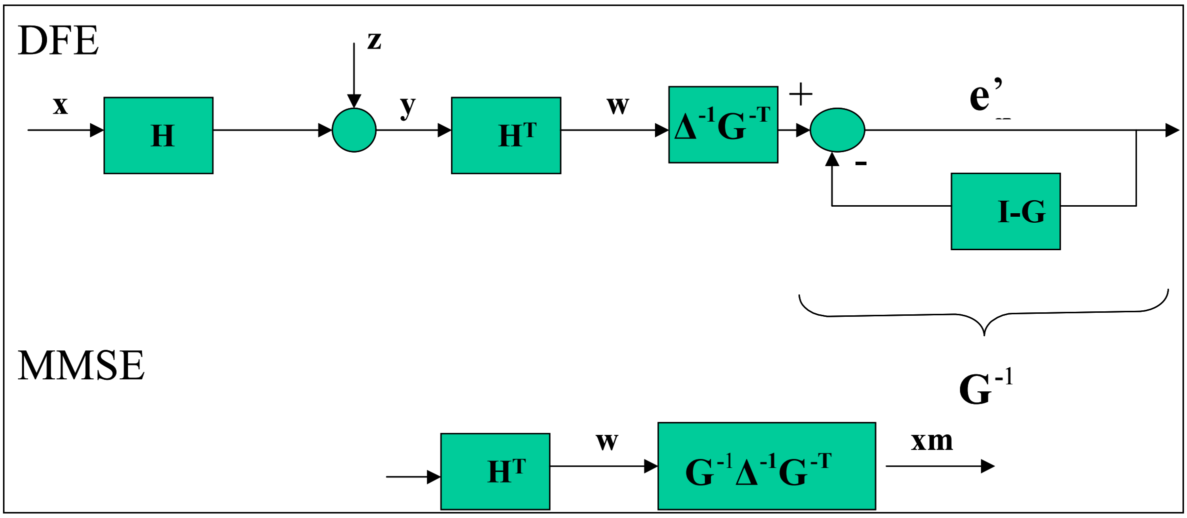

Figure IX-6.- DFE and MMSE systems.

In order to obtain the specific rates for each user when the DFE is used, the block Cholesky factorization[2] may be computed explicitly as

$$
\left(\mathbf{R}_x^{-1} + \mathbf{H}^T\mathbf{H}\right)^{-1} = \begin{bmatrix} \mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1 & \mathbf{H}_1^T\mathbf{H}_2 \\ \mathbf{H}_2^T\mathbf{H}_1 & \mathbf{R}_2^{-1} + \mathbf{H}_2^T\mathbf{H}_2 \end{bmatrix}^{-1} = \mathbf{G}^{-1}\mathbf{\Delta}^{-1}\mathbf{G}^{-T} \qquad \text{(IX.28)}
$$

where

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[2] Aside from the SVD decomposition, other matrix factorizations are going to be considered along this chapter : Cholesky factorization consists in $\mathbf{A} = \mathbf{B}\mathbf{B}^T$ where A is square and B is a lower triangular matrix; LU factorization consists in $\mathbf{A} = \mathbf{L}\mathbf{D}\mathbf{U}^H$ where A is square, L is lower triangular, D is diagonal and U^H is upper triangular; QR factorization $\mathbf{A} = \mathbf{Q}\mathbf{R}$ where A does not need to be square, Q is orthonormal and R is upper triangular.

$$\mathbf{G} = \begin{bmatrix} \mathbf{I} & \left(\mathbf{R}_x^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1} \mathbf{H}_1^T\mathbf{H}_2 \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1}$$

$$\boldsymbol{\Delta}^{-1} = \begin{bmatrix} \left(\mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1} & 0 \\ 0 & \left(\mathbf{R}_2^{-1} + \mathbf{H}_2^T\mathbf{H}_2 - \mathbf{H}_2^T\mathbf{H}_1\left(\mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1}\mathbf{H}_2^T\mathbf{H}_2\right)^{-1} \end{bmatrix} \tag{IX.29}$$

Therefore

$$R_1 = I(X_1^{'}; X_1) = \frac{1}{2}\log\frac{\left|\mathbf{R}_1\right|}{\left|\left(\mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1}\right|} = \frac{1}{2}\log\left|\mathbf{H}_1^T\mathbf{R}_1\mathbf{H}_1 + \mathbf{I}\right| \tag{IX.30}$$
$$= I(X_1; Y / X_2)$$

$$R_2 = I(X_2^{'}; X_2) = \frac{1}{2}\log\frac{\left|\mathbf{R}_2\right|}{\left|\left(\mathbf{R}_2^{-1} + \mathbf{H}_2^T\left(\mathbf{I} + \mathbf{H}_1\mathbf{R}_1\mathbf{H}_1^T\right)^{-1}\mathbf{H}_2\right)^{-1}\right|} =$$
$$= \frac{1}{2}\log\frac{\left|\mathbf{H}_1^T\mathbf{R}_1\mathbf{H}_1 + \mathbf{H}_2^T\mathbf{R}_2\mathbf{H}_2 + \mathbf{I}\right|}{\left|\mathbf{H}_1^T\mathbf{R}_1\mathbf{H}_1 + \mathbf{I}\right|} = \tag{IX.31}$$
$$= I(X_2; Y)$$

$$R_1 + R_2 = I(X_1, X_2; Y) = \frac{1}{2}\log\left|\mathbf{H}_1^T\mathbf{R}_1\mathbf{H}_1 + \mathbf{H}_2^T\mathbf{R}_2\mathbf{H}_2 + \mathbf{I}\right| =$$
$$= I(X_2; Y) + I(X_1; Y / X_2)$$

The third equation says that the total throughput cannot exceed the capacity of a point-to-point AWGN channel with the sum of the received powers of the 2 users. This is a valid constraint since the signals of the two users are independent. Without this third equation, the capacity region would have been a rectangle and each user could simultaneously transmit at the point-to-point AWGN channel with the sum of the received powers of the 2 users. This is a valid constraint since the signals of the two users are independent.

Note that, in general, for Ntot users,

$$\sum_{i=1}^{N_{tot}} R_i = \log\left|\mathbf{I}+\sum_{i=1}^{Ntot}\mathbf{H_i^H R_i H_i}\right| =$$

$$= \log\left|\mathbf{I}+\mathbf{H}_1^{\mathbf{H}}\mathbf{R}_1\mathbf{H}_1\right| + \ldots + \log\frac{\left|\mathbf{I}+\sum_{i=j}^{1}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|}{\left|\mathbf{I}+\sum_{i=j-1}^{1}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|} + \ldots + \log\frac{\left|\mathbf{I}+\sum_{i=Ntot}^{1}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|}{\left|\mathbf{I}+\sum_{i=Ntot-1}^{1}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|}$$

(IX.32)

Usually, the users are ordered $|\mathbf{h}_1| \le |\mathbf{h}_2| \le \ldots \le |\mathbf{h}_{Ntot}|$, in that case, the rate achieving sum MAC capacity is

$$R_j^{sum} = \log\frac{\left|\mathbf{I}+\sum_{i=j}^{Ntot}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|}{\left|\mathbf{I}+\sum_{i=j+1}^{Ntot}\mathbf{H}_i^{\mathbf{H}}\mathbf{R}_i\mathbf{H}_i\right|} \qquad \text{(IX.33)}$$

For only multi-antenna at reception, if Ntot=2 users, with the DFE

$$R_1 = I(X_1;Y/X_2) = \log\left(1+\frac{P_1\|\mathbf{h}_1\|^2}{N_o}\right)$$
$$R_2 = I(X_2;Y) = \log\left(1+P_2\mathbf{h}_2^H\left(N_o I + P_1\mathbf{h}_1\mathbf{h}_1^H\right)^{-1}\mathbf{h}_2\right) \qquad \text{(IX.34)}$$

In conclusion, the DFE achieves de sum capacity of the MAC channel.

Other receiver structures

We observe that the user rates Ri that achieve sum capacity require certain degree of interference among users in this way the transmission benefits from all the multiplexing gain that is offered by the multi-antenna channel, as we have commented before. This solution might cause, however, big difference among users depending on their channel. Any receiver structure that would try to null the interference (as for instance Zero Forcer), thus trying to make the reception more fair for all users, would decrease the rate. In any case, for low SNR or SNIR scenario, or if fairness is required, the DFE might not be realistic because, due to error propagation, the sum rates will not be obtained, thus requiring either a more sophisticated Maximum Likelihood detection or some interference cancellation scheme. Suboptimal theoretically speaking but more realistic if the actual throughput is the main concern, as indicated in next figure.

In general, there are Ntot! corner points on the boundary of the capacity region and each corner point is specified by an ordering of the Ntot users and the corresponding rates are achieved by an DFE receiver with that ordering of cancelling users.

IX.2.3 FADING CHANNEL

If the communication is over several coherence intervals of the user channels the new capacity region is

$$R_k \le E\left[\log\det\left(\mathbf{I}_{nr} + \frac{1}{N_o}\mathbf{H}\mathbf{R}_k\mathbf{H}^H\right)\right] \qquad k = 1...Ntot$$
$$\sum_{k=1}^{Ntot} R_k \le E\left[\log\det\left(\mathbf{I}_{nr} + \frac{1}{N_o}\sum_{k=1}^{Ntot}\mathbf{H}_k\mathbf{R}_k\mathbf{H}_k^{\mathbf{H}}\right)\right] \qquad \text{(IX.35)}$$

If only there are multi-antenna at reception, the regions are defined by

$$R_k < \log\left(1 + \frac{|\mathbf{h}_k|^2 P_k}{N_o}\right)$$
$$\sum_{k=1}^{Ntot} R_k < \log\det\left(\mathbf{I}_{nr} + \frac{1}{N_o}\mathbf{H}\mathbf{R}\mathbf{H}^H\right) = \log\det\left(\mathbf{I}_{nr} + \frac{1}{N_o}\sum_k P_k\mathbf{h}\mathbf{h}_k^H\right) \qquad \text{(IX.36)}$$
$$with \quad \mathbf{R} = diag\{P_i\}$$

With a sufficiently random and well-conditioned channel matrix **H**, the performance gain is significant.

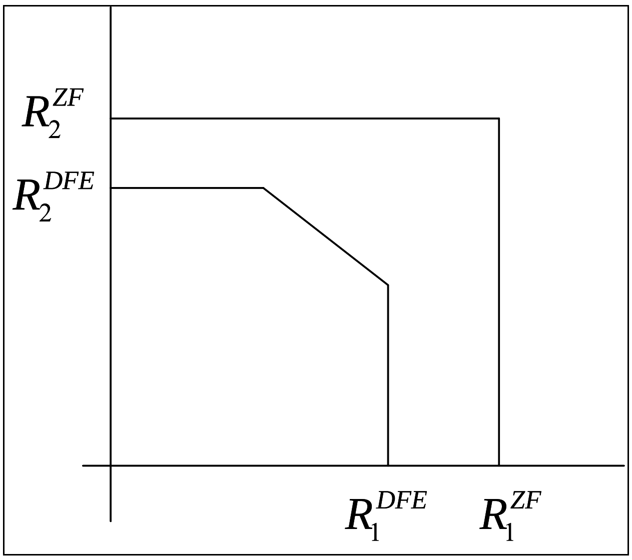

Figure IX-7.- Throughput regions (that account for BER) for the ZF and DFE

Only CSIR

In the case of i.i.d Rayleigh fading model, the capacity achieving power allocation is equal powers to the data streams (as in the point-to-point MIMO)

Full CSIT and CSIR

In a MIMO MAC channel, the situation of full CSI is an unrealistic one due to the increase in number of parameters to feedback (so that the users can change their transmit strategies as a function of the time-varying channels). A more realistic situation is when only the receiver has multiple antenna.

Only multi-antenna at reception

Now the users can vary their transmit power as a function of the channel realizations; still subject to an average power constraint. If we assume 2 users $E\left[P_k(\mathbf{h}_1,\mathbf{h}_2)\right]\leq P \qquad k=1,2$

In the point-to-point channel, we have seen that the power variations are waterfilling over the channel states. To get some insight into how the power variations are done in the uplink with multiple receive antennas, let us focus on the sum capacity

$$\begin{aligned} C_{sum} &= \max_{P_k} E\left[\log\det\left(\mathbf{I}_{nr}+\frac{1}{N_o}\mathbf{HRH}^H\right)\right]= \\ &= \max_{P_k} E\left[\log\det\left(\mathbf{I}_{nr}+\frac{1}{N_o}\sum_{k=1}^{Ntot}P_k\mathbf{hh}_k^H\right)\right] \end{aligned} \qquad \text{(IX.37)}$$

where the power allocations are subject to the average constraints. Firstly, note that in the MAC channel with a single receive antenna at the base-station

$$\log\left(1+\frac{\sum_{k=1}^{Ntot}P_k\left|h_k\right|^2}{N_o}\right) \qquad \sum_k P_k = P \qquad \text{(IX.38)}$$

where, a global power constraint is considered, the power allocation that maximizes sum capacity allows only the best user to transmit

$$P_k=\begin{cases}\left(\dfrac{1}{\lambda}-\dfrac{N_o}{\max_i\left|h_i\right|^2}\right)^+ & if\ \left|h_k\right|^2=\max_i\left|h_i\right|^2 \\ 0 & \end{cases} \qquad \text{(IX.39)}$$

Thus, following a so-called **opportunistic strategy**.

In the MAC channel each user is received as a vector ($\mathbf{h}_k$ for user k) at the base-station and there is no natural ordering of the users to bring this argument forth here. Still the optimal allocation of powers can be found using the Lagrangian techniques, but there is no closed form solution. However, with both nr and Ntot large and comparable, there is a simple policy close to the optimal one: every user transmits and the power allocated is waterfilling over its own channel state

$$P_k(\mathbf{H}) = \left(\frac{1}{\lambda} - \frac{I_o}{|\mathbf{h}_k|^2}\right)^+ \qquad k = 1...Ntot$$
$$I_o = \frac{P_k^*(\mathbf{H})\|\mathbf{h}_k\|^2}{SINR_k} \qquad \text{(IX.40)}$$

where, Io itself is a function of the power allocations of the other users (which themselves depend on the power allocated to user k). However if Ntot and nr are large enough, Io converges to a constant in probability. As usual, the water level λ is chosen such that the average power constraint is met.

If we compare the last proposed waterfilling allocation with the opportunistic one with one receive antenna. The important difference is that when there is only one user transmitting, waterfilling is done over the channel quality with respect to the background noise. However, here all the users are simultaneously transmitting, using a similar waterfilling power allocation policy. The waterfilling is done over the channel quality (the receive beamforming gain) with respect to the background interference plus noise: this is denoted by the term Io. Note that the multiuser diversity gain is lost, which is called **hardening effect**. The traditional receive beamforming power gain is balanced by the loss of the benefit of the multiuser diversity gain (which is also a power gain) due to the “hardening” of the effective fading distribution: $\|\mathbf{h}_k\|^2 \approx n_r$. In particular, at high SNR the waterfilling policy simplifies to the constant power allocation at all times (if nr>Ntot).

A different result is obtained if nr is fixed and Ntot goes to infinite. In this case we can still talk of multiuser diversity gain, which is achieved by carrying out an opportunistic policy based on the users SNIR, and it is the basis of many practical schedules as we are going to see in the section dedicated to the BC channel.

IX.3. MIMO BROADCAST CHANNEL.

In our discussion of receiver architectures for point-to-point communications and the MAC channel, we boosted the performance of linear receivers by adding successive cancellation. Is there something analogous in the BC channel as well?

IX.3.1 SUM CAPACITY IN SISO BC CHANNEL

This section characterizes the sum capacity of a class of Gaussian vector broadcast channels where a single transmitter with multiple transmit terminals sends independent information to multiple receivers. Coordination is allowed among the transmit terminals, but not among the receive terminals. The sum capacity is shown to be a saddle-point of a Gaussian mutual information game, where a signal player chooses a transmit covariance matrix to maximize the mutual information and a fictitious noise player chooses a noise correlation to minimize the mutual information.

The sum capacity is achieved using a precoding strategy for Gaussian channels with additive side information noncausally known at the transmitter. The optimal precoding structure is shown to correspond to a decision-feedback equalizer that decomposes the broadcast channel into a series of single-user channels with interference pre-subtracted at the transmitter.

The figure illustrates a N user (out of Ntot users) BC channel, where independent messages are jointly encoded by the transmitter x, and the receivers are each responsible for decoding the messages, respectively.

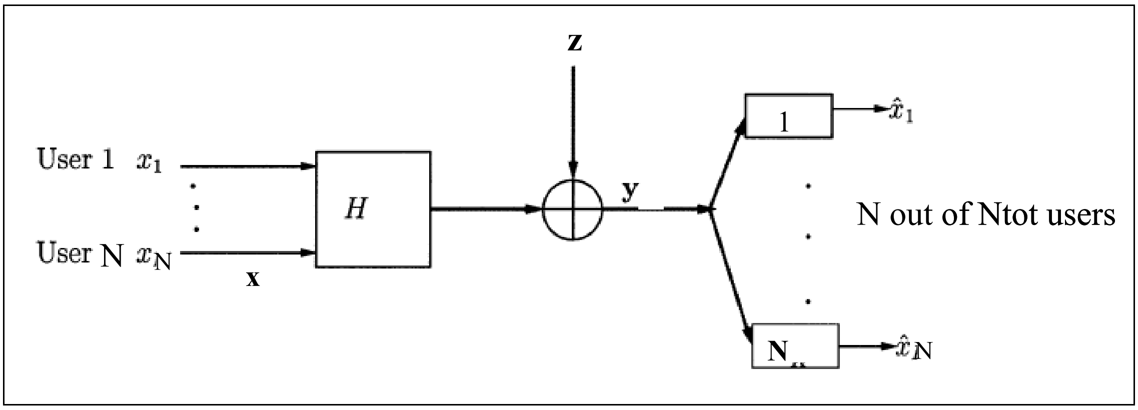

Figure IX-8.- Broadcast channel

The sum capacity result has also been obtained in simultaneous and independent work by D.Tse and A. Goldsmith These two separate pieces of work arrive at essentially the same result via a duality relation between the multiple-access channel capacity region and the dirty-paper precoding region for the broadcast channel. The proof technique contained in this section is different in that it reveals an equalization structure for the optimal broadcast strategy. This decision-feedback equalizer viewpoint leads directly to a path for implementation, thus connecting with the structures that have been given in chapter VI. It also makes the capacity result amenable to practical coding schemes.

Further, the result in this section is in fact more general than that of D. Tse and A. Goldsmith. The presented results apply to broadcast channels with arbitrary convex input constraints, while the results of Tse and Goldsmith appear to be applicable for broadcast channels with a total power constraint only. First we address the sum capacity optimization, the capacity region for the vector broadcast channel is addressed later on in this section.

The BC channel can be formulated as

$$\begin{aligned}\mathbf{y}_1 &= \mathbf{H}_1\mathbf{x}+\mathbf{z}_1 = \mathbf{H}_1(\mathbf{x}_1+\mathbf{x}_2)+\mathbf{z}_1\\ \mathbf{y}_2 &= \mathbf{H}_2\mathbf{x}+\mathbf{z}_2 = \mathbf{H}_2(\mathbf{x}_1+\mathbf{x}_2)+\mathbf{z}_2\end{aligned} \qquad \text{(IX.41)}$$

or, under a superuser formulation (where the number of scheduled users N coincides with the total number of users Ntot)

$$\mathbf{y}_{n_r Nx1} = \mathbf{H}_{(nr\,x\,N)xnt}\mathbf{x}_{nt\,x1} + \mathbf{z}_{n_r Nx1}$$
$$\mathbf{y}=\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{bmatrix} \quad \mathbf{H}=\begin{bmatrix}\mathbf{H}_1\\ \mathbf{H}_2\end{bmatrix} \quad \mathbf{x}=\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} \quad \mathbf{z}=\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix} \qquad \text{(IX.42)}$$

This section characterizes the maximum sum rate R1+R2. The development here is restricted to the two-user case for simplicity.

When a Gaussian broadcast channel has a scalar input and scalar outputs, it can be regarded as a degraded broadcast channel (see chapter VI) for which the capacity region is well established. Intuitively, this means that one user's signal is a noisier version of the other user's signal. The capacity region for a degraded broadcast channel is achieved using a superposition coding and interference subtraction scheme due to Cover. The "dirty paper" result by Costa gives us another way to derive the degraded Gaussian BC channel capacity. Let us go first for the Cover's scheme. Consider the Gaussian scalar broadcast channel

y1=x+z1

y2=x+z2 (IX.43)

where x is the scalar transmitted signal subject to a power constraint P. Assume that $\sigma_1 < \sigma_2$. Then , z2 can be rewritten as z2=z1+z', where z' is $N(0,\sigma_2^2-\sigma_1^2)$ is independent of z1. Since z2' has the same distribution as z2, y2 is now equivalent to y1+z'. Thus, y2 can be regarded as a degraded version of y1. The capacity region for a degraded broadcast channel si achieved by dividing the total power into P1=aP and P2=(1-a)P (0<a<1) and to construct two independent messages, one codeword is chosen from each codebook, and their sum is transmitted. Because y2 is a degraded version of y1, the codeword intended for y2 can also be decoded by y1. Thus, y1 can subtract the effect of the codeword intended for y2 and can effectively get a cleaner channel with noise power σ_1^2 instead of $\sigma_1^2+P_2$.

Recalling chapter VI, the following rate pair is achievable:

$$\begin{aligned}R2 &= \frac{1}{2}.\log\left(1+\frac{P_2}{\sigma_2+P_1}\right)\\ R1 &= \frac{1}{2}.\log\left[1+\frac{P_1}{\sigma_1}\right]\end{aligned} \qquad \text{(IX.44)}$$

It can be shown that, when $\sigma_1 > \sigma_2$ the above rate region is smaller than the true capacity region formulated next

$$R1 = \frac{1}{2}.\log\left(1 + \frac{P_1}{\sigma_1 + P_2}\right)$$
$$R2 = \frac{1}{2}.\log\left[1 + \frac{P_2}{\sigma_2}\right] \quad \text{(IX.45)}$$

Thus, the decoding order matters. This successive cancellation scheme can be carried out at the transmitter instead of doing it at the receiver, thus resulting the so-called Dirty Paper coding. It was proposed by Costa and it also achieves capacity in the BC SISO channel. In DP, the code for x2 is written on a paper got in dirt by the code of signal x1. In this way, the degraded channel is not necessary.

When a Gaussian BC channel has a vector input and vector outputs, it is no longer necessarily degraded, and superposition coding is no longer capacity achieving. For instance, if a linear superposition of signals is transmitted at the base-station

$$\mathbf{x}(m) = \sum_{k=1}^{Ntot} u_k(m)\,\mathbf{b}_k \quad \text{(IX.46)}$$

Then each user's signal will be projected differently onto different users, and there is no guarantee that there is a single user who would have sufficient SIRN to decode everyone else's data. However, the "dirty-paper" result by Costa can be extended to the vector case to presubtract multiuser interference at the transmitter, again with no increase in transmit power. Next, the rest of the section is devoted to obtain the precoding scheme that emulates dirty paper coding, thus achieving the sum capacity in a broadcast channel:

- **First**, we establish the relationship between the BC sum capacity and that in a cooperative scheme, both for MIMO. As a result, the optimal transmitter correlation matrix is designed.

- **Second**, from the optimal **R**x, a capacity lossless precoder is designed in order to obtain independent reception process for each signal xi, thus suitable for BC. The final scheme is called DP precoder and is based on the GDFE.

- **Third**, the optimal power loading at the transmitter is obtained

- **Finally**, we obtain the sum capacity as an aggregation of each stream rate and comment its relationship with the MAC sum capacity thanks to the existing duality among them.

IX.3.2 BC SUM CAPACITY AND COOPERATIVE SUM CAPACITY FOR MIMO

Let us consider a 2 user BC channel

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}$$

or

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x} + \mathbf{z}_i \qquad i = 1,2 \tag{IX.47}$$

$$\mathbf{y}_1 = \mathbf{H}_1(\mathbf{x}_1 + \mathbf{x}_2) + \mathbf{z}_1 = \mathbf{s}_1 + \alpha\mathbf{s}_2 + \mathbf{z}_1$$
$$\mathbf{y}_2 = \mathbf{H}_2(\mathbf{x}_1 + \mathbf{x}_2) + \mathbf{z}_2 = \alpha\mathbf{s}_1 + \mathbf{s}_2 + \mathbf{z}_2$$

Because y1 and y2 cannot coordinate in a broadcast channel, the BC capacity does not depend on the joint noise distributions and only on the marginals. This is so because two broadcast channels with the same marginals but with different joint distribution can use the same encoder and decoders and maintain the same probability of error.

Let us first show an example that illustrates Sato's bound: the least favourable noise correlation depends on the structure of the channel (as α depends on the channel).

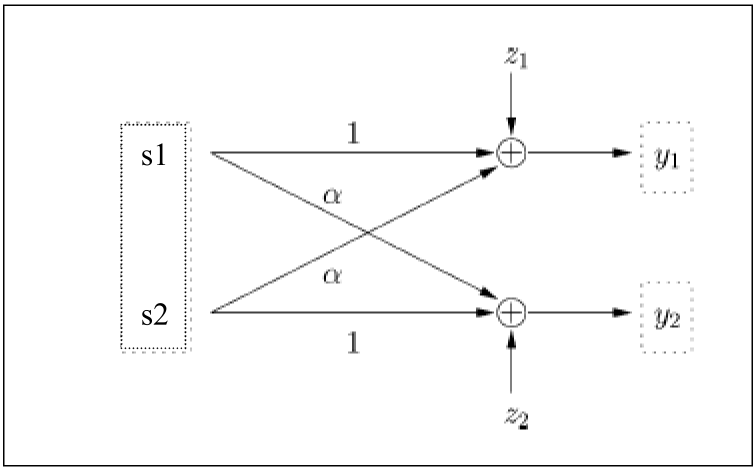

Figure IX-9.- Simple BC channel

Consider the two-user two-terminal broadcast channel shown in above figure where the channel from s1 to y1 and the channel from s2 to y2 have unit gain, and the crossover channels have a gain a. Assume that s1 and s2 are independent Gaussian signals and z1 and z2 are Gaussian noises all with unit variance. The broadcast channel capacity is clearly bounded by I(X1,X2;Y1,Y2). This mutual information is a function of the crossover channel gain and the correlation coefficient r between z1 and z2. Consider the case α=0. In this case, the least favourable noise correlation is r=0 . This is because if z1 and z2 were correlated, decoding of y1 would reveal z1 from which z2 can be partially inferred. Such inference is possible, of course, only if y1 and y2 can cooperate. In a broadcast channel, where receivers y1 and y2 cannot take advantage of such correlation, the capacity with correlated and is the same as with uncorrelated z1 and z2. Thus, regardless of the actual correlation between z1 and z2 , the broadcast channel capacity is bounded by the mutual information evaluated assuming uncorrelated noise. Consider another case α=1 . The least favourable noise here is the perfectly correlated noise with r=1. This is because r=1 implies z1 and z2 equals. So, one of y1 and y2 is superfluous. If z1 and z2 were not perfectly correlated, collectively would reveal more information than y1 or y2 alone would. Since r=1 is the least favourable noise correlation, the broadcast channel sum capacity is bounded by the mutual information assuming r=1.

The previous explanation justifies that the cooperative capacity of the Gaussian vector channel with a least favourable noise bounds the capacity for the Gaussian broadcast channel

$$R_1 + R_2 \leq \min I(X;Y) = \min_{p(z)} I(X;HX+Z) \qquad \text{(IX.48)}$$

Sato's outer bound states that the broadcast channel sum capacity is bounded by (cannot be better than) the capacity of any discrete memoryless channel whose noise marginal distributions are equal to p(zi). The tightest outer bound is then the capacity of the channel with the least favourable noise correlation. Therefore,

$$\begin{aligned} C_{sum}^{BC} &\leq \min_{R_z} \max_{R_x} \frac{1}{2}\log\frac{\left|\mathbf{H}\mathbf{R}_x\mathbf{H}^T + \mathbf{R}_z\right|}{\left|\mathbf{R}_z\right|} = \min_{R_z} C_{sum}^{Coop} \\ s.t.\quad & tr(\mathbf{R}_x) \leq P \qquad \text{(IX.49)} \\ & \mathbf{R}_{z_i}\, known \\ & \mathbf{R}_x, \mathbf{R}_z \geq 0 \end{aligned}$$

Additionally, although the actual noise distribution may not have the same joint distribution as the least favourable noise, because the marginal distributions are the same, a precoder designed for the worst noise and to require independent receivers, is oblivious of the correlation between zi's and performs as well as with the actual noise. Therefore,

$$C_{sum}^{BC} \geq \max_{R_x} \min_{R_z} \frac{1}{2}\log\frac{\left|\mathbf{H}\mathbf{R}_x\mathbf{H}^T + \mathbf{R}_z\right|}{\left|\mathbf{R}_z\right|} \qquad \text{(IX.50)}$$

If the function is convex-concave it has a saddle point and therefore $\min_x \max_y f(x,y) = \max_y \min_x f(x,y)$. This is our case[3], therefore

$$C_{sum}^{BC} = \max_{R_x} \min_{R_z} \frac{1}{2} \log \frac{\left| \mathbf{H R_x H^T} + \mathbf{R}_z \right|}{\left| \mathbf{R_z} \right|} \qquad \text{(IX.51)}$$

In order to obtain the optimal transmitter correlation matrix the max-min problem has to be solved.

The task of finding the least favourable noise correlation can be formulated as the following optimization problem.

$$\begin{aligned} &\min_{R_z} \frac{1}{2} \log \frac{\left| \mathbf{H R_x H^T} + \mathbf{R}_z \right|}{\left| \mathbf{R_z} \right|} \\ &s.t. \quad \mathbf{R}_z^{(i)} = \mathbf{I} \quad i = 1..N \\ &\qquad \mathbf{R}_z \geq 0 \end{aligned} \qquad \text{(IX.52)}$$

To solve the problem we derive the Lagrangian

$$\begin{aligned} L &= \log \left| \mathbf{H R_x H^T} + \mathbf{R_z} \right| - \log \left| \mathbf{R_z} \right| + \sum_{i=1}^{2} tr\left(\mathbf{\Phi}_i \left(\mathbf{R}_z^{(i)} - \mathbf{I} \right) \right) - tr\left(\mathbf{\Psi} \mathbf{R}_z \right) \\ \frac{\partial L}{\partial R_z} &= 0 = \left(\mathbf{H R_x H^T} + \mathbf{R}_z \right)^{-1} - \mathbf{R}_z^{-1} + \begin{bmatrix} \mathbf{\Phi}_1 & 0 \\ 0 & \mathbf{\Phi}_2 \end{bmatrix} - \mathbf{\Psi} \end{aligned} \qquad \text{(IX.53)}$$

and the least favourable noise is when its correlation matrix fulfils that

$$\mathbf{R}_z^{-1} - \left(\mathbf{H R_x H^T} + \mathbf{R_z} \right)^{-1} = \begin{bmatrix} \mathbf{\Phi}_1 & 0 \\ 0 & \mathbf{\Phi}_2 \end{bmatrix} \qquad \text{(IX.54)}$$

or equivalently

$$\mathbf{R_z} + \mathbf{R_z} \left(\mathbf{H R_x H^T} \right)^{\mathbf{-1}} \mathbf{R_z} = \begin{bmatrix} \mathbf{\Phi_1^{-1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi_2^{-1}} \end{bmatrix}. \qquad \text{(IX.55)}$$

Note that although the actual noise distribution may not have the same joint or cross distribution as the least favourable noise, the marginal distributions coincide. To interpret the obtained condition we can rewrite it in terms of the equivalent channel

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[3] The function $\frac{1}{2} \log \frac{\left| \mathbf{H R_x H^T} + \mathbf{R}_z \right|}{\left| \mathbf{R_z} \right|}$ is concave in Rz and $\min_{R_z} \frac{1}{2} \log \frac{\left| \mathbf{H R_x H^T} + \mathbf{R}_z \right|}{\left| \mathbf{R_z} \right|}$ is convex in Rx

$$\tilde{\mathbf{H}} = \mathbf{R}_z^{-1/2}\mathbf{H} \text{ as } \left(\mathbf{I} + \tilde{\mathbf{H}}\mathbf{R}_x\tilde{\mathbf{H}}^T\right)^{-1} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 \end{bmatrix} \quad \text{(IX.56)}$$

Then the condition can be seen as imposing block diagonal structure on the correlation matrix of the received signal after the whitening filter.

In order to find **R**x, note that the presented optimization problem suggests the following game-theoretical interpretation for the Gaussian vector broadcast channel. There are two players in the game. A signal player chooses an Rx to maximize the mutual information I(X;HX+Z) subject to the constraint tr(Rx)<=P . A noise player chooses a fictitious noise correlation Rz to minimize the mutual information subject to the constraint Rzi=I. A Nash equilibrium in the game is a set of strategies such that each player's strategy is the best response to the other player's strategy. The Nash equilibrium in this mutual information game exists, and the Nash equilibrium corresponds to the sum capacity of the Gaussian vector broadcast channel.

This max min scheme can be easily solved whenever a saddle point exist. Such a saddle point will be the Gaussian Broadcast channel sum capacity, and its calculations depend on both Rx and Rz. The saddle-point property of the Gaussian broadcast channel sum capacity implies that the capacity achieving is such that Rx is the water-filling covariance matrix for Rz ,and Rz is the least favourable noise covariance matrix for Rx. In fact, the converse is also true. This is because the mutual information is a concave–convex function, and the two KKT conditions, corresponding to the two optimization problems are, collectively, sufficient and necessary at the saddle-point. Thus, the computation of the saddle-point is equivalent to simultaneously solving the water-filling problem and the least favourable noise problem.

One might suspect that the following algorithm can be used to find a saddle-point numerically. The idea is to iteratively compute the best input covariance matrix for a given noise covariance, then compute the least favourable noise covariance matrix for the given input covariance. If the iterative process converges, both KKT conditions are satisfied, and the limit must be a saddle-point of $\frac{1}{2}\log\frac{\left|\mathbf{H}\mathbf{R}_x\mathbf{H}^T + \mathbf{R}_z\right|}{\left|\mathbf{R}_z\right|}$. Although such an iterative min-max procedure is not guaranteed to converge for a general game even when the payoff function is concave–convex, the iterative procedure appears to work well in practice for this particular problem. The convex–concave nature of the problem also suggests that general-purpose numerical convex programming algorithms can be used to solve for the saddle-point with polynomial complexity. Finally, the main sum capacity result can be easily generalized to broadcast channels with an arbitrary convex input constraint. This is so because the saddle-point for the mutual information expression is Gaussian as long as the input and noise constraints are convex.

Once Rx and Rz have been obtained by convex optimization, as for instance using the interior-point method, it just remain to make a connection between the transmitted symbols and the input data u, this is accomplished by the precoding matrix B.

IX.3.3 PRECODER DESIGN

Towards non-cooperative receivers

Consider a Gaussian vector channel **y**=**Hx** + **z**. Assume that H is a square matrix. If the noise covariance matrix Rz is not block-diagonal, a noise whitening filter is required as a first step at reception. Suppose that the noise covariance matrix has an eigenvalue decomposition

$$\mathbf{R}_z = \mathbf{Q}^T \mathbf{\Lambda} \mathbf{Q} \qquad \text{(IX.57)}$$

If in addition, the transmitter covariance matrix Rx is also not block-diagonal, then a Gaussian source **u** and a transmit filter **B** can be created such that **R**u=**I** and **x=Bu**. Let the SVD of the optimal **R**x obtained in the previous section be

$$\mathbf{R}_x = \mathbf{V\Sigma V}^T = \mathbf{B^H B} \qquad \text{(IX.58)}$$

The appropriate transmit filter has the form

$$\mathbf{B} = \mathbf{V}\sqrt{\mathbf{\Sigma}}\mathbf{M} \qquad \text{(IX.59)}$$

where **M** is an arbitrary orthonormal matrix. It consists on a beamforming matrix **V**, a power allocation matrix **Σ** and a precoding matrix **M** whose whole is to carry out a proper interference cancellation in order to achieve capacity by decoupling the BC channels. The dimensions of **M** are the same dimension as $\mathbf{R}_z$, thus, number of receiving antennas. So, the rank of $\mathbf{R}_x$ is always equal to or lower than the rank of the superuser channel. When **R**x is of strictly lower rank, zeros can be padded in the channel to make the effective channel matrix a square matrix

$$\mathbf{R_x} = \mathbf{V\Sigma_o V^T} \qquad \mathbf{\Sigma} = [\sqrt{\mathbf{\Sigma}_0}\ 0] \qquad \text{(IX.60)}$$

In order to design **B**, let us take the configuration of the figure with MMSE reception, which is capacity lossless. The goal is to obtain a precoder **B** such that reception can be independently done by each receiver

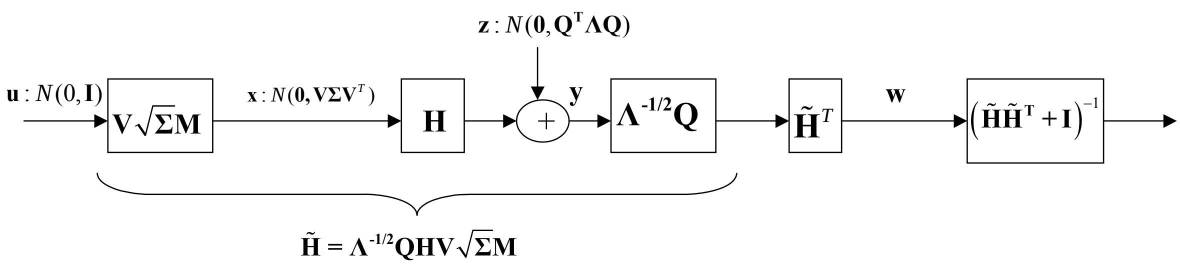


Figure IX-10.- MMSE configuration

Note that the transmit filter and the noise whitening filter create the following effective channel

$$\tilde{\mathbf{H}} = \mathbf{\Lambda}^{-1/2}\mathbf{QHV}\sqrt{\mathbf{\Sigma}}\mathbf{M} \qquad \text{(IX.61)}$$

Intuitively, the transmitting matrix **M** should transform this equivalent channel into a block diagonal channel so that each receiver could carry out independent reception. This is precisely the condition that we have obtained in order to solve the maxmin problem formulated before, where

$$\left(\mathbf{I} + \tilde{\mathbf{H}}\mathbf{R}_x\tilde{\mathbf{H}}^T\right)^{-1} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 \end{bmatrix} \qquad \text{(IX.62)}$$

being **Φ**i the dual variables of the optimization problem.

If the DFE receiver of the figure is considered because it allows maximal rate transfer to each parallel channel, then the whole system up to obtaining signal v is block diagonal (a more detailed solution of the problem is described in Appendix A).

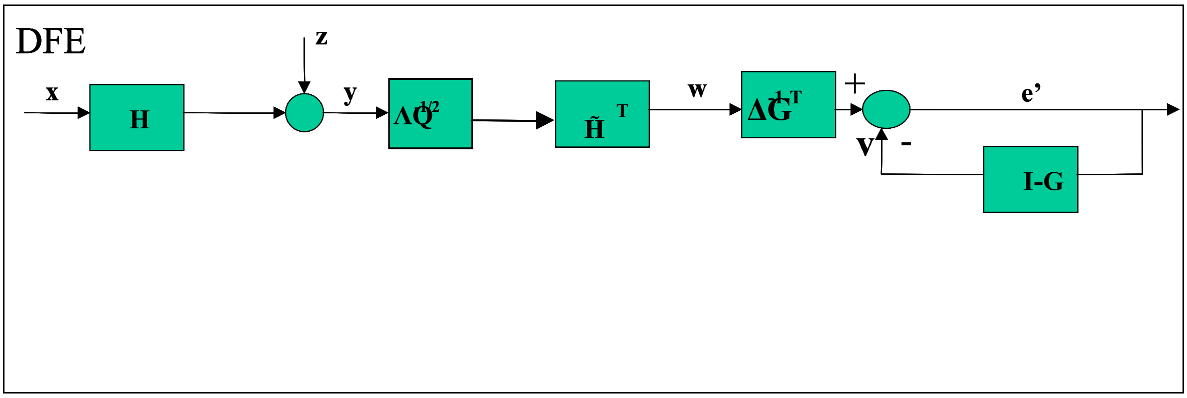


Figure IX-11.- DFE receiver

There are still one question to answer before obtaining a complete precoding design: to complete the decoupled receiver design by transferring $\mathbf{G}^{-1}$ to the transmitter if possible.

Complete decoupled receiver design that achieves Csum: GDFE precoder versus DP precoder

In order to finally get a decoupled receiver design, the feedback filtering at reception can be transferred to the transmitter side, so called **decision feedback precoder**, which results in a similar concept to DP coding.

The final design of the transmitter is then the one proposed in the figure

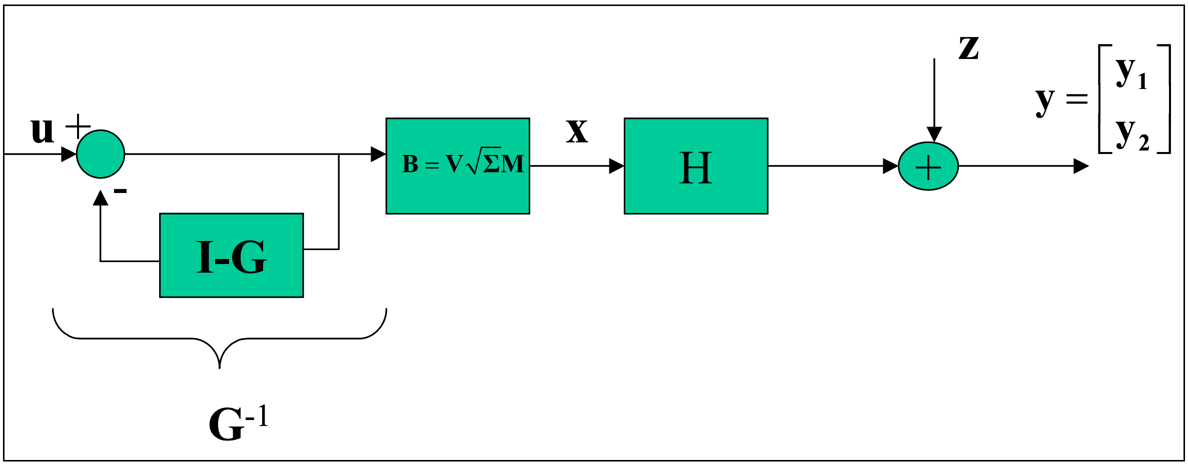


Figure IX-12.- GDFE precoder and matrix **B** for achieving BC sum capacity

Therefore, matrix **B** is completed with $\mathbf{G}^{-1}$ implemented in a feedback way, in order to preserve capacity as appendix B shows. Thanks to the feedback implementation, the precoder follows a Dirty Paper philosophy. For instance, the transmitter first picks a codeword for receiver 2 with full (noncausal) knowledge of the codeword intended for receiver 1. Therefore, receiver 2 does not see the codeword intended for receiver 1 as interference. Similarly, the codeword for receiver 3 is chosen such that receiver 3 does not see the signals intended for receivers 1 and 2 as interference. This process continues for all K receivers. Receiver 1 subsequently sees the signals intended for all other users as interference, receiver 2 sees the signals intended for users 3 to K as interference, etc. Note that the ordering of the users clearly matters in such a procedure.

There is one aspect left and it is regarding the question whether the structure of the GDFE precoder is capacity lossless or not. The answer is yes, it is capacity loss-less and the prove can be found in the work done by Cioffi and also considered in the appendix B of this chapter.

To sum up, with the least favourable noise, there exists a GDFE structure with a block –diagonal feedforward filter. This, together with a precoder B, eliminates the need for coordination at the receiver. **Thus, the precoding GDFE achieves the BC sum capacity and the precoding matrix B has been obtained**.

Algorithm for achieving Csum at broadcast:

Solved the maxmin problem and get the best **R**x for the worst **R**z

From the convex optimization problem get the dual variables $\mathbf{\Phi}_i$

From **R**x and $\mathbf{\Phi}_i$, obtain $\mathbf{B} = \mathbf{V}\sqrt{\mathbf{\Sigma}}\mathbf{M}$ that decouples in an optimal way part of the lossless MMSE receiver

The rest of the receiver is $\mathbf{G}^{-1}$, which is moved to the transmitter. When implemented as a feedbackward filtering, the final transmitter structure emulates the DP precoding philosophy. Recall that:

$$\mathbf{G} = \begin{bmatrix} \mathbf{I} & \left(\mathbf{R}_x^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1}\mathbf{H}_1^T\mathbf{H}_2 \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \qquad \text{(IX.63)}$$

IX.3.4 BC SUM CAPACITY AND MAC-BC DUALITY

The DFE transmit filter **B** designed for the least favourable noise also identifies the set of sum capacity-achieving **R**i. For example, for the 2 user case,

$$\begin{aligned} R_1 &= \frac{1}{2}\log\frac{\left|\mathbf{H}_1\mathbf{R}_1\mathbf{H}_1^T + \mathbf{H}_1\mathbf{R}_2\mathbf{H}_1^T + \mathbf{R}_{z1}\right|}{\left|\mathbf{H}_1\mathbf{R}_2\mathbf{H}_1^T + \mathbf{R}_{z1}\right|} = I(X_1;Y) \\ R_2 &= \frac{1}{2}\log\frac{\left|\mathbf{H}_2\mathbf{R}_2\mathbf{H}_2^T + \mathbf{R}_{z2}\right|}{\left|\mathbf{R}_{z2}\right|} = I(X_1;Y/X_2) \end{aligned} \qquad \text{(IX.64)}$$

which are the rates that are obtained in the appendix B for DFE precoding (reversing the orders of user 1 and 2) and are also called dirty paper rates.

For the general case of K users, the rate for user i that achieves sum capacity is

$$R_i = \frac{1}{2}\log\frac{\left|\mathbf{H}_i\left(\sum_{k=i}^{K}\mathbf{R}_k\right)\mathbf{H}_i^T + \mathbf{I}\right|}{\left|\mathbf{H}_i\left(\sum_{k=i+1}^{K}\mathbf{R}_k\right)\mathbf{H}_i^T + \mathbf{I}\right|} \qquad \text{(IX.65)}$$

where users are ordered in increasing order. One important feature to notice about the dirty paper rate equations is that the rate equations are neither a concave nor a convex function of the covariance matrices. This makes finding the dirty paper region very difficult, because generally the entire space of covariance matrices which meet the power constraint must be searched over. This justifies our focus under a filtering perspective that departs from the MAC channel instead of directly obtaining the precoders for each user from the general maxmin problem.

The sum capacity is

$$C_{sum}^{BC} = \sum_{i=1}^{Ntot} R_i \qquad \text{(IX.66)}$$

resulting

$$C_{sum}^{BC}=\sum_{i=1}^{N_{tot}} R_i=\log\left|\mathbf{I}+\mathbf{H}_{Ntot}^{\mathbf{H}}\mathbf{R}_{Ntot}\mathbf{H}_{Ntot}\right|+\ldots+\log\frac{\left|\mathbf{I}+\mathbf{H}_1^{\mathbf{H}}\left(\sum_{i=1}^{Ntot}\mathbf{R}_i\right)\mathbf{H}_1\right|}{\left|\mathbf{I}+\mathbf{H}_1^{\mathbf{H}}\left(\sum_{i=1}^{Ntot-1}\mathbf{R}_i\right)\mathbf{H}_1\right|}$$

$$=\log\left|\mathbf{I}+\sum_{i=1}^{Ntot}\mathbf{H}_{\mathbf{i}}^{\mathbf{H}}\mathbf{R}_i^{MAC}\mathbf{H}_{\mathbf{i}}\right| \qquad with \quad \sum_i Tr\left[\mathbf{R}_i^{MAC}\right]\le P_T$$

(IX.67)

Note also that the same rates sum capacity was obtained for the MAC channel with DFE reception. This is also called MAC-BC duality [44]: the capacity region of the BC channel can be obtained with the union of MAC capacity regions with equal average power constraints and not allowing to the transmitters to cooperate in the MAC. One key point is that to achieve the same rate vector in the BC and MAC, the decoding order must in general be reversed, i.e., if user 1 is decoded last in the BC then user 1 is decoded first in the MAC. Exploiting duality, in [43], Goldsmith proposes an alternative algorithm for the iterative design of the precoder in a MIMO BC channel.

$$R^{DP}=\sum_{i=1}^{Ntot}\log\left(1+\frac{\left|w_{ii}\right|^2}{1+\sum_{j>i}\left|w_{ij}\right|^2}\right)=\sum_{i=1}^{Ntot}\log\left(1+SNIR_i^{DP}\right) \qquad \text{(IX.68)}$$

$$\mathbf{x}=\mathbf{Bu} \quad w_{ij}=\left[\mathbf{HB}\right]_{ij}$$

In order to get some insight into the sum rate expression, note that in the case of only multiple antenna at the transmitter with MMSE decoding, the sum rate results in

(**)

Being the last term of the equality very useful when designing practical precoding schemes for BC with the aim of sum rate maximization. Caire and Shamai proposed in [7], for the case of only antennas at transmission, that the components of u should be generated by successive dirty-paper encoding with Gaussian codebooks and B should maximized (**) over all precoding matrices B satisfying the trace or power constraint. For the 2 user case with 1 antenna per user they obtained that

$$R\begin{cases}\log(1+\left|\mathbf{h}_1\right|^2 A) & A\le A_1\\ \log\dfrac{\left(A\left|\mathbf{HH}^H\right|+trace(\mathbf{HH}^H)\right)^2-4\left|\mathbf{h_2}\,\mathbf{h_1^H}\right|^2}{4\left|\mathbf{HH}^H\right|} & A>A_1\end{cases} \qquad \text{(IX.69)}$$

where $\left|\mathbf{h_1}\right|^2\ge\left|\mathbf{h}_2\right|^2$ and $A_1=\dfrac{\left|\mathbf{h}_1\right|^2-\left|\mathbf{h}_2\right|^2}{\left|\mathbf{HH}^H\right|}$

As a conclusion,

$$C_{SUM} = R^{DP} = \max_{k=1..N \; \sum_{k=1}^{N} Tr\Sigma_k \le P} \log\det\left(\mathbf{I} + \sum_{k=1}^{N} \mathbf{H_k}\boldsymbol{\Sigma}_k\mathbf{H_k^H}\right)$$
$$\boldsymbol{\Sigma}_\mathbf{k} = E\left\{\mathbf{x_k}\mathbf{x_k^H}\right\} = \mathbf{B_k}\mathbf{P}_k\mathbf{B}_k^\mathbf{H} \quad \mathbf{B}_k : DFE\ precoding(or\ DP) \tag{IX.70}$$

or in the case of just multiple antenna at the transmitter

$$C_{SUM} = R^{DP} = \max_{p_1,p_2\cdots \sum_{k=1}^{N} p_k \le NP} \log\det\left(\mathbf{I} + \sum_{k=1}^{N} \mathbf{h}_k p_k \mathbf{h_k^H}\right) \tag{IX.71}$$

Multiuser diversity

If both the transmitter and receivers know the channel perfectly in a BC with Ntot single-antenna receivers with average transmit power of nt SNR and the transmitter has nt antennas, then for sufficiently large Ntot, the sum rate capacity scales like

$$C_{SUM}\big|_{Ntot\to\infty} \approx E\left\{C_{sum}\right\}_{Ntot\to\infty} \approx nt \log\log(Ntot\ SNR) \tag{IX.72}$$

Where nt and SNR are fixed.

In order to prove it observe that

$$\begin{aligned}
&E\left\{\max_{p_1,p_2\cdots \sum_{k=1}^{N} p_k \le NP} \log\det\left(\mathbf{I} + \sum_{k=1}^{N_{tot}} \mathbf{h_k} p_k \mathbf{h_k^H}\right)\right\} \\
&\le nt\ E\left\{\max_{p_1,p_2\cdots \sum_{k=1}^{N} p_k \le NP} \log\left(1 + \frac{\sum_{k=1}^{N_{tot}} Tr(\mathbf{h_k}\mathbf{h_k^H} p_k)}{nt}\right)\right\} \\
&\le ntE\left\{\max_{p_1,p_2\cdots \sum_{k=1}^{N} p_k \le NP} \log\left(1 + \frac{\max_{1\le i\le Ntot} Tr(\mathbf{h_k}\mathbf{h_k^H})\sum_{k=1}^{N_{tot}} p_k}{nt}\right)\right\} \\
&= ntE\left\{\log\left(1 + P\max_{1\le i\le Ntot} Tr(\mathbf{h_k}\mathbf{h_k^H})\right)\right\}
\end{aligned} \tag{IX.73}$$

Where we have used the inequality $\det(\mathbf{A}_{MxM}) \le \left(\frac{tr(\mathbf{A})}{M}\right)^M$. As Ntot goes to inf. The max behaves like log Ntot+O(log log Ntot).

Compared to the single user capacity of nt log(1+SNR), we observe that the sum-rate increases double-logarithmically in Ntot.. Thus, the multiuser diversity gain increases SNR by a factor of logNtot. This is precisely the basis of the so-called opportunistic schemes, which just need SNIR feedback instead of the whole knowledge of **H**. Due to its importance in practical scheduler we devote later on a specific section to study these schemes.

Next low complexity alternatives to the optimal precoding are proposed. Note that if only the transmitter can be designed taking into account the superuser channel **H**, a two matrix decomposition of the channel is desirable, rather than a 3 matrix decomposition as the SVD. Cholesky, QR or LU decompositions are possible candidates that may lead to designs different from the one proposed. When full CSIR or CSIT is available, optimal structures in MAC or BC carry out a successive interference cancellation or signal encoding respectively. However, other solutions can be found in between that trade-off performance and complexity in implementation and in required CSI. When studying these alternative schemes, note that duality between MAC and BC can be carried out, thus transferring receiving filters to the transmitter and the other way round. When doing that one must be careful in not incurring in any capacity loss or transmitting power increase.

IX.3.5 LOW COMPLEXITY PRECODING STRUCTURES

Chapter VI was precisely devoted to this subject. Low complexity precoding structures were obtained based on the idea extending the "dirty paper" concept of the degraded BC to the non-degraded one (that appears when either multiantenna are present at transmission or reception). Intuitively, this extension relates with moving the feedbackward part of the DFE at the transmitter. This implies a "triangular interference cancelling", that allows for interference suppression without increase in the transmitted power, in contrast to a straight forward Zero Forcing precoding. Matching intuition with theory, this section has shown the optimality, in terms of BC sum capacity, of moving the feedbackward filtering of the DFE to the transmitter and has given the optimal design procedure for the precoding.

Due to the design complexity of the optimal precoding, in the literature there are various different practical implementation schemes. Although Chapter VI sums up the basics of these different schemes, we comment on some examples.

Tomlinson-Harashima

The basis of most of the practical precoders is the structure used by the Tomlinson-Harashima precoder. In that case, an alternative to design the DFE precoding is by considering a **QR decomposition of the channel**

$$\mathbf{H} = \mathbf{Q}^H\mathbf{R} \qquad \text{(IX.74)}$$

Being **R** lower triangular and modifying the receiver design accordingly to next figure, where now the output is **w.**

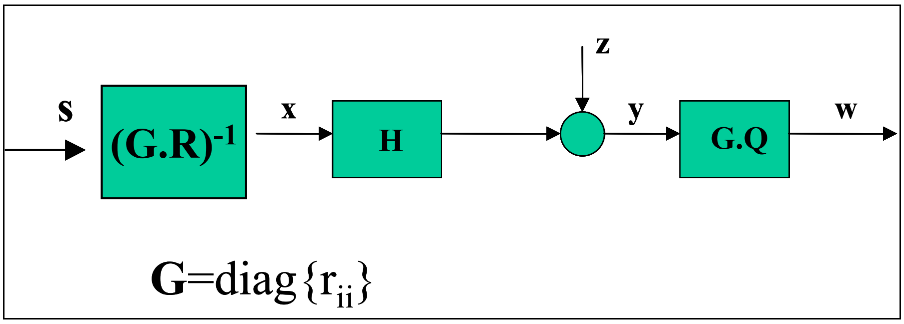


Figure IX-13.- Tomilson-Harashima precoder scheme.

$$\mathbf{w} = \mathbf{GQHx} + \mathbf{GQz} = \mathbf{GQQ^H Rx} + \mathbf{GQz} = \mathbf{GRx} + \mathbf{GQz} =$$

$$= \mathbf{s} + \mathbf{GQz} \text{ if } \mathbf{B} = (\mathbf{GR})^{-1} \qquad \text{(IX.75)}$$

In order to study if it is capacity lossless the correlation of the error **e=w-s** should be studied. Observe that $\mathbf{R_e} = \mathbf{GQQ}^H\mathbf{G^H}\sigma^2$, thus diagonal and capacity lossless. However, if we want to obtain parallel receivers, the RQ decomposition of the channel should be considered, the lower triangular matrix R changes, and a different precoding structure is obtained, where the receivers are decoupled, thus suitable for the broadcast channel

$$\mathbf{H} = \mathbf{RQ^H} \qquad \text{(IX.76)}$$

where **R** is a lower triangular matrix of dimension (Ntot x m) and $\mathbf{Q}^H$ is unitary of dimension m x nt (where m=rank(**H**))

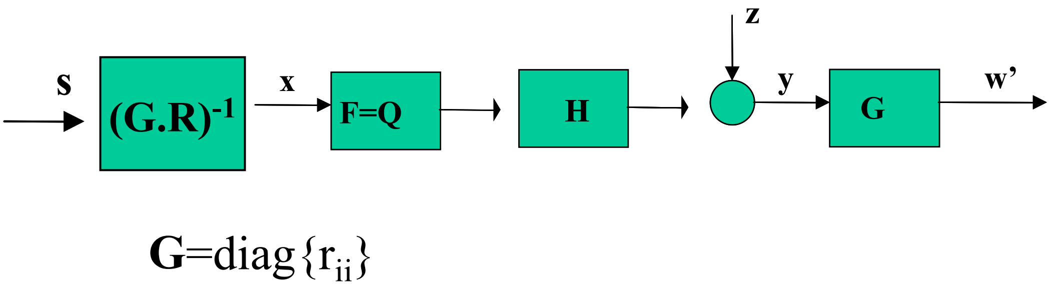


Figure IX-14.- Tomilson-Harashima precoder scheme (2).

where now the output is **w'**

$$\mathbf{w}' = \mathbf{GHFx} + \mathbf{Gz} = \mathbf{GRQ^H Qx} + \mathbf{Gz} = \mathbf{GRx} + \mathbf{Gz} = \mathbf{s} + \mathbf{Gz} \text{ if } \mathbf{B} = (\mathbf{GR})^{-1}$$

still being capacity lossless.

Zero Forcing precoder

Other examples are the studies of Caire and Shamai who compare different possibilities for the case of nt transmitting antennas and N users, with 1 antenna per user. In the precoding scheme, **x**=**Bu**, they obtained **u** by successive dirty-paper encoding so that nulling the interference terms i>j, while the remaining i<j terms are forced to zero by letting **B**=**Q**, where **Q** is the unitary matrix that results from the QR decomposition of the channel, **H**=**RQ** (where **R** is lower triangular). This suboptimal ZF-DP coding strategy is shown to achieve asymptotically optimal throughput for high SNR if the channel matrix has full row rank, while for vanishing SNR, it reduces to simple maximal ratio combining beamforming to the best user, which is shown to be also optimal in general, for low SNR. Next figure (figure 5 from [7]) plots some comparative results for nt=4 and N=4 users and depicts the importance of channel knowledge at the transmitter. Note that ZF beamforming consists of inverting the channel matrix at the transmitter in order to create orthogonal channels between the transmitter and the receivers without receivers' cooperation. Although it was one of the first BC architectures to be studied in the literature because of it simplicity, it is not optimal as the figure shows. We will come to the problem of transmit beamforming design later on.

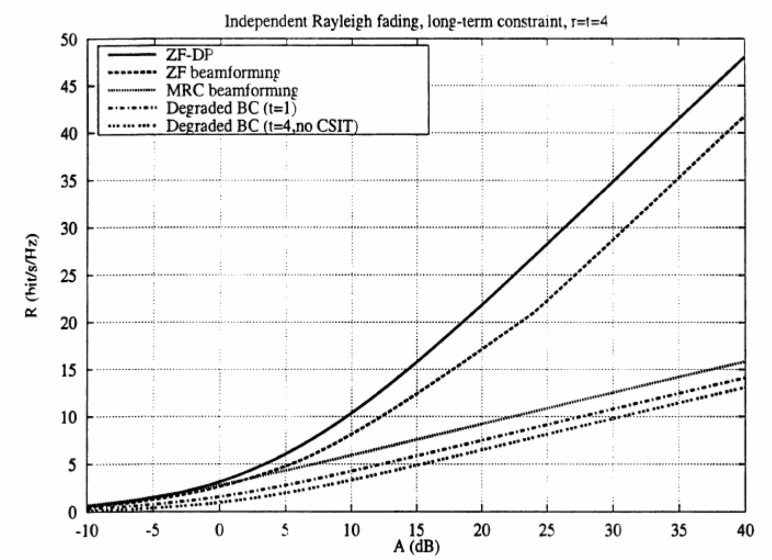


Figure IX-15.- Throughput versus SNR comparison

The practical implementation not only cares about the complexity and the tractability of the designed precoders/receivers, but also on the other system aspects as system fairness, users access control and system delay. For instance, next table (from [38]) compares ZF, DP-QR (with random ordering of users) and SVD precoding (called cooperative in the table). In all of them uniform power allocation is considered. The appendix compares the aforementioned techniques for the instantaneous channel case.

Technique	Gain	Mean	Standard Deviation	Asymptotic IF
Cooperative	λ_k^2/K	Q/K	$\sqrt{Q/K}$	$1/(1+\xi)$
Dirty Paper	d_k^2/K	$(2Q-K+1)/2K$	$\sqrt{Q+\frac{1}{12}(K-5)(K-1)}/K$	$(2-\xi)^2/\left[(2-\xi)^2+\xi^2/3\right]$
Zero Forcing	α_k^2/K	$(Q-K+1)/K$	$\sqrt{Q-K+1}/K$	1

Table IX-1.- Comparative mean SNR and variance among users, Q is the number of antenna and N the number of users

Fairness

One of the aspects in the table is the index of fairness (IF), which is defined as the ratio between the mean and the variance. Although ZF is the precoder with worst performance it is the most fair among users. When getting into multiuser systems, a new aspect to study is the global performance of the system versus the individual performance. Depending on the desired fairness, different criteria design are to be chosen, which will not necessary coincide with the maximization of the sum rate.

Next figure is an example of the performance of the three precoders. Again, although the cooperative technique is the one that offers better performance for a specific SNR, it is also the most unfair among users. In the scheduling section, which takes into account system practical aspect, we come back to this fairness issue.

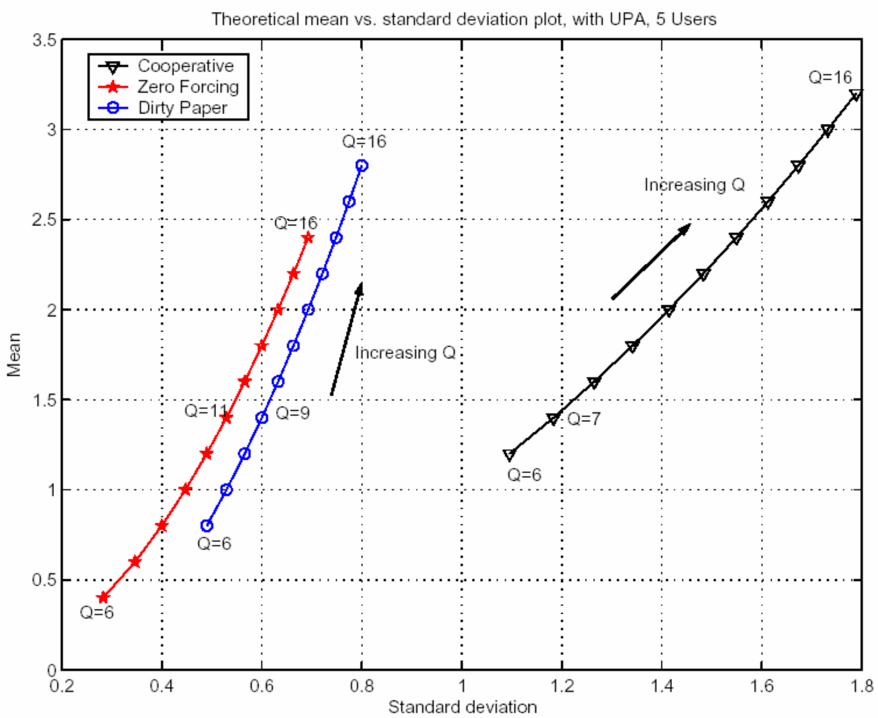


Figure IX-16.- Comparative results for 5 users and varying number of antenna.

When considering practical precoders one aspect to design is the power allocated to each user. Next figure compares the sum rate results when a ZF precoder is used with different design criteria.

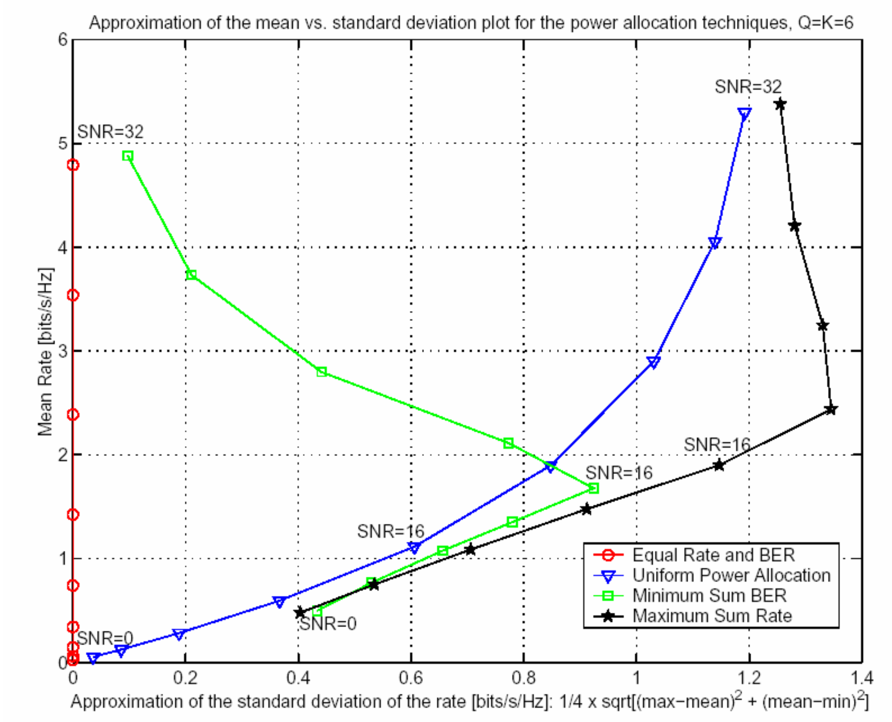


Figure IX-17.- Outage mean rate vs. the approximation of the standard deviation at 90% of SNR

Note that the sum rate criterion is not necessarily the best one if fairness issues are to be taken into account. Note also that when designing the precoder in MIMO transmission the global system performance in terms of rate do not give the same results as if BER were the goal and it is important to know how to use the spatial degrees of freedom properly in terms of both, diversity and multiplexing (or rate) gain. In SISO transmission this aspect was not a concern because better BER implied better rate and the other way round.

Access Control

In a cooperative transmission, since for any unitary matrix **Q** (permutation matrices are unitary), the matrix **QH** has the same singular values of **H**, the sum capacity in a cooperative system is independent of the user ordering. On the other hand, ZF beamforming depends on the choice of the unordered active user set and ZF-DP depends on the ordered active user set. Note that in both cases, only m=rank(**H**) users can be served simultaneously. The access control or user selection problem is still to be solved in many practical situations and offers an important degree of freedom that can be exploited in order to improve the performance of suboptimal systems. Ideally, if the number of users goes to infinite, a subset of users with mutually orthogonal spatial signatures could be found and ZF beamforming would amount to a unitary transformation, thus involving no power penalty and perfect user separation. This can be seen as another manifestation of the ubiquitous principle of multiuser diversity, that plays a central role in multiuser channels with fading as we will seen later on. In [38;46, 47] the authors address the problem when ZF is applied. Note that the grouping of users requires an exhaustive search over the entire user set. This makes a low complexity implementation of the optimal ZF challenging and userful.

To finish this section, next, diversity optimizing techniques or transmit beamforming are presented.

Transmit beamforming

In the literature, practical schemes concentrate on the diversity advantage, that means, the increase of the effective SNIR at the receivers. Moreover, they are not based on information-theoretic issues, but rather strategies that could be currently implemented. In these schemes, the transmitted signal **x** has been substituted by a beamforming matrix **B**. The general SNIR for the kth user in the multiuser MISO system, γk, can be expressed as

$$\gamma_k = \frac{\mathbf{b}_k^H \mathbf{R}_k \mathbf{b}_k}{\sum_{i \neq k} \mathbf{b}_i^H \mathbf{R}_k \mathbf{b}_i + \sigma_k^2} \qquad \text{(IX.77)}$$

The different strategies consist on transmit beamforming design and power assignment. The optimal beamforming strategy [46] in terms of rate is the one that

$$\begin{aligned} R_{BF} &= \max_{b_k, P_k} \sum_{i=1}^{Ntot} \log(1 + SNIR^{BF}) \\ s.t. &\sum_{i=1}^{Ntot} \left|\mathbf{b}_k\right|^2 P_k \leq P \end{aligned} \qquad \text{(IX.78)}$$

which is optimal for large number of users, as it achieves the same rate as DP coding [46]

$$E\left\{R_{BF}\right\} \approx nt \log(1 + \frac{P}{nt} \log Ntot) \qquad \text{(IX.79)}$$

but it is difficult to carry out in practice. Note that the SINR of each user in the BC depends in general on all the transmit signatures of the users. Hence, it is not meaningful to pose the problem of choosing the transmit beamformers to maximize each of the SINR separately. A more sensible formulation is to minimize the total transmit power needed to meet a given set of SINR requirements.

$$\begin{aligned} &\min_{b_k} \sum_{k=1}^{K} \mathbf{b}_k^H \mathbf{b}_k \\ &s.t. \gamma_k \geq \gamma_{thres} \qquad k = 1 \ldots K \end{aligned} \qquad \text{(IX.80)}$$

The optimal transmit signatures balance between focusing energy in the direction of the user of interest and minimizing the interference to other users. This transmit strategy can be thought of as performing transmit beamforming. Implicit in this problem formulation is also a problem of allocating powers to each of the users.

Taking into account the uplink-downlink duality (see appendix C), the transmit beamforming problem can be solved by looking at the uplink dual. Since for any choice of transmit signatures, the same SINR can be met in the uplink dual using the transmit beamformer as receive filters and the same total transmit power, the BC problem is solved if we can find receive filters that minimize the total transmit power in the uplink dual. The receive filters are always chosen to be the MMSE filters given the transmit powers of the users; the transmit powers are iteratively updated so that the SINR requirement of each user is just met. This MMSE beamformers can now be used as the optimal transmit beams in the BC and afterwards the optimal power allocation can be found. The optimal beamforming in MAC and BC are the same if power constraint is the same, the difference between both channels is the power that has to be allocated to each user. This duality is going to be used to obtain the BC capacity region in next section.

Note that if fairness issues come into play, other strategies are possible as that in [38; 48] the design is based on the maximization of the minimum SINR subject to a power constraint

$$\begin{aligned} &\max_{b_k} \min_{k} \gamma_k \\ &\sum_{k=1}^{K} p_k \leq P \end{aligned} \qquad \text{(IX.81)}$$

Conclusions

Along this section we have shown that aside from the theoretical analysis, practicality aspects are really important in multiuser MIMO systems. The main aspects are:

The ordering of the users clearly matters in such a procedure and needs to be optimized in the capacity computation

For latency and degrees of freedom reasons (N< Ntot). Then the throughput can be further optimized with respect to the active user set and SDMA (Spatial diversity multiple access) has to be combined with other strategies, as for instance TDMA. These aspects refer to access control policies

The real situation is to have partial CSIT, which make optimal strategies such as DP unfeasible, thus requiring also for an access control policy.

When not only optimality but users priority and QoS come into play, fairness has to be considered.

Finally, in delay is also an important parameter to control in the network and come into play when users queues of finite length are considered. Although it is out of the scope of this chapter, the multiuser MIMO system design is narrowly related with the users buffer control. Some examples are shown in the last section of the chapter that considers the scheduling problem.

Basically, in the design of a scheduler or practical broadcast system 3 questions have to be answered:

Transmitter architecture

Power allocation

Access control (i.e. number of users to give access and order)

In the section devoted to scheduling these aspects are going to be considered. Next, before getting into the problem of partial CSIT we study the BC capacity region.

IX.3.6 CAPACITY REGION

When multiple users share the same channel, the channel capacity can no longer be characterized by single number. Since there is an infinite number of ways to divide the channel between many users, the multiuser channel capacity is characterized by a rate region.

We have seen that in the MIMO BC, DP coding can be applied at the transmitter when choosing codewords for different users. The ordering of the users clearly matters in such a procedure, and needs to be optimized in the capacity calculation. Let $\pi(.)$ denote a permutation of the user indices and $\mathbf{R_x} = \left[\mathbf{R_1} \ldots \mathbf{R_K}\right]$ denote a set of positive semi-definite covariance matrices with $Tr\left(\mathbf{R_1} + \ldots + \mathbf{R_K}\right) \leq P$. Under DP coding, if user $\pi(1)$ is encoded first, followed by user $\pi(2)$, etc., then the following rate vector is achievable

$$R(\pi, \mathbf{R}_i) : R_{\pi(k)} = \frac{1}{2}\log \frac{\left|\mathbf{H}_{\pi(k)}\left(\sum_{j=k}^{K}\mathbf{R}_{\pi(j)}\right)\mathbf{H}_{\pi(k)}^{\mathbf{T}} + \mathbf{I}\right|}{\left|\mathbf{H}_{\pi(k)}\left(\sum_{j=k+1}^{K}\mathbf{R}_{\pi(j)}\right)\mathbf{H}_{\pi(k)}^{\mathbf{T}} + \mathbf{I}\right|} \qquad \text{(IX.82)}$$

with $k = 1, .., K$

The capacity region C is then the convex hull of the union of all such rates vectors over all permutations and all positive semi-definite covariance matrices satisfying the average power constraints

$$C_{BC}(P, \mathbf{H}) = Co\left(\bigcup_{\pi, \mathbf{R}} R(\pi, \mathbf{R})\right) \qquad \text{(IX.83)}$$

The transmitted signal is $\mathbf{x} = \mathbf{x_1} + \ldots + \mathbf{x_K}$. The DP coding implies that $\mathbf{x_1}, \ldots, \mathbf{x_K}$ are uncorrelated, and thus $\mathbf{R_x} = \mathbf{R_1} + \ldots + \mathbf{R_K} \leq P$.

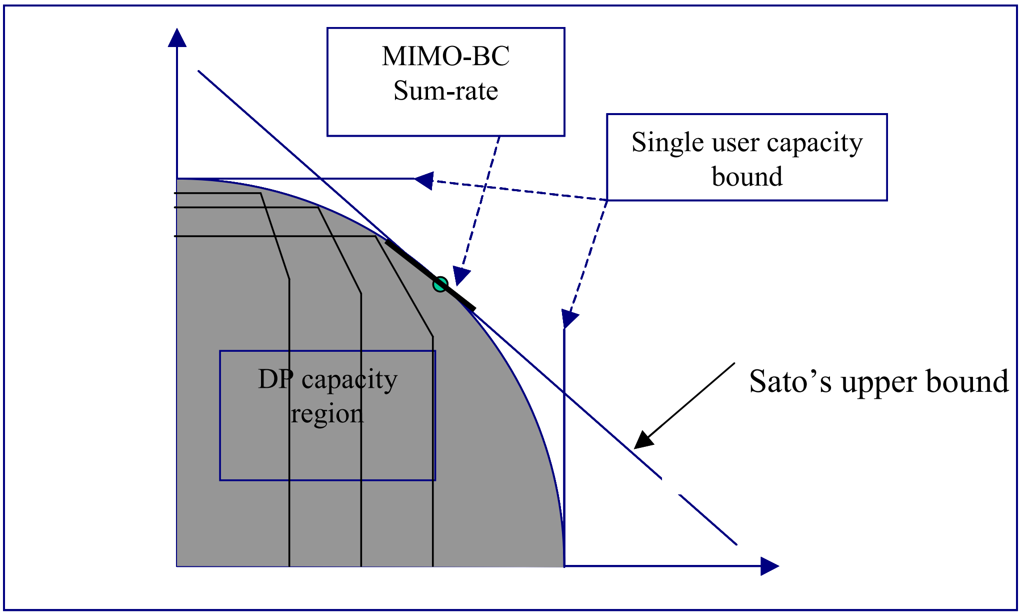


Figure IX-18.- Capacity region for the BC

As we have already commented, one important feature to notice about the rate equations is that they are neither a concave nor convex function of the covariance matrices. However, by exploiting the duality between the MIMO BC and the MIMO MAC that can be exploited to greatly simplify this calculation. The figure outlines the capacity region for the BC. In a separated way, Tse (uplink-downlink duality) proved that the DP or DFE achievable region achieves the sum rate capacity of the MIMO Gaussian BC. Also Shamai (Enhanced channel and Minkowski's inequality) showed that DP coding or DFE precoding achievable rate region is the capacity region of the Gaussian MIMO BC.

The capacity region for the BC is obtained as the union of capacity regions of the dual MAC, where the union is taken over all individual power constraints that sum up to the BC power constraints

$$C_{BC}(\bar{P},\mathbf{H}) = \bigcup_{\{P_i\}_i^K : \sum_{i=1}^{K} P_i \le \bar{P}} C_{MAC}\left\{\mathbf{R}_1,\ldots,\mathbf{R}_K;\mathbf{H}\right\} \tag{IX.84}$$

This leads to the conclusion that the uplink (MAC) and downlink (BC) channels differ only due to the fact that power constraints are placed on each transmitter in the MAC instead of on all transmitters jointly. As shown in the figure, every point on the boundary of the BC capacity region is a corner point of the dual MAC for some set of powers with the same sum power. **Thus, the dirty paper BC achievable region equals the sum power MIMO MAC capacity region.** Successive encoding in the transmitter or successive decoding in the receiver is then required to separate the different signals that are superimposed at the channel output in a BC or MAC channel, respectively. The fundamental trade-off between users is here parameterized by two components: I) the poser allocation, which should be performed jointly, and ii) the encoding/decoding order, with Ntot! Possible orderings of the users. The figure depicts that dirty paper coding achieves the Sato upper bound, and therefore, equals the sumrate cpacity of the MIMO BC [48].

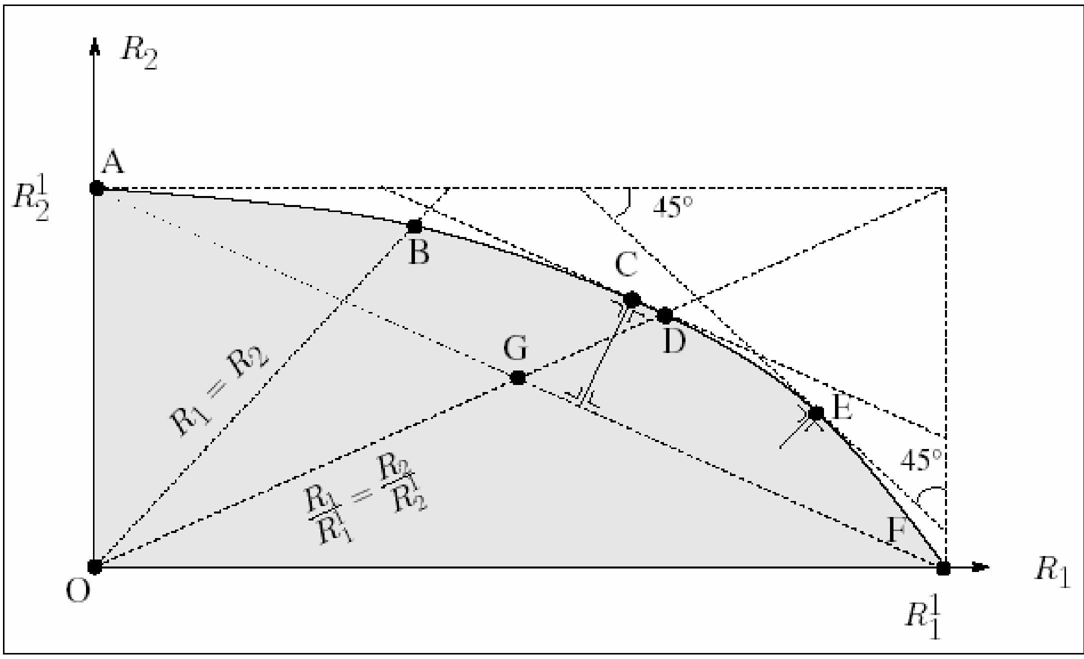
Interestingly, both capacity regions are exactly the same (duality property) **as soon as the power constraint is set on the total transmitted power**.

The boundary of the global capacity region can be traced out by means of a set of relative priority coefficients $\sum_k \xi_k = 1$, which control practical aspects such as fairness and priority aspects that appear in multiuser systems when sharing resources. Since the MAC capacity region is convex, it is well known from convex theory that the boundary of the capacity region can be fully characterized by maximizing the function $\xi_1 R_1 + ... + \xi_K R_K$ over all rate vectors in the capacity region and for all nonnegative priorities. Each boundary point of the capacity region maximizes the linear combination of the user rates $R_\xi = \sum_k \xi_k R_k$. The maximum aggregate rate is known to be

$$R_\xi = \sum_{k=1}^{Ntot} \xi_k \log_2\left(1 + SNIR_k\right) \qquad \text{(IX.85)}$$

For the BC capacity region, the same applies by duality.

For a fixed set of priorities, this is equivalent to finding the point on the capacity region boundary that is tangent to a line whose slope is defined by the priorities. The structure of the MAC capacity region implies that all boundary points of the capacity region are corner points of polyhedrons corresponding to different sets of correlation matrices. Furthermore, the corner point should correspond to successive decoding in order of increasing priority, i.e., the user with the highest priority should be decoded last and, therefore, sees no interference. Thus, the problem of finding the boundary point on the capacity region associated with priorities $\xi_1 ... \xi_K$ assumed to be in descending order.



Figu

Figure IX-19.- Two-user capacity region and specific points of the boundary

In the next figure, the boundary of the capacity region is the curve ABCDEF. The extreme points F and A, on this boundary, correspond to the single-user capacities R_{11} and R_{12} of users 1 and 2, respectively. Point E, with a local tangent at 45 degrees, gives the maximum sum-rate max (R1 + R2). This setting generally results in unfair situations where the users with the best channels have a much higher rate than the others, which is not desirable in practical applications. Point B, on the other hand, gives the maximum common rate or symmetric capacity. When the single-user rates are very different, the common rate constraint is generally a waste of resources as it forces the users with the best channels to lower their rate dramatically to reach the level of the weakest channels. The balanced capacity, given by point D, satisfies the relation $R1=R_{11} = R2=R_{12}$. It appears as a smart compromise between the symmetric capacity B and the maximum sum-rate E. The line AGF represents the rate distributions obtained by using Time-Division Multiple-Access (TDMA). Balanced rates $0.5R_{11}$ and $0.5R_{12}$ are obtained if time slots of equal duration are allocated to each user (point G). Higher balanced rates (point D) can be achieved by allowing a simultaneous transmission of signals by all users, with an appropriate power an spectrum allocation. In any case, the maximum balanced rates can be written

$$R_k = g\frac{R_k^1}{Ntot} \quad \text{(IX.86)}$$

Additional requirements in terms of minimum throughput should be considered for some applications, where customers could pay for a minimum guaranteed service (e.g: a video connection), plus a best-effort service (e.g: Internet connection) with a variable rate that depends on network conditions. The balanced capacity criterion could then be applied on the variable rate only.

IX.3.7 THE FADING CHANNEL

With full CSI, both the base-station and the users track the channel fluctuations and, in this case, the extension of the linear beamforming strategies combined with Costa precoding to the fading channel is natural. Now we can vary the power and transmit signature allocations of the users, and the Costa precoding order as a function of the channel variations. Linear beamforming combined with Costa precoding achieves the capacity of the fast fading downlink channel with full CSI, just as in the time-invariant downlink channel.

Due to the duality, we have a connection between the strategies for the downlink channel and its dual uplink channel.

Next, aspects related with the practical implementation, as partial CSIT and real-time schedulers as fairness and access control are considered. Fairness comes to the scene when optimality has to be trade-off with users' priorities. Concerning access control, note that up to now we have not addressed the problem of real-time access and how users can be ordered in practice when partial CSIT is only available, thus dirty paper implementation is not realistic.

As we have already said, in the design of a scheduler or practical broadcast system 3 questions have to be answered:

Transmitter architecture

Power allocation

Access control (i.e. number of users to give access and order)

IX.3.8 SUM CAPACITY WITH PARTIAL CSIT

When studying the sum rate in the BC channel we obtained that when the number of users goes to infinite and only nt antennas are at the transmitter, the dirty paper precoding behaves as

$$E\left\{R^{DP}\right\}\Big|_{Ntot\to\infty} \approx nt \log\log \rho Ntot \qquad \text{(IX.87)}$$

Thus offering not only multiplexing diversity but also multiuser diversity.

Another comment on the asymptotic behaviour if Ntot goes to infinite is that the optimal transmitter structure that maximizes the sum capacity in multiuser MIMO is then the beamforming [46].

However, having full CSIT requires a lot of feedback and practically it is unrealistic. This motivates the question of how much partial side information is needed in the transmitter that provides us a linear scaling of the throughput with nt and reduces the amount of feedback. If we resort to the asymptotic analysis for Ntot going to infinite, it would be desirable that the new scheme would also take advantage of the multiuser diversity. This is precisely the basis of the so-called opportunistic schemes, which just need SNIR feedback instead of the whole knowledge of **H**.

Multiuser diversity and opportunistic transmission

R. Knopp obtained in [41] that the power control scheme that maximizes the information ergodic capacity of the MAC in a SISO multiuser communication, is such that only the user with best channel transmits at a time. Note that for a scalar BC channel the same result applies. In the conventional TDMA scheme, the base station transmits to only a single user at a time. In this case, the maximum sum-rate, achieved by sending to the user with the largest channel gain, is given by

$$R_{TDMA} = \max_{k \in \{1 \ldots Ntot\}} \log\left(1 + P\left|\mathbf{h}_k\right|^2\right) \qquad \text{(IX.88)}$$

and if Ntot goes to infinite, we observe that the sum-rate increases double-logarithmically in Ntot; thus, taking advantage of the Multiuser diversity. The multiuser diversity effect comes from the fact that when there are many users that fade independently, at any one time there is a high probability that one of the users will have a strong channel. By allowing only that user to transmit, the shared channel resource is used in the most efficient manner and the total system throughput is maximized (see fogire). The larger the number of users, the stronger tends to be the best channel, and the more the multiuser diversity gain.

The work by Knopp set the basis of the so-called opportunistic schemes: the sender opportunistically transmits only when its channel is near its peaks (“riding the peaks”). These schemes are the only implementable schedulers or BC schemes in the actual systems. Therefore, one section is devoted to them, which will comment on the main system issues when implementing them. Opportunistic schemes are the best proof of the benefit of letting PHY and MAC layer interact to face the problems of the mobile channel.

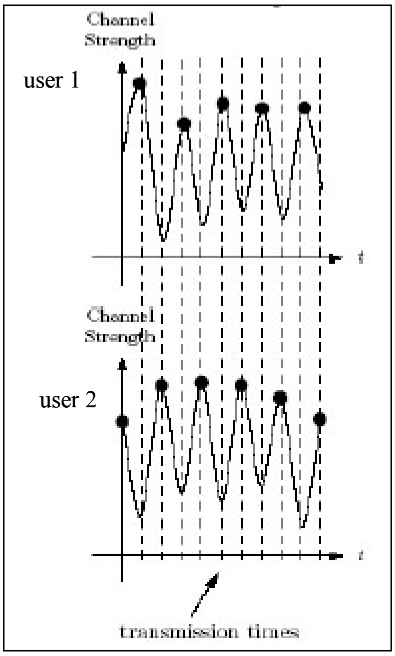


Figure IX-20.- Opportunistic principle for 2 users

The question we pose now is if we can extend the opportunistic waterfilling to MIMO BC channels? Can the MIMO BC channel enjoy of both Multiuser diversity and multiplexing gain? The answer is given by Hassibi in the so-called opportunistic orthogonal SDMA, that extends the opportunistic results of SIMO to the MIMO case.

Opportunisitc SDMA and single antenna receivers

The conceptual idea is to have multiple beams, each orthogonal to one another, at the same time. Separate pilot symbols are introduced on each of the beams and the users feedback the SINR of each beam. Transmissions are scheduled to as many users as there are beams at each time slot. If there are enough users in the system, the user who is beamformed with respect to a specific beam (and orthogonal to the other beams) is scheduled on the specific beam.

Let us consider

$$\mathbf{y}_{Nx1} = \mathbf{A}^{H}_{N\,x(nr\,x\,N)}\mathbf{H}_{(nr\,x\,N)xnt}\mathbf{x}_{nt\,x1} + \mathbf{w}_{Nx1} \qquad \text{(IX.89)}$$

Let us consider Ntot>=nt, otherwise, we use only Ntot of the transmit antennas. Under this considerations we specify N as the number of served users. Note that usually, the number of served users is nt, smaller than Ntot, thus requiring an access control mechanism, which is provided in a natural and low complexity way by the opportunistic scheme that is going to be introduced. On the other hand, although in the DFE precoders Ntot users can be served simultaneously the user ordering stage implies a lot of computational effort.

At each time m, let $\mathbf{B}(m)=\left[\mathbf{b}_1(m),...,\mathbf{b}_{nt}(m)\right]$ be an nt x nt beamforming unitary matrix, with the columns or beams orthogonal. The vector signal sent out from the antenna array at time m is

$$\mathbf{x}_{nt\,xN}=\mathbf{B}(m)\mathbf{u}=\sum_{m=1}^{nt}\mathbf{b}_m(m)u_m \qquad \text{(IX.90)}$$

The unitary matrix **B**(m) is varied such that the individual components do not change abruptly in time. For simplicity we consider the scenario when the channel coefficients are not varying over the time-scale of communication (slow fading)

The ith rx knows (**Hi** **b**m) m=1..nt (by training). Therefore, the ith rx can compute the following nt SINRs by assuming that the um is the desired signal and the other signals are interference as follows

$$SINR_{i,m}=\frac{\left|\mathbf{H_i b_m}\,u_{im}\right|^2}{1/SNR+\sum_{u\neq m}\left|\mathbf{H_i b_m}\,u_{im}\right|^2}\quad m=1..nt \qquad \text{(IX.91)}$$

Note that on average the SINRs behave like

$$SINR_{i,n}\approx\frac{1}{\frac{1}{SNR}+(nt-1)}\approx\frac{1}{nt-1} \qquad \text{(IX.92)}$$

Therefore if the beams are assigned randomly, the rate or throughput will be

$$R=E\left\{\sum_{i=1}^{nt}\log\left(1+SINR_{i,m}\right)\right\}\leq nt\log\left(1+\frac{1}{nt-1}\right)\approx 1 \qquad \text{(IX.93)}$$

Observe that there is no nt-fold in the system throughput and, therefore, CSIT is crucial. As an alternative Hassibi presented an scheme where nt orthogonal beams are assigned to nt users depending on the feedback SINR's. In that case the same asymptotic multiplexing and diversity gain as dirty paper (DP) is obtained when Ntot goes to infinite.

$$R=E\left\{\sum_{i=1}^{nt}\log\left(1+\max_{1\leq i\leq Ntot}SINR_{i,m}\right)\right\}\approx ntE\left\{\log\left(1+\max_{1\leq i\leq Ntot}SINR_{i,m}\right)\right\} \qquad \text{(IX.94)}$$

Intuitively, if the number of users is large the probability of finding nt users placed at the pointing directions of the nt orthogonal beams is high, thus almost nulling the interference among them

$$nt\,E\left\{\log\left(1+\max_{1\leq i\leq Ntot}SINR_{i,m}\right)\right\}\approx ntE\left\{\log\left(1+P\max_{1\leq i\leq Ntot}Tr(\mathbf{h_k h_k^H})\right)\right\} \qquad \text{(IX.95)}$$

Next figure compares the result of the opportunistic SMDA scheme with those of an opportunistic beamforming that serves one user at a time. Note that the opportunistic SDMA does not obtain higher throughput than the single beamformer if the number of users is small, this is due to the interference cased by the simultaneous nt beams.

There are some system requirements to support multiple beams. First, multiple pilot symbols have to be inserted (one for each beam) to enable coherent downlink reception; thus, the fraction of pilot symbol power increases. Second, the receivers now track nt separate beams and feedback SINR of each on each of the beams. On a practical note, the receivers could feedback only the best SINR and the identification of the beam that yields this SINR; this restriction probably will not degrade the performance by much. Thus, with affordable feedback the proposed opportunistic SDMA utilizes all the spatial degrees of freedom.

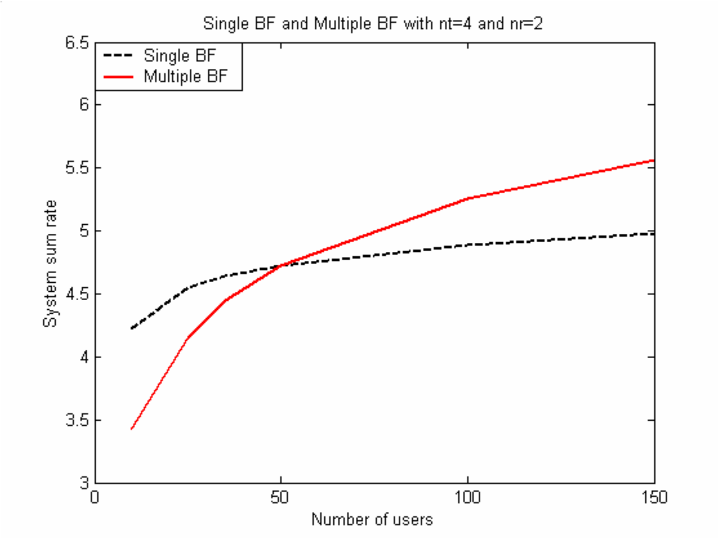


Figure IX-21.- Opportunistic SDMA vs. opportunistic single beam.

The amount of multiuser diversity gain depends crucially on the tail of the fading distribution SNIRk: the heavier the tail, the more likely there is a user with a very strong channel, and the larger the multiuser diversity gain. For instance, because of the line-of-sight component, the Rician fading distribution is less “random” and has a lighter tail that the Rayleigh distribution with the same average channel gain. As a consequence, it can be seen that the multiuser diversity gain is significantly smaller in the Rician case compared to the Rayleigh case.

Finally, the opportunistic schemes that chose the best transmitting antenna would be optimal in those situations where maximal ratio combining transmission is optimal (e.g.if users are located in orthogonal positions) and for each user there is a predominant antenna in the equivalent channel response. Otherwise, these antenna selection schemes are suboptimal in multiuser scenario. However, due to its practicality and also due to the fact that they can help to diminish the hardening effect, they are an interesting option. In point to point MIMO antenna selection arise as optimal also when there is partial channel state information at the transmitter and rate is to be optimized. The same might happen in the multiuser case and it is a topic to be further researched.

Partial CSIT and Multi-antenna transmitters and receivers

In particular, we can ask what impact multiple receive antennas have on multiuser diversity, an important outcome. In general, any deterministic component that would be introduced in the system (e.g. receiving beamforming in a BC channel) reduces the multiuser gain and therefore, the benefit of using opportunistic schemes (the so-called hardening effect of the fading distribution).

With multiple transmit antennas at the base-station and multiple receive antennas at each of the users, with full CSI we split the information for user k into independent data streams, modulates them on different spatial signatures and then transmit them. The spatial signatures and power allocation to the users (and the further allocation among the data streams within a user) can be done as a function of the channel fluctuations. Linear strategies can be carried out or, if computational complexity is not a problem, Costa precoding (i.e., dirty paper or DFE precoding) can be incorporated.

Without CSI (i.e., only CSIR) the transmitter has no access to the channel fluctuations. One of the important conclusions is that time sharing among the users achieves the capacity region in the symmetric BC channel with CSIR alone. Note that since the statistics of the user channels are identical, if user k can decode its data reliably, then all the other users can also successfully decode user k’s data, concluding that the sum of the rates at which the users are being simultaneously reliably transmitted is bounded by

$$\sum_{i=1}^{Ntot} R_i \le E\left[\log\left(1+\frac{SNR|\mathbf{h}|^2}{nt}\right)\right] \qquad \text{(IX.96)}$$

This implies that the total spatial degrees of freedom in the BC channel are restricted to one instead of the min (nt, Ntot) that can be reached with full CSIT. Thus lack of CSI at the base station causes a drastic reduction in the degrees of freedom of the channel.

With partial CSIT opportunistic schemes are an attractive alternative but it would be desirable to obtain intermediate solutions between those schemes and the optimal DP. To that purpose we recall the superuser transmitter formulation of precoding matrix **B**

$$\mathbf{B}_{ntxN} = \mathbf{V}_{ntxN}\sqrt{\mathbf{\Sigma}}_{ntxN}\mathbf{M}_{NxNtot} \qquad \text{(IX.97)}$$

Where we extend the design of each of the matrixes as follows: **V** is the beamforming matrix, build up with no channel state information. It can contain either nt orthogonal beams; thus, N=nt, or more than nt beams. In this case, Grassmanian manifold can be used to optimized the design of the quasiorthogonal beams. The role of more beams than antennas is when the low number of antennas hinders capacity and transmission to more users than antennas increase capacity if thanks to multiantennas at reception allow for interference cancellation at the receiver.

Matrix **Σ** accounts for the power allocation, which is a topic still unaddressed in opportunistic SDMA. Finally matrix **M** controls the selection of users and the association of each user with a transmitting beam. For instance, in an opportunistic system **M** will chose those users whose spatial channel is as much orthogonal as possible, thus achieving null interference.

Next figure plots a MIMO BC system with nt transmit antennas and Ntot>= nt users each with Nk receive antennas. The plot shows the proposed general scheduler.

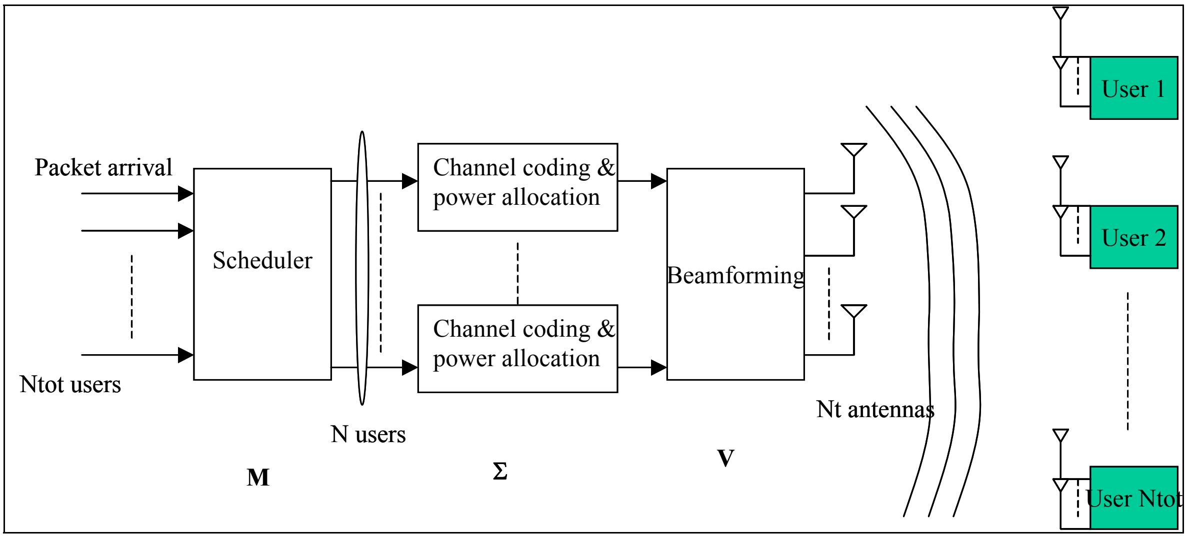


Figure IX-22.- MIMO BC scheduler

IX.4. SYSTEM ASPECTS: SCHEDULING

IX.5. CONCLUSIONS

While the wealth of references and results gives the impression that the problem of MIMO multiuser is by now mature from both the information theory and communications aspects. We believe that such an interpretation is misleading. In fact the many unsolved problems, some strongly motivated by practical applications and implications remain and some, such as the role of the CSI at the transmitted and optimal MIMO BC strategies in the presence of partial CSI , are explicitly mentioned in the sequel. We believe that this topic still calls for intensive research addressing many fundamental problems, which are not yet fully understood.

IX.A. APPENDIX: DETAILED COMPUTATION OF THE OPTIMAL PRECODING IN BC

Departing from section IX.3.3.1 and before getting into the design, let us reformulate the MMSE noise correlation matrix $\left(\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}+\mathbf{I}\right)^{-1}$. Let us consider the Cholesky factorization of the MMSE noise matrix

$$\mathbf{G}^{-1}\boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}=\left(\tilde{\mathbf{H}}^T\tilde{\mathbf{H}}+\mathbf{I}\right)^{-1}=\left(\mathbf{M}^T\sqrt{\boldsymbol{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\boldsymbol{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\boldsymbol{\Sigma}}\mathbf{M}+\mathbf{I}\right)^{-1} \quad \text{(IX.98)}$$

Now, choose a square matrix **C**, such that

$$\mathbf{C}^T\mathbf{C}=\left(\sqrt{\boldsymbol{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\boldsymbol{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\boldsymbol{\Sigma}}+\mathbf{I}\right)^{-1} \quad \text{(IX.99)}$$

in general **C=UR**, where **U** is an orthonormal matrix and **R** an upper triangular one.

Then the Cholesky factorization can be written as

$$\mathbf{G}^{-1}\boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}=\mathbf{M}^T\mathbf{R}^T\mathbf{U}^T\mathbf{U}\mathbf{R}\mathbf{M} \quad \text{(IX.100)}$$

where **URM** is upper triangular.

Now the GDFE is used for the receiver implementation. Although, we show later in section IX.3.3. 2.and IX.3.3. 3. that GDFE has nice optimal features, note that the GDFE error e'i that is obtained when recovering each of the transmitted signals is decoupled, in contrast to the MMSE error. Therefore, we expect that GDFE would help in the decoupled receiver design better than the MMSE.

Recall that the feedforward filter, which we call **F** is

$$\mathbf{F}=\boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\tilde{\mathbf{H}}^T\boldsymbol{\Lambda}^{-1/2}\mathbf{Q}$$

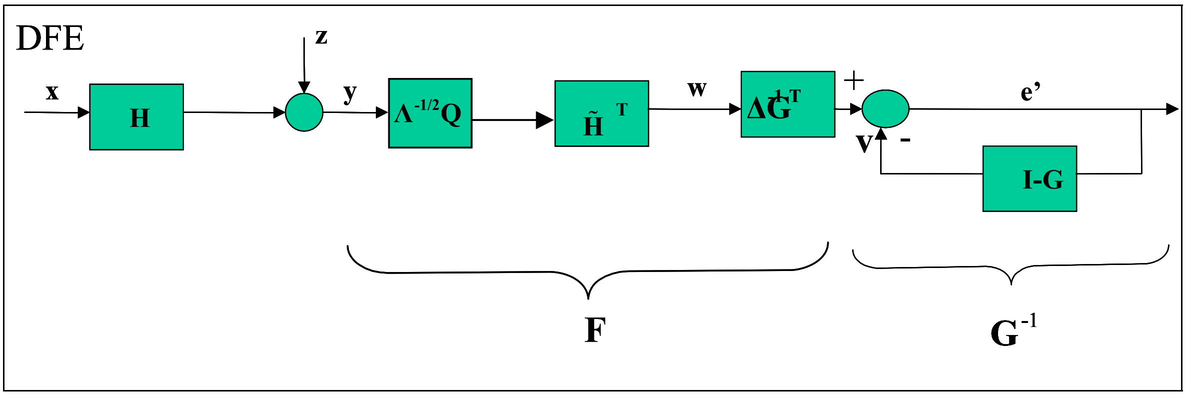


Figure IX-23.- GDFE reception

Then, it can be further computed as follows

$$\begin{aligned}\mathbf{F} &= \mathbf{\Lambda}^{-1/2}\mathbf{URMM}^{T}\sqrt{\mathbf{\Sigma}}\mathbf{V}^{T}\mathbf{H}^{T}\mathbf{Q}^{T}\mathbf{\Lambda}^{-1}\mathbf{Q} \\ &= \mathbf{\Lambda}^{-1/2}\mathbf{UR}\sqrt{\mathbf{\Sigma}}\mathbf{V}^{T}\mathbf{H}^{T}\mathbf{Q}^{T}\mathbf{\Lambda}^{-1}\mathbf{Q}\end{aligned} \tag{IX.101}$$

Now a block diagonal structure is imposed on **F**

$$\begin{aligned}\mathbf{F} &= \mathbf{\Lambda}^{-1/2}\mathbf{UR}\sqrt{\mathbf{\Sigma}}\mathbf{V}^{T}\mathbf{H}^{T}\mathbf{Q}^{T}\mathbf{\Lambda}^{-1}\mathbf{Q} = \\ &= \mathbf{\Lambda}^{-1/2}\begin{bmatrix}\mathbf{\Phi}_1^{-1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_1^{-1/2}\end{bmatrix}\end{aligned} \tag{IX.102}$$

where **UR** can be obtained. Finally, an appropriate transmit filter $\mathbf{B} = \mathbf{V}\sqrt{\mathbf{\Sigma}}\mathbf{M}$ is found by obtaining an **M** that makes **URM** block upper-triangular. This is possible by the following QR-factorization: $\mathbf{R}^{T}\mathbf{U}^{T} = \mathbf{MK}$, where **K** is lower triangular and **M** is orthogonal. Then $\mathbf{URM} = \mathbf{K}^{T}$ is upper-triangular. Note that

$$\mathbf{R}^{T}\mathbf{R} = \left(\sqrt{\mathbf{\Sigma}}\mathbf{V}^{T}\mathbf{H}^{T}\mathbf{Q}^{T}\mathbf{\Lambda}^{-1}\mathbf{QHV}\sqrt{\mathbf{\Sigma}} + \mathbf{I}\right)^{-1} \tag{IX.103}$$

There are still the question to answer before obtaining a complete precoding design:

- the design of the block diagonal matrix

<u>Design of the block diagonal matrix</u>

Next we show that the condition under which there exists a suitable **UR** to make the feedforward filter **F** block-diagonal, and therefore suitable for non-cooperative receivers as in the BC channel, is the same as the diagonalization condition on the noise covariance matrix

$$\mathbf{R}_z^{-1} - \left(\mathbf{R}_z + \mathbf{HR}_x\mathbf{H}^{T}\right)^{-1} = \begin{bmatrix}\mathbf{\Phi}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2^{-1}\end{bmatrix} \; (**) \tag{IX.104}$$

where $\mathbf{\Phi}_i^{-1}$ are positive semi-definite matrices and are the dual variables associated with the block-diagonal constraints of the max-min problem.

In order to get some insight in the meaning of constraint (**), note that this conditions is equivalent to $\left(\mathbf{I} + \tilde{\mathbf{H}}\mathbf{R}_x\tilde{\mathbf{H}}^{T}\right)^{-1} = \begin{bmatrix}\mathbf{\Phi}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2\end{bmatrix}$, thus imposing block diagonal structure on the correlation matrix of the received signal after the whitening filter.

The condition under which there exists a suitable **UR** to make the feedforward filter **F** block-diagonal is the same as the diagonalization condition on the noise covariance matrix

$$\begin{bmatrix} \mathbf{\Phi}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2^{-1} \end{bmatrix} = \mathbf{R}_z^{-1} - \left(\mathbf{R}_z + \mathbf{H}\mathbf{R}_x\mathbf{H}^T\right)^{-1} = \mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q} - \left(\mathbf{Q}^T\mathbf{\Lambda}\mathbf{Q} + \mathbf{H}\mathbf{V}\mathbf{\Sigma}\mathbf{V}^T\mathbf{H}^T\right)^{-1} =$$

$$= \mathbf{Q}^T\mathbf{\Lambda}^{-1/2}\left(\mathbf{I} - \left(\mathbf{I} + \mathbf{\Lambda}^{-1/2}\mathbf{Q}\mathbf{H}\mathbf{V}\mathbf{\Sigma}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1/2}\right)^{-1}\right)\mathbf{\Lambda}^{-1/2}\mathbf{Q}^T =$$

$$= \mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\mathbf{\Sigma}}\left(\mathbf{I} + \sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\mathbf{\Sigma}}\right)^{-1}\sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q} \tag{IX.105}$$

As $\mathbf{R}^T\mathbf{R} = \left(\sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\mathbf{\Sigma}} + \mathbf{I}\right)^{-1}$ (IX.106)

Then

$$\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{H}\mathbf{V}\sqrt{\mathbf{\Sigma}}\mathbf{R}^T\mathbf{R}\sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q} = \begin{bmatrix} \mathbf{\Phi}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2^{-1} \end{bmatrix}$$

(IX.107)

$$\mathbf{R}\sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q} = \mathbf{U}\begin{bmatrix} \mathbf{\Phi}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2^{-1} \end{bmatrix}$$

Therefore,

$$\mathbf{F} = \mathbf{\Lambda}^{-1/2}\mathbf{U}\mathbf{R}\sqrt{\mathbf{\Sigma}}\mathbf{V}^T\mathbf{H}^T\mathbf{Q}^T\mathbf{\Lambda}^{-1}\mathbf{Q} =$$

$$= \mathbf{\Lambda}^{-1/2}\begin{bmatrix} \mathbf{\Phi}_1^{-1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_1^{-1/2} \end{bmatrix} \tag{IX.108}$$

IX.B. APPENDIX: DECISION FEEDBACK PRECODER IS CAPACITY LOSSLESS

Recalling in the figure the DFE receiver structure that has been presented in the MAC section. Next we show that the feedforward filter can be implemented in the transmitter (if CSIT is available) without losing capacity with respect to the DFE.

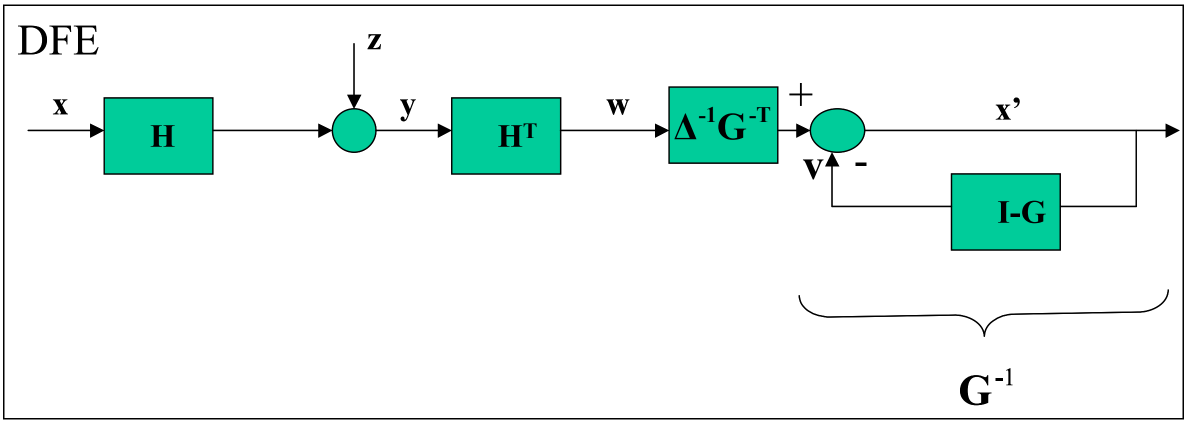


Figure IX-24.- DFE receiver structure

Let us depart from the MAC channel model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} = [\mathbf{H}_1 \ \mathbf{H}_2]\begin{bmatrix}\mathbf{x}_1 \\ \mathbf{x}_2\end{bmatrix} + \begin{bmatrix}\mathbf{z}_1 \\ \mathbf{z}_2\end{bmatrix} (*) \qquad \text{(IX.109)}$$

and consider the output of the feedforward filter, **v=[v1 v2]**^{T} . The goal is to compute the achievable rates of the two subchannels: form **x1** to **v1** and from **x2** to **v2**. If they are the same as the rates from **x1** to **x1'** and from **x2** to **x2'**, which are the rates of the lossless MMSE, no capacity loss is incurred by implementing a DFE precoder. In other words, we are interested in proving that:

$$\begin{aligned} R_1 &= I(X_1;V_1) \overset{?}{=} I(X_1;X'_1) \\ R_2 &= I(X_2;V_2) \overset{?}{=} I(X_2;X'_2) \end{aligned} \qquad \text{(IX.110)}$$

To prove it note that

$$\mathbf{v} = \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{H}^T\mathbf{H}\mathbf{x} + \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{H}^T\mathbf{z} = \boldsymbol{\Delta}^{-1}\mathbf{G}^{-T}\mathbf{H}^T(\mathbf{H}\mathbf{x} + \mathbf{z}) \qquad \text{(IX.111)}$$

If

$$\mathbf{x} = \mathbf{x}' + \mathbf{e}' \qquad \text{(IX.112)}$$

then

$$\mathbf{x'} = \mathbf{v} + (\mathbf{I} - \mathbf{G})\mathbf{x} \qquad \text{(IX.113)}$$

Note that, $\mathbf{x}_2^{'} = \mathbf{v}_2$, thus

$$R_2 = I(X_2;V_2) = I(X_2;X'_2) = I(X_2;Y) \qquad \text{(IX.114)}$$

Now, consider the subchannel from **x**1 to **v**1 with **x**2 available at the transmitter instead of at the receiver $I(X_1;V_1/X_2)$

By substituting for the transmission model in (*) as it was done for the MAC channel, we obtain

$$\begin{aligned}\mathbf{x}_1^{'} &= \mathbf{\Delta}_{11}^{-1}\mathbf{w}_1 - \mathbf{G}_{22}\mathbf{x}_1 \\ \mathbf{v}_1 &= \mathbf{\Delta}_{11}^{-1}\mathbf{w}_1 + \mathbf{e}_1^{'} = \left(\mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1}\mathbf{H}_1^T\left(\mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{z}_1\right)\end{aligned} \qquad \text{(IX.115)}$$

being **w** the output to the matched filter to the channel

$$\mathbf{w} = \mathbf{H}^T\mathbf{H}\mathbf{x} = \begin{bmatrix}\mathbf{H}_1^T\mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_1^T\mathbf{H}_2\mathbf{x}_2 \\ \mathbf{H}_2^T\mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2^T\mathbf{H}_2\mathbf{x}_2\end{bmatrix} \qquad \text{(IX.116)}$$

and we obtain

$$\mathbf{x}_1^{'} = \left(\mathbf{R}_1^{-1} + \mathbf{H}_1^T\mathbf{H}_1\right)^{-1}\mathbf{H}_1^T(\mathbf{H}_1^T\mathbf{x}_1 + \mathbf{z}_1) \qquad \text{(IX.117)}$$

Observe that $\mathbf{v_1}=\mathbf{x_1}'+\mathbf{H_2}\ \mathbf{x_2}$ as $\mathbf{x_1}'$ and $\mathbf{x_2}$ are independent (since, x1, x2 and z1 are jointly independent)

$$R_1 = I(X_1;V_1/X_2) = I(X_1;X_1') = I(X_1;Y/X_2) \qquad \text{(IX.118)}$$

where, the last equality was shown in the MAC channel section and proves that a precoder designed in order to obtain $R_2=I(X_2;Y)$ and $R_1=I(X_1;Y/X_2)$ achieves the same capacity as a DFE equalizer: $R_2=I(X_2;X_2')$ and $R_1=I(X_1;X_1')$

Therefore,

$$I(X_1 X_2;Y) = I(X_2;Y)+I(X_1;Y/X_2) = I(X_2;X_2')+I(X_1;X_1') \qquad \text{(IX.119)}$$

or, depending on the precoding order

$$I(X_1 X_2;Y) = I(X_1;Y)+I(X_2;Y/X_1) = I(X_1;X_1')+I(X_2;X_2') \qquad \text{(IX.120)}$$

Interference cancellation may occur at the transmitter by pre-subtracting x2 form x1. Pre-subtraction achieves the exact same capacity as a decision feedback equalizer. In conclusion, the DFE precoding as shown in the next figure is capacity lossless.

IX.C. APPENDIX: UPLINK-DOWNLINK SINR DUALITY

Restricting to linear beamforming strategies for the downlink and to linear detection strategies for the uplink, let **F** denote the linear transmitter matrix and $\mathbf{F}^H$ denote the linear receiver matrix, respectively. Without loss of generality, we can assume the normalization $\left[\mathbf{FF}^H\right]_{jj}=1$ for all j. With this normalization, the input power constraint is given simply by $\sum_k P_k^{bc} \le P$. The SINR for user i in the BC channel is

$$SINR_{bc,i} = \frac{P_i^{bc}\phi_{ii}}{1+\sum_{j\neq i} P_j^{bc}\phi_{ij}} \qquad \phi_{ij} = \left|\mathbf{HF}\right|_{ij}^2 \tag{IX.121}$$

In the dual MAC, the output of the linear detector of user i is given by the i-th element of the vector $\mathbf{F}^H\mathbf{y}_{MAC}$. The SINR for user i is given by

$$SINR_{mac,i} = \frac{P_i^{mac}\phi_{ii}}{1+\sum_{j\neq i} P_j^{mac}\phi_{ij}} \tag{IX.122}$$

Suppose that target SINRs γ1…γm are required in both the BC and its dual MAC. The system of equations $SINR_i \ge \gamma_i$ can be written in compact matrix form as follows.

$$\text{Let } \mathbf{a} = \left[a_1 \cdots a_1\right]^T \qquad a_i = \frac{\gamma_i}{(1+\gamma_i)\phi_{ii}} \tag{IX.123}$$

Then the SINR equations for the BC take on the form

$$\left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\right]\mathbf{p}^{bc} \ge \mathbf{a} \tag{IX.124}$$

For the MAC take on the form

$$\left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}^T\right]\mathbf{p}^{MAC} \ge \mathbf{a} \tag{IX.125}$$

The SINR vector γ is feasible for both BC and MAC with linear processing matrix **F** if and only if the non-negative matrix diag(**a**)**Φ** has Perron-Frobenious eigenvalue ρ(diag(**a**)**Φ**)<1. In this case, the solutions

$$\mathbf{p}_{opt}^{bc} = \left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\right]^{-1}\mathbf{a} \qquad \text{(IX.126)}$$

and

$$\mathbf{p}_{opt}^{mac} = \left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}^{T}\right]^{-1}\mathbf{a} \qquad \text{(IX.127)}$$

of the BC and MAC power allocation equations are the componentwise minimal power allocation that meets the $\boldsymbol{\gamma}$ with equality. Moreover $\sum_i p_{opt,i}^{bc} = \sum_i p_{opt,i}^{mac}$

AS an immediate corollary we get that, a SINR vector $\boldsymbol{\gamma}$ is feasible if and only if

$$\rho(diag(\mathbf{a})\mathbf{\Phi}) < 1$$
$$and$$
$$\sum_i p_{opt,i}^{bc} = \mathbf{1}^{T}\left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\right]^{-1}\mathbf{a} \le P$$

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