PHY-MAC Cross-layer: multiuser systems

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3. Multiuser capacity and opportunistic communications

What are the “optimal” multiple access schemes?

Multiuser information theory answer some aspects of this question and show that, compared with the point-to-point setting, the multiuser scenario offer more opportunities to exploit: which user(s) to transmit from/to, amount of power and rate to allocate among them.

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Introduction

When considering the different multiple access techniques (TDMA/FDMA,CDMA, OFDM) designed to share the channel among several users. A natural question is: what are the “optimal” multiple access schemes?

Ex: in bursty networks: FDMA or TDMA are inefficient and combination with CDMA or SDMA may be useful and handle collisions by means of protocols

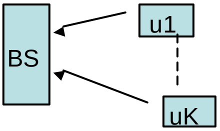
Information theory can be generalized from the point-to-point scenario, to the multiuser ones, providing limits to multiuser communications and suggesting optimal multiple access strategies. New techniques and concepts such as successive cancellation, superposition coding and multiuser diversity emerge.

A new design principle for wireless systems appears taking advantage of multiuser diversity in an opportunistic way. Instead of the classical approach of channel averaging, one should exploit channel fluctuations.

1. MAC AWGN channel: capacity region

Capacity via successive interference cancellation

Consider 2 users $y[m]=x1[m]+x2[m]+w[m]$



Pk: **average** (along the degrees of freedom) power constraint for user k

The capacity region C is the set of all pairs (R1, R2) such that simultaneously User 1 and user2 can reliably communicate at rate R1 and R2, respectively. Because signaling dimensions can be allocated to different users in an infinite number of different ways, multiuser channel capacity is defined by a rate region rather than a single number. This region describes all user rates that can be simultaneously supported by the channel with arbitrarily small error probability.

In Information theory the reliability is given by a BER as low as desired in Network theory we will see that reliability is given by a bounded delay. Thus reliability will be synonymous of stability.

The capacity region C characterizes the optimal tradeoff achievable by any multiple access scheme. Ex: in OFDM, this tradeoff can be achieved by varying the number of sub-carriers allocated to each user.

From the capacity region, one can derive other scalar performance measures:

The symmetric capacity

$$C_{sym} = \max_{(R_1,R_2)\in C} R$$

Is the maximum common rate at which both users can simultaneously reliably communicate

The sum capacity

$$C_{sum} = \max_{(R_1,R_2)\in C} R_1 + R_2$$

Is the maximum total throughput that can be achieved

Three conditions characterize the capacity region C of the MAC

$$R_1 < \log\left(1+\frac{P_1}{N_o}\right) = I(x_1;y|x_2)$$

$$R_2 < \log\left(1+\frac{P_2}{N_o}\right) = I(x_2;y|x_1)$$

$$R_1+R_2 < \log\left(1+\frac{P_1+P_2}{N_o}\right) = I(x_2,x_1;y) = I(x_1;y)+I(x_2;y|x_1) \neq I(x_1;y|x_2)+I(x_2;y|x_1)$$

The first two conditions say that the rate of the individual user cannot exceed the Capacity of the point-to-point link with the other user absent from the system

Note that the 3rd constraint says that the total throughput cannot exceed the Capacity of a point-to-point AWGN channel with the sum of the received powers Of the 2 users. This is a valid constraint since the signals of the two users are Independent.

Without the 3rd constraint, the capacity region would have been a rectangle, and Each user could simultaneously transmit at the point-to-point capacity as if the Other user did not exist. The 3rd constraint says that there mus t be a tradeoff Between the performance of the two users.

The general MAC case with K users

$$\sum_k R_k \le \log\left(1+\frac{\sum_k P_k}{N_o}\right)$$

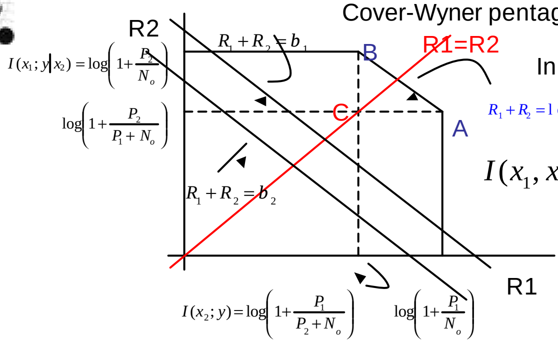
The capacity region of the uplink channel is the convex hull of the union of the pentagons over all possible independent input distributions subject to the appropriate individual average cost constraints

$$C = convex\ \ hull\ \ of\ \left(\bigcup_{P_{x1},P_{x2}} C(P_{x1},P_{x2})\right)$$

The convex hull operation means that we also include the convex combinations. This is natural since the convex combinations can be achieved by time-sharing

In the MAC with single tx antenna, there is a unique set of input distributions (with average power constraint on the two users) that simultaneously max the three different constraints. In general, no single pentagon may dominate over the Other pentagons, and in this case the overall capacity region may not be a pentagon.

Cover-Wyner pentagon



In the corner point B:

$$R_1+R_2=\log\left(1+\frac{P_1+P_2}{N_o}\right)$$

$$I(x_1,x_2;y)=I(x_1;y)+I(x_2;y|x_1)$$

by applying the chain rule for mutual information

Note that user i can achieve its single-user bound while at the same time user j can get a non-zero rate thanks to the successive interference cancellation (SIC) receiver.

If the goal of the system is to maximize the sum rate, any point on AB is equally fine. On the other hand, some operating points are not fair.

Best MAC-SISO policy to max. Csum is simultaneous transmission, each user with is maximum power

All the other rate points on the segment AB can be obtained by time/freq.-sharing between multiple access strategies in point A and point B. The points on AB are Pareto optimal. They dominate, component wise any other point in the capacity region. The preferred operating point in AB depends on the system objective.

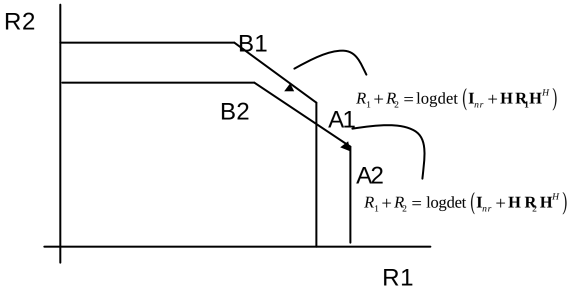
A fair scheme is the one that goes for the symmetric capacity (point C)

$$C_{sym} = \max_{(R_1,R_2)\in C} R$$

Note that with the conventional receiver CDMA, the near-far problem turns into a near-far advantage. Users closer to the base-station can be allowed to take Advantage of the stronger channel and transmit at a higher rate while not degrading the performance of the users in the edge of the cell.

This advantage is less apparent in providing voice service where the required data rate of a user is constant over time, but it can be important for providing data services where users can take advantage of the higher data rates when they are closer to the base-station.

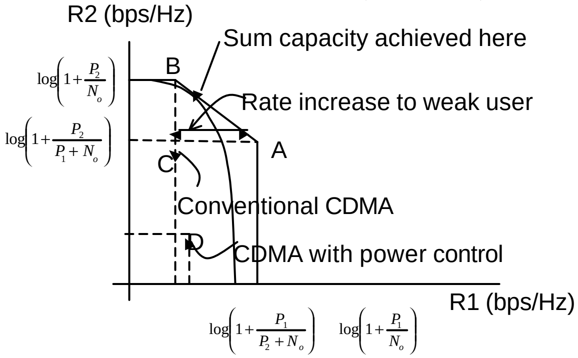
An example is provided by the MAC with multiple transmit antennas at the users.Then there is no single choice of covariance matrices that simultaneously max the constraints: the capacity region is the convex hull of the union of pentagons created by all the possible covariance matrices (subjt.to power constrains on users). In this case the overall capacity region may not be a pentagon.



In the orthogonal multiple access (either time or frequency)

$$R_1 < aW\log\left(1+\frac{P_1}{aN_o}\right) bits/s$$

$$R_2 < (1-a)W\log\left(1+\frac{P_2}{(1-a)N_o}\right) bits/s$$



This schemes are suboptimal
Except for one point: when

$$a = \frac{P_1}{(P_1+P_2)}$$

But results in highly unfair

Intuitively, to exploit the available degrees of freedom all users must access the channel simultaneously and their signals should be separable at the base station. To get this benefit, more complex signal processing is required at the receiver to extract the signal of each user from the aggregate.

For example is multiple receive antennas (nr) are available, there is no further degree-of-freedom gain beyond having nr users performing SDMA simultaneously. This suggests a nearly-optimal multiple access strategy where the users are divided into groups of nr users with SDMA within each group and orthogonal multiple access between the groups.

On the other hand, at low SNR, the channel is power-limited rather than degrees-of-freedom-limited and SDMA provides little performance gain over orthogonal multiple access

The general K-user MAC capacity is described by $2^K - 1$ constraints, one for each possible non-empty subset of users S

$$\sum_{k \in S} R_k < \log\left(1 + \frac{\sum_{k \in S} P_k}{N_o}\right) \qquad \forall S \subset \{1, \ldots, K\}$$

As before, the right hand side corresponds to the maximum sum rate that can be achieved by a single transmitter with the total power of the users in S and with no other users in the system.

$$C_{sum} = \log\left(1 + \frac{\sum_{k \in S} P_k}{N_o}\right) bps/Hz$$

There are exactly K! corner points, each one corresponding to a successive cancellation order among the users

The equal received power case (P1=….=PK=P) is particularly simple

The sum capacity is

$$C_{sum} = \log\left(1 + \frac{KP}{N_o}\right) bps/Hz$$

The symmetric capacity is

$$C_{sym} = \frac{1}{K} \log\left(1 + \frac{KP}{N_o}\right) bps/Hz$$

Which can be obtained by orthogonal multiplexing: each user is allocated a Fraction 1/K of the total degrees of freedom

Observe that the sum capacity is unbounded as the number of users grows. In contrast, if the conventional CDMA receiver (MF) is used, the sum rate is

$$K\log\left(1+\frac{P}{(K-1)P+N_o}\right)bps/Hz \underset{K\to\infty}{\approx}$$

$$K\frac{P}{(K-1)P+N_o}\log_2 e \approx \log_2 e = 1.442\,bps/Hz$$

Thus it is an interference-limited system.

Note that the above comparison pertains to a single-cell scenario, since the only external effect modeled is white Gaussian noise. In a cellular network, the out-of-cell interference must be considered, and as long as the out-of-cell signals cannot be decoded, the system would still be interference-limited, no matter what the receiver is.

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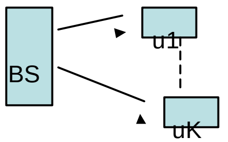
2. Broadcast AWGN channel

Assuming two users

$$y_k[m] = h_k x[m] + w_k[m] \qquad k = 1,2$$

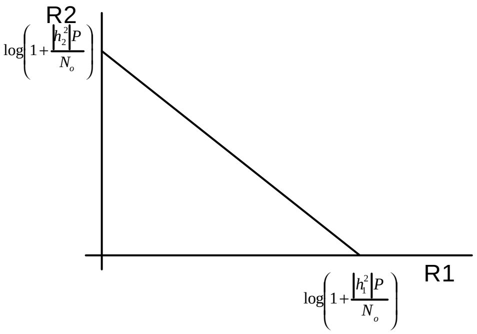


There is average power constraint P for x[m]
We assume that hk is known to both the transmitter
And the user k

To establish the capacity region, we have the single-user bounds

$$R_k < \log\left(1+\frac{P|h_k|^2}{N_o}\right) k = 1,2$$

Which give 2 extreme points, further we can share the degrees of freedom (time And bandwidth) between the users in an orthogonal manner to obtain any rate Pair on the line joining these two extreme points. Can we achieve a rate pair Outside this triangle by a more sophisticated communication strategy?



Case of symmetric channel $|h_1|=|h_2|=|h|$

Therefore, if user 1 can successfully decode its data, then user 2 should also Be able to decode successfully the data of user 1 (and vice versa). Thus,

$$R_1+R_2<\log\left(1+\frac{(P_1+P_2)|h|^2}{N_o}\right)$$

Thus, the triangle is again the capacity region for the symmetric channel

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The rate pairs in the capacity region can be achieved by strategies used on Point-to-point AWGN channels and sharing the degrees of freedom (time and Bandwidth) between the two users. However, the symmetry between the two Channels suggests a natural, and alternative approach to orthogonalization of The degrees of freedom. Each user can perform successive interference cancellation.

$$x[m]=x_1[m]+x_2[m]$$

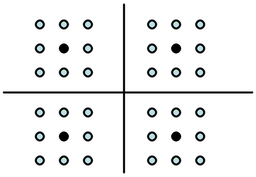
$$R_1=\log\left(1+\frac{P_1|h_1|^2}{P_2|h_1|^2+N_o}\right)=$$

$$\log\left(1+\frac{(P_1+P_2)|h_1|^2}{N_o}\right)-\log\left(1+\frac{P_2|h_1|^2}{N_o}\right)$$

The performance of user 2 (the weaker one) is then

$$R_2=\log\left(1+\frac{P_2|h_2|^2}{N_o}\right)$$

For the general case: superposition coding achieves capacity



32-QAM con QPSK

32-QAM

$$R_1 = \log\left(1+\frac{P_1|h_1|^2}{P_2|h_1|^2+N_o}\right)$$

QPSK

$$R_2 = \log\left(1+\frac{P_2|h_1|^2}{N_o}\right)$$

On the other hand, orthogonal schemes achieve, for each power split P=P1+P2 and degree-of-freedom split α

$$R_1 = a\log\left(1+\frac{P_1|h_1|^2}{aN_o}\right)$$

$$R_2 = (1-a)\log\left(1+\frac{P_2|h_2|^2}{(1-a)N_o}\right)$$

One can show that superposition coding is strictly better than the orth. Schemes. In these ones, a significant fraction of the degrees of freedom to the weak user are Needed to achieve near single-user performance, degrading the strong user

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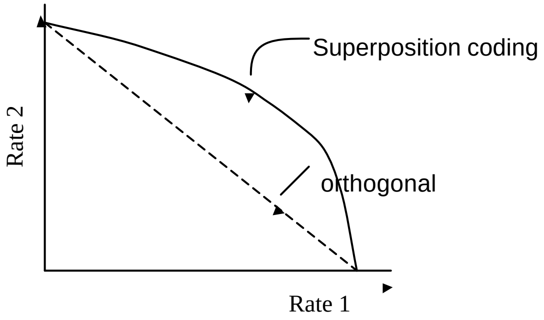

Best policy in BC-SISO: one user at a time

For the general K-users case

$$\sum_{k=1}^{K} R_k < \log\left(1+\frac{P\,|h|^2}{N_o}\right)$$

In general with the ordering $|h_1| \le |h_2| \le \dots \le |h_K|$ the boundary of the capacity Region is given by the parameterized rate tuple

The cancellation order at every
Rx is always to decode the weaker
Users before the own data

$$R_k = \log\left(1+\frac{P_k|h_k|^2}{(\sum_{j=k+1}^{K} P_j)|h_k|^2+N_o}\right) \qquad k=1\dots K$$

For all possible splits $P=\sum_{k=1}^{K} P_k$ of the total power at the base station

Note that if the goal is max Csum , in the MAC, we required all the users to Transmit simultaneously, but in the Broadcast, **it will be achieved by transmitting only to a single user, the one with bigger SNR**

Some considerations of the SIC implementation:

-In broadcast, the complexity at each mobile implementing the SIC scales with The number of users in the cell. In addition, the mobile has to decode information Not intended for him. One solution, in practice is to break the users in the cell into Groups, with each group containing a small number of users with disparate Channels. Within each group superposition coding is done, and across the Groups, transmission is kept orthogonal.

-The effect of error propagation degrades the BER by a factor at most the number Of users K

- A/D quantization error: it needs to have high dynamics

3. MAC fading channel

$$y[m]=\sum_{k=1}^{K} h_k[m]x_k[m]+w[m] \qquad E[|h_k[m]|]=1$$

We concentrate on the symmetric case

Slow fading channel: $h_k[m]=h_k$

It implies that the outage capacity has to be studied

$$p_{out}^{MAC}=P\left\{\log\left(1+\frac{P}{No}\sum_{k\in S}|h_k|^2\right)<|S|R \quad S\subset\{1\ldots K\}\right\}<e$$

At low SNR the orthogonal access is close to optimal and their outage is equal to the point-to-point channel, but scale down by the number of users

At high SNR, the symmetric outage capacity for moderate number of users is approximately equal to that of point-to-point

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Fast MAC fading

With only CSIR, there is no dynamic power allocation

$$C_{sum}=E\left[\log\left(1+\frac{\sum_{k=1}^{K}|h_k|^2 P}{N_0}\right)\right]<\log\left(1+\frac{KP}{N_o}\right)$$

Jensen's inequality

$$\sum_{k=1}^{K} E[P_k(h)]=KP \qquad (*)$$

It impacts on the corresponding capacity region

Hence without CSIT, fading always penalizes, unless the number of users is large

$$\frac{1}{K}\sum_{k=1}^{K}|h_k|^2\rightarrow 1 \qquad R_k=E\left[\log\left(1+\frac{|h_k|^2 P}{\sum_{i>k}^{K}|h_i|^2 P+N_o}\right)\right]\approx_{SNR\downarrow\downarrow}\frac{P}{(K-k)P+N_o}\log_2 e$$

If an orthogonal multiple access schemes (optimal in the AWGN symmetric case) is used, the sum rate achieved is

$$\sum_{k=1}^{K}\frac{1}{K}E\left[\log\left(1+\frac{K|h_k|^2 P}{N_o}\right)\right]=E\left[\log\left(1+\frac{K|h_k|^2 P}{N_o}\right)\right]$$

Which is always less than (*) for K>=2

With full channel state information (symmetric channel case):

For a given realization of the channel gains hk,l, the sum capacity (bits/symbol) of this parallel channel is, as for the point-to-point case over L coherence periods

$$\max_{P_{k,l}:k=1\ldots K,l=1\ldots L} \frac{1}{L}\sum_{l=1}^{L}\log\left(1+\frac{\sum_{k=1}^{K}P_{k,l}\left|h_{k,l}\right|^{2}}{N_0}\right) \qquad \frac{1}{L}\sum_{l=1}^{L}P_{k,l}=P$$

Due to the iid of hkl, we can use orthogonal multiple access to achieve the maximum sum rate. Contrast this with the case when only the receiver has CSI, where orthogonal access is strictly suboptimal for fading channels.

Next in order to get the optimal power allocation we relax the power constraint and replace it by

$$\frac{1}{L}\sum_{l=1}^{L}\sum_{k=1}^{K}P_{k,l}=KP$$

Then

$$\max_{P_{k,l}}\left(1+\frac{\sum_{k=1}^{K}P_{k,l}\left|h_{k,l}\right|^{2}}{N_o}\right) \qquad \sum_{k=1}^{K}P_{k,l}=Cte. \qquad (*)$$

This quantity is maximized by giving all that power to the user with the strongest channel gain, that is a generalization of the single user **opportunistic waterfilling**

$$P_{k,l}=\begin{cases}\left(\frac{1}{l}-\frac{N_o}{\max_i\left|h_{i,l}\right|^{2}}\right)^{+} & if \quad \left|h_{k,l}\right|=\max_i\left|h_{i,l}\right| \\ 0 & else\end{cases}$$

Taking L go to inf. And applying to the ergodicity of the fading process, we get The optimal capacity-achieving power allocation strategy, which allocates powers To the users as a function of the joint channel state $h=[h_1,\ldots,h_K]$

With λ chosen to satisfy $\sum_{k=1}^{K}E\left[P_k^{opt}(h)\right]=KP$

The resulting sum capacity is

$$C_{sum} = E\left[\log\left(1+\frac{P_{k^{opt}}\left|h_{k^{opt}}\right|^2}{N_o}\right)\right]$$

With kopt being the index of the user with the strongest channel at joint channel State h.

Note that as users are symmetric, the individual power constraints in (*) are automatically satisfied and we have solved the original problem as well.

Comment: “users” is not a new dimension, in addition to the time dimension, over Which dynamic power allocation can be performed. Therefore, the solution is not Waterfilling over the joint time/user space. Having multiple users does not provide Additional degrees of freedom in the system: the users are just sharing the Time/frequency degrees of freedom already existing in the channel. The problem is:

-How to partition the total resource (power) across time/frequency (degrees of freed.)

-How to share resources across the users in each of the degrees of freedom

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For asymmetric channels , Csum may not be the better cost function, but another one within the capacity region, since the user with the statistically better channel (due to its position in the cell, for instance) may get a much higher rate at the expense of the other users.

It turns out that, orthogonal multiple access in not optimal. Instead, users tx simultaneously and are jointly decoded with a SIC, even though the rates and powers are still dynamically allocated as a function of the channel states.

4. Downlink fading channel

$$y_k[m] = h_k[m]x[m] + w_k[m] \qquad E[|x[m]|] = 1 \quad k = 1 \ldots K$$

Symmetric channels are considered

With only CSIR

We have the single-user bounds, in terms of the point-to-point fading channel capacity

$$R_k < E\left[\log\left(1 + \frac{|h|^2 P}{N_0}\right)\right]$$

As in the AWGN case, if fading statistics are symmetric and by the assumption Of ergodicity, we can say that if user k can decode its data reliably, then all the Other users can also successfully decode user k's data. We obtain the single "super-user" capacity

$$\sum_{k=1}^{K} R_k < E\left[\log\left(1 + \frac{|h|^2 P}{N_0}\right)\right]$$

And, as in the AWGN case: the rate pairs in the capacity region can be Achieved by both orthogonalization schemes and superposition coding.

What about the asymmetric channel?

While orthogonalization schemes can be used, the applicability of superposition decoding is not so clear. In the asymmetric fading case, users in general have different fading distributions and there is no longer a complete ordering of the Users: non-degraded channel

With CSIT and CSIR

If the max of Csum is the goal. The optimal strategy is exactly the same as in the Csum of the MAC. In the broadcast channel we have again an opportunistic scheme

$$C_{sum} = E\left[\log\left(1+\frac{P_{opt}(h)(\max_{k=1\ldots K}\left|h_k\right|^2}{N_o}\right)\right]$$

$$P_{opt} = \left(\frac{1}{l} - \frac{N_o}{\max_i \left|h_i\right|^2}\right)^+$$

And λ should fulfill the average power constraint

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5. Multiuser diversity

Let us consider the sum capacity of the MAC and broadcast flat fading channel

$$C^{MAC}{}_{sum} = E\left[\log\left(1+\frac{P_{k^{opt}}\left|h_{k^{opt}}\right|^2}{N_o}\right)\right]$$

$$C^{Broad}{}_{sum} = E\left[\log\left(1+\frac{P_{opt}(h)(\max_{k=1\ldots K}\left|h_k\right|^2}{N_o}\right)\right]$$

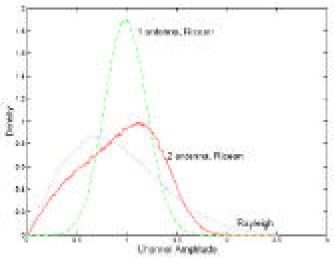
Compared to a single tx user, the multiuser gain comes from two effects:
-the increase in total tx power in the case of the MAC
-the effective channel gain at time m that is improved from $\left|h_1[m]\right|^2$ to $\max_{1\le k\le K}\left|h_k[m]\right|^2$

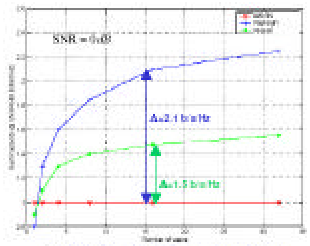
MULTIUSER DIVERSITY

The amount of multiuser diversity gain depends strongly on the tail of the Fading distribution mod(hk); the heavier the tail, the more likely there is a user With a very strong channel. For this reason, the multiuser diversity gain is Smaller in the Rician case compared to the Rayleigh case.

In contrast to classical diversity techniques, multiuser diversity:

-Aims at maximizing throughput but not at improving the reliability
-Exploits channel fading instead of counteracting it
-Is system wide, across the users in the network and not point-to-point communications





Conclusion: Some **system aspects** (in cellular) of multiuser diversity are:

Conventional multiple access:
-Averages out fast channel fluctuations try to make the individual point-to-point links as close to AWGN as possible, with a reliable channel quality that is constant along time
-Track slow fluctuations if CSIT. No need to track fast ones
-Power control the slow fluctuations
-Can support tight delay
-Role of tx. Antennas it point-to-point diversity
-Power gain in downlink with multiple rx antennas
-Interferences are managed by averaging

Opportunistic communications:
-Exploit channel fluctuations
-Track as many fluctuations as possible if CSIT
-Rate control to all fluctuations
-Needs some laxity concerning delay constraints
-The role of Tx antennas is to increase fluctuations
-Interferences are opportunistically avoided (opportunistic nulling)

Separate very-low latency signals from flexible latency data

Some system aspects (in cellular) of multiuser diversity are:

- Feedback channel from the users so that their channel qualities can be tracked
- The ability of the BS to schedule tx among users as well as to adapt the data Rate as a function of the instantaneous channel quality

These features are already present in the designs of many third-generation Systems. Nevertheless, in practice there are several considerations to take into Account before realizing such gains.

1. Fairness and delay, because the usual situation is that of asymmetric channels And the individual needs should not be forgotten. Also multiuser diversity max Long term fading, and delay is also an important concern in this systems

2. Channel measurement and feedback: both the error in channel state Measurement and the delay in feeding it back constitute a significant bottleneck In extracting the multiuser diversity gain

3. Slow and limited fluctuations

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1. Fair scheduling, delay and multiuser diversity:

A possible solution is the proportional fair scheduler

$$\max_k \frac{R_k[m]}{T_k[m]}$$

$$T_k[m+1]=\begin{cases}(1-1/t_c)T_k[m]+(1/t_c)R_k[m] & k=k_{opt}\\ (1-1/t_c)T_k[m] & k\neq k_{opt}\end{cases}$$

The algorithm schedules a user when its instantaneous channel quality is high Relative to its own average channel condition over the time scale tc. In short, Data are transmitted to a user when its channel is near its won peaks.

Parameter tc is tied to the latency time-scale of the application.

With a very long tc, the algorithm maximizes $\sum_{k=1}^{K}\log T_k$

Multiuser diversity and superposition coding: to benefit from both aspects, The users in a cell are divided into two classes

6. An example of practical Use of the capacity region

Wireline Multi-access Channel

-Introduction

-Single-user capacity & Time Division

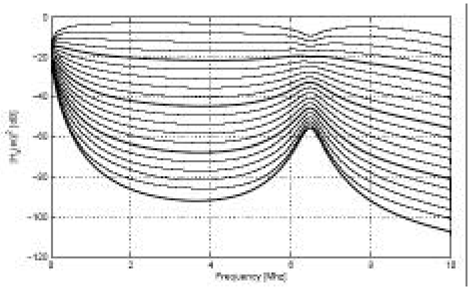
-Successive encoding/decoding

-Balanced capacity

-Mean vs. variance trade-off

Introduction

Wireline channels (e.g. the powerline access channel or the ADSL channel) are quite different from their wireless counterparts. They can be considered as static channels (or slowly time-variant on the scale of seconds or even minutes), so that a perfect channel knowledge can be assumed, both at transmitter(s) and receiver(s). They are generally strongly frequency-selective channels, due to the combination of cable losses and multipath propagation. Remote users benefit from different channel qualities (in terms of attenuation), depending on the position of their modem along the line.



We have seen that the full characterization of the *K*-user capacity region is not tractable for a large number of users. Some specific rate distributions on the boundary of the capacity region, corresponding to some desirable working points, should be identified.

The mapping between the resource allocation and the obtained rate distribution is dependent on the communication scenario. In every scenario, a specific constrained optimization problem can be obtained, and can be solved by appropriate methods, depending, among others, on the shape of the rate region (convexity, etc.):

- The powerline access network (PLC) is a SISO channel: the same physical cable is shared by all customers. The downlink can be modeled as a broadcast channel, with a single power constraint for all users, while the uplink is a multiple-access channel, with individual power constraints.

- Digital Subscriber Lines (DSL) can be modeled as a SIMO (downlink) or a MISO (uplink) multiuser channel: there is one twisted pair for each customer, but a coordinated access to all twisted pairs is theoretically possible at the central office. Several multiuser techniques are currently under study in the context of DSL systems, ranging from crosstalk cancellation to dynamic spectrum management (which can be centralized or distributed), or simply the basic power backoff techniques.

Single-user capacity and Time-Division:

A K-user multi-access channel is fully defined by the set of KN channel gains $\{h_{kn}^2\}$ and the set of N received noise powers $\{\sigma_n^2\}$ associated with the N frequency bins of width b. The total available bandwidth is $B = Nb$. Assuming a power budget of $\overline{P}$, the single-user capacity R_k^1 associated with user-k is given by

$$R_k^1 = b \sum_{n=1}^{N} \log_2 \left(1 + \frac{P_{kn} h_{kn}^2}{\sigma_n^2}\right)$$

where

$$P_{kn} = \left(\frac{1}{\mu} - \frac{\sigma_n^2}{h_{kn}^2}\right)_+$$

and

$$\textstyle\sum_n P_{kn} = \overline{P}.$$

Time-Division is a possible strategy to allow a multiple access to the system. In this scenario, time slots of fixed duration are allocated to the different users, in a round-robin fashion. The capacity region is

$$\{\{R_k\} : R_k \leq \alpha_k R_k^1\}$$

The trade-off between users is parametrized by the timing coefficients *ak*. The obvious way of ensuring a fair share of the channel is to allocate time slots of equal duration to all users, i.e. *ak* = 1/*K*.

Successive encoding/decoding

For a given power budget *P*, a larger capacity region can be obtained by allowing a simultaneous access to the channel by the different users. Successive encoding in the transmitter(broadcast channel) or successive decoding (SD) in the receiver (multiple access channel) is then required to separate the different signals that are superimposed at the channel output. The fundamental trade-off between users is here parametrized by two components: (i) the power allocation *fPkng*, which should be performed jointly for all users, and (ii) the encoding/decoding order, with *K*! possible orderings of the users.

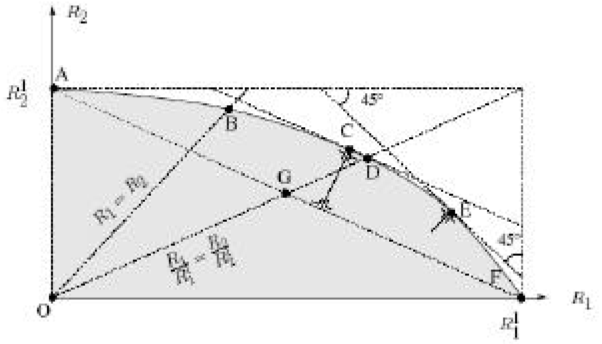
The boundary of the global capacity region can be traced out by means of a set of relative priority coefficients *ak* with sum(*ak)* = 1. Each boundary point of the capacity region maximizes the linear combination of the user rates $R_\alpha = \sum_k \alpha_k R_k$
For a multiple-access channel with successive decoding in the receiver, a decoding order *(1,2,….K)* and a power allocation, the maximum aggregate rate is known to be:

$$R_\alpha = b\sum_{n=1}^{N}\sum_{k=1}^{K}\alpha_k \log_2\left(1+\frac{P_{kn}h_{kn}^2}{\sigma_n^2+\sum_{l=1}^{k-1}P_{ln}h_{ln}^2}\right)$$

Actually, the coefficients ®*k* do not bring any information about the distribution of the maximum user rates *R¤k* . Even with a higher relative priority *ak* >1/*K*, a given user *k* with a poor channel quality could obtain a lower data rate *R¤k* than the other users, or even a zero data rate. The only interpretation that can be associated with the relative priorities *ak* is that $\mathrm{dR}^* \perp \alpha$,which means that the boundary of the capacity region is normal to the vector *a* in the neighborhood of the point R¤.

Balanced capacity

The *balanced capacity* of a multiuser channel, is an example of a specific rate distribution that satisfies a fairness criterion. It is defined as the distribution of maximum simultaneously achievable data rates that are proportional to the single user rates. It is a specific point of the boundary of the capacity region for which the coexistence with the other users has the same relative cost for every user



The Figure illustrates an arbitrary two-user capacity region:

- The boundary of the capacity region is the curve *ABCDEF*.

- The extreme points *F* and *A*, on this boundary, correspond to the single-user capacities *R*11 and *R*12 of users 1 and 2, respectively.

- Point *E*, with a local tangent at 45 degrees, gives the *maximum sum-rate* max (*R*1 + *R*2). This setting generally results in unfair situations where the users with the best channels have a much higher rate than the others, which is not desirable in practical applications.

- Point *B*, on the other hand, gives the maximum common rate or *symmetric capacity*. When the single-user rates are very different, the common rate constraint is generally a waste of resources as it forces the users with the best channels to lower their rate dramatically to reach the level of the weakest channels.

- The *balanced capacity*, given by point *D*, satisfies the relation *R*1/*R*11 = *R*2/*R*12. It appears as a smart compromise between the symmetric capacity *B* and the maximum sum-rate *E*.

- The line AGF represents the rate distributions obtained by usingTime-Division Multiple-Access (TDMA). Balanced rates 12*R*11 and 12*R*12 are obtained if time slots of equal duration are allocated to each user (point *G*).

-Higher balanced rates (point *D*) can be achieved by allowing a simultaneous transmission of signals by all users, with an appropriate power an spectrum allocation. In any case, the maximum balanced rates can be written:

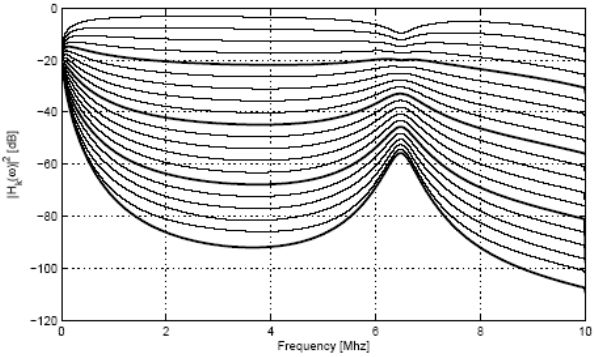
$$R_k = g\frac{R_k^1}{K}$$

where $g < 1$ is the rate gain with respect to the TDMA strategy (*OD*=*OG* on Fig.).

Additional requirements in terms of minimum throughput should be considered for some applications. Customers could pay for a minimum guaranteed service (like for instance a video connection), plus a best-effort service (e.g. Internet connection) with a variable rate that depends on network conditions. The balanced capacity criterion could then be applied on the variable rate only.

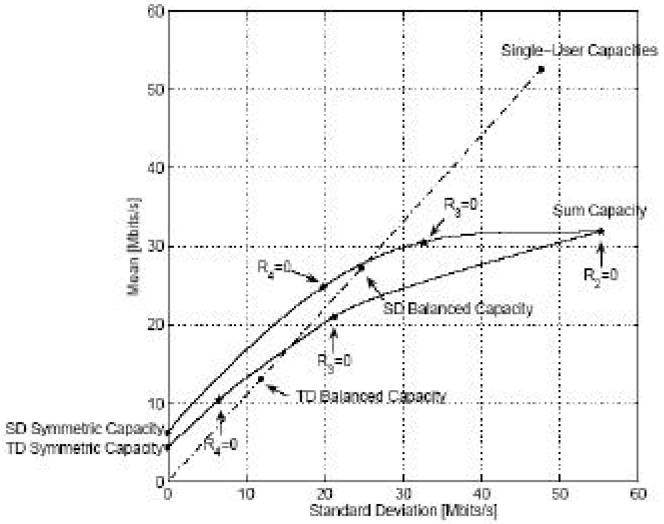
Mean vs. variance trade-off

We propose to analyze the 4-user capacity region associated with a wireline access channel whose frequency responses are given by the Figure (dark lines, i.e. users in positions 5, 10, 15, and 20, with respect to the cable head-end). The bandwidth *B* is 10 MHz, the white noise level is chosen as -120 dBm/Hz, and the transmission power budget is *P* = 10 mW.



The complete analysis of the capacity region is not feasible as it requires an exhaustive 4-dimensional exploration of the feasible rate combinations. The focus should be set on specific rate distributions. Two extreme options are obviously the sum capacity (best global performance) and the symmetric capacity (absolute fairness). In the trade-off between performance and fairness, which depends on the operator policy, the operating point should be selected on a well-chosen trajectory on the boundary of the capacity region, joining these two extreme rate distributions.

The mean vs. variance diagram offers a convenient way of illustrating this trajectory.



The figure gives the mean vs. variance representation of the considered 4-user capacity region.
The upper curve gives the limit for the true capacity region (using successive encoding/decoding,i.e. rates) while the lower curve gives the limit for the TDMA capacity region.

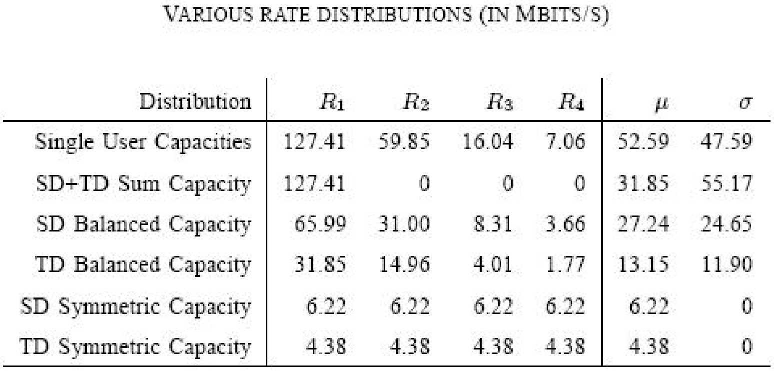
In both cases, starting from the symmetric capacity, the mean and standard deviation of the rate distribution simultaneously increase until the sum capacity is reached.

In this example, the sum capacity is obtained by allocating the whole power *P* to the best user. At intermediate points on the trajectory, the rates of users 4 and 3 go successively to zero, as shown in the Figure. The mean and standard deviation of the single user capacities are also given.

By definition, the balanced rate distribution is located on a straight line joining the single user rate distribution to the origin. As expected, the balanced capacity is rather close to the mean vs. variance upper limit. A slightly better mean is possible for an equivalent standard deviation, but the rate distribution obtained in that case is such that *R*4 = 0.

This reminds that the standard deviation is only a partial measure of the fairness associated with a rate distribution.

Table I gives the details associated with specific rate distributions.



VARIOUS RATE DISTRIBUTIONS (IN MBITS/S)

Distribution	R_1	R_2	R_3	R_4	μ	σ
Single User Capacities	127.41	59.85	16.04	7.06	52.59	47.59
SD+TD Sum Capacity	127.41	0	0	0	31.85	55.17
SD Balanced Capacity	65.99	31.00	8.31	3.66	27.24	24.65
TD Balanced Capacity	31.85	14.96	4.01	1.77	13.15	11.90
SD Symmetric Capacity	6.22	6.22	6.22	6.22	6.22	0
TD Symmetric Capacity	4.38	4.38	4.38	4.38	4.38	0

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