

# PHY-MAC Cross-layer: multiuser systems

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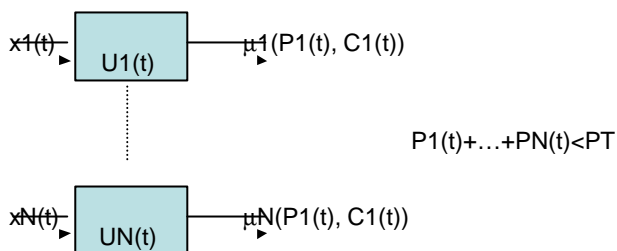
## 6. Energy optimal control for Time Varying wireless networks (multiuser)

From PIMRC05 (Berlin, September 2005), Marc Realp and Ana Perez

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### Introduction



Problems to be addressed:

- Power and server allocation in multiuser systems:  $N$  users but  $K < N$  servers
- Which is the power and server allocation policy that stabilizes the system?
- A bound on average delay is established

## In other words....

A controller allocates power to each of the N queues at every instant of time  
In reaction to channel state information

In order to:

Estabilize the system and achieve maximum throughput and maintain  
Acceptable low levels of unfinished work in all the queues

## Power and Server allocation

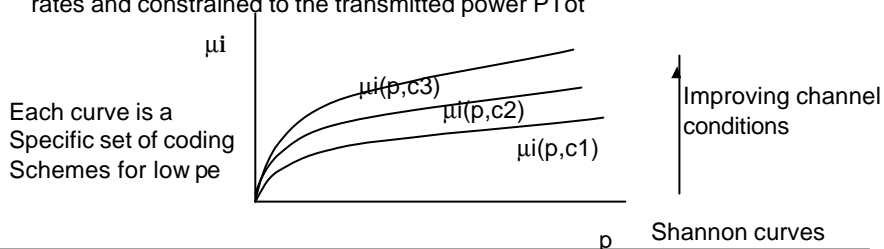
The channel states are

$$C(t) = (C_1(t) \dots C_N(t)) \quad C(t) \in S_1 \times S_2 \times \dots \times S_N$$

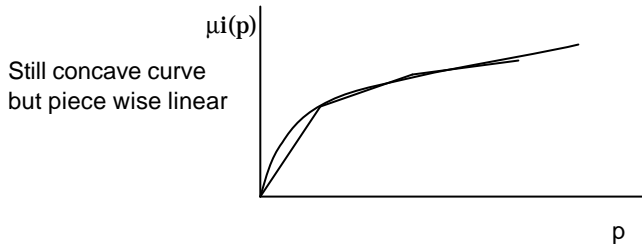
Constant during T, with transitions at  $t=KT$

$p_C$  Are the time average probabilities for each state C

$P(t) = (P_1(t), \dots, P_N(t))$  is the power allocation vector, that controls the server rates and constrained to the transmitted power  $P_{Tot}$



- Although in practice: there is only a finite bank of coding schemes



How does the server allocation problem relates to the power allocation problem?

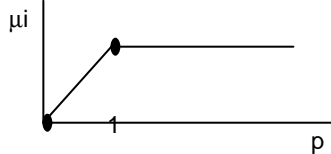
- Every time slot the servers are scheduled to serve K of the N queues
- Queue  $i$  tx at  $\mu_i$  whenever a server is allocated to it

$$\sum p_i \leq K$$

$$\tilde{m}(p) = \begin{cases} m_1 p & p \in [0,1] \\ m & p > 1 \end{cases}$$



Transformed into a power allocation problem



- Example of server allocation algorithm

$N=3$      $K=2$

$(\mu_1 \mu_2 \mu_3) = (1 \ 1 \ 1/2)$

$L=1$  (length of arrival packets)

Bernoulli process

$(p_1 \ p_2 \ p_3) = (p \ p \ (1-p)/2 + \epsilon)$

$p < 1/2$

$0 < \epsilon < pp/2$

Policy: serve the fastest queue (non empty)

-1 and 2 have priority

-3 is served with  $(1-p)/2$     **Unstable because it cannot support its input rate**

## Stability and downlink capacity region

### Stability criterion for 1 queue

$X(t)$ : ergodic input process with rate  $\lambda$

It is the total amount of bits that arrived during  $[0, T]$

$\mu(t)$ : instantaneous bit processing rate in the server

$U(t)$ : unprocessed bits in the queue at time  $t$

$$\underline{I} = \lim_{t \rightarrow \infty} \frac{X(t)}{t} \quad \underline{m} = \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t \underline{m}(t) dt$$

$$\underline{m}(t) \leq \underline{m}_{\max} \quad \forall t \rightarrow 0 \leq \underline{m} \leq \underline{m}_{\max}$$

$$g(M) = \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t 1_{[U(t) > M]} dt \quad \text{OVERFLOW function}$$

$g(M)$  measures the fraction of time that the unfinished work in a queue is above a certain value  $M$

The network of a single queue is stable if  $g(M)$  tends to 0 as  $M$  tends to inf.

↓ Then

$$\underline{I} \leq \underline{m}$$

Proof:

As  $\underline{I}_i \leq \underline{m}_i \quad \forall i \in \{1 \dots N\}$  (for all queues)

We upperbound  $\underline{m}_i$  as follows

$|T_{\underline{C}}(t)|$  Total length of intervals when the channel is in state  $\underline{C}$

$|C|$  Total number of states in the system

$$\frac{|T_{\underline{c}}|}{\tilde{t}} \leq \underline{p}_{\underline{c}} + \mathbf{e} \quad \forall \underline{c} \quad (1)$$

$$\underline{m}_i \leq \frac{1}{\tilde{t}} \int_0^{\tilde{t}} \underline{m}_i(p_i(\mathbf{t}), c_i(\mathbf{t})) dt + \mathbf{e} \quad \forall i \in \{1 \dots N\}$$

Thus, under power decisions  $\mathbf{P}(\mathbf{t})$  we have for all  $i$

$$I_i \leq \underline{m}_i \leq \sum_{\underline{c}} \frac{|T_{\underline{c}}|}{\tilde{t}} \frac{1}{|T_{\underline{c}}|} \int_{t \in T_{\underline{c}}} \underline{m}_i(p_i(\mathbf{t}), c_i(\mathbf{t})) dt + \mathbf{e}$$

$$\leq \sum_{\underline{c}} \frac{|T_{\underline{c}}|}{\tilde{t}} \underline{m}_i \left( \frac{1}{|T_{\underline{c}}|} \int_{t \in T_{\underline{c}}} p_i(\mathbf{t}) dt, c_i(\mathbf{t}) \right) + \mathbf{e}$$

From concavity of  $\mu_i$

$$\leq \sum_{\underline{c}} (\underline{p}_{\underline{c}} + \mathbf{e}) \underline{m}_i \left( \frac{1}{|T_{\underline{c}}|} \int_{t \in T_{\underline{c}}} p_i(\mathbf{t}) dt, c_i(\mathbf{t}) \right) + \mathbf{e}$$

From (1)

$$I_i \leq \sum_{\underline{c}} \underline{p}_{\underline{c}} \underline{m}_i(p_i^{\underline{c}}, c_i) + \mathbf{e}(1 + |\underline{c}| m_{\max})$$

Thus the capacity region (at network level) is:

$$I_i \leq \sum_{\underline{c}} \underline{p}_{\underline{c}} \underline{m}_i(p_i^{\underline{c}}, c_i) \quad i \in \{1 \dots N\}$$

## Theorem on the stability on the downlink

Definition (from a network theory point of view):

It is the compact set of points such that all queues of the system can be stabilized (with some power allocation policy) whenever the vector of input rates  $\lambda=(\lambda_1 \dots \lambda_N)$  is strictly in the interior of the set, and conversely

Theorem of the capacity region at the network level

The capacity region of the downlink channel with  $P_{TOT}$  and rate power curves  $m_i(p_i, c_i)$  (cross-layer) is the set of all input rate vectors  $\lambda$  such that exists pic satisfying

$$\sum_i p_{ic} \leq P_{TOT} \quad \forall C$$

$$I_i \leq \sum_C p_c m_i(p_{ic}, c_i) \quad i \in \{1 \dots N\}$$

## Stabilizing power allocation algorithm

BUT  $\pi$  and  $\lambda_i$  need to be known

↓  
Solution to get a stabilizing and feasible power allocation algorithm

In essence, the policy learns the system parameters indirectly by basing power allocation decisions both on channel state and queue backlog information

At the beginning of each slot, observe  $U(t)$ ,  $C(t)$  and allocate  $P(t)$  such that

$$\sum_i p_i(t) \leq P_{Tot}$$

$$\max_P \sum_i q_i U_i m_i(p_i, c_i)$$

↙ ↘  
The slowest

↙ ↘  
The fastest

**Corollary: downlink delay bound**

$$\sum q_i \bar{U}_i \leq \frac{TB}{2e} \quad B, e > 0$$

By Little's theorem

$$\sum q_i l_i \bar{D}_i \leq \frac{TB}{2e}$$

**Comments on implementation**

The problem is a non-linear optimization one. But as  $\mu_i$  are concave in the power parameters: the problem can be computed efficiently

For

$$m_i(p_i, \mathbf{a}_i) = \log(1 + \mathbf{a}_i p_i)$$

The solution is found by

L: set of downlinks  $i: 1 \dots N$  such that  $U_i(t) > 0$

$$p_i = \frac{q_i U_i(t) \left( P_{Tot} + \sum_{j \in \Lambda} \frac{1}{\mathbf{a}_j} \right)}{\sum_{j \in \Lambda} q_j U_j(t)} - \frac{1}{\mathbf{a}_i} \quad \text{if } i \in \Lambda$$

$$p_i = 0 \quad \text{otherwise}$$

Process ending at most in  $N-1$  iterations



### ●Proof of the stability

For stability analysis, the following Lyapunov function is defined

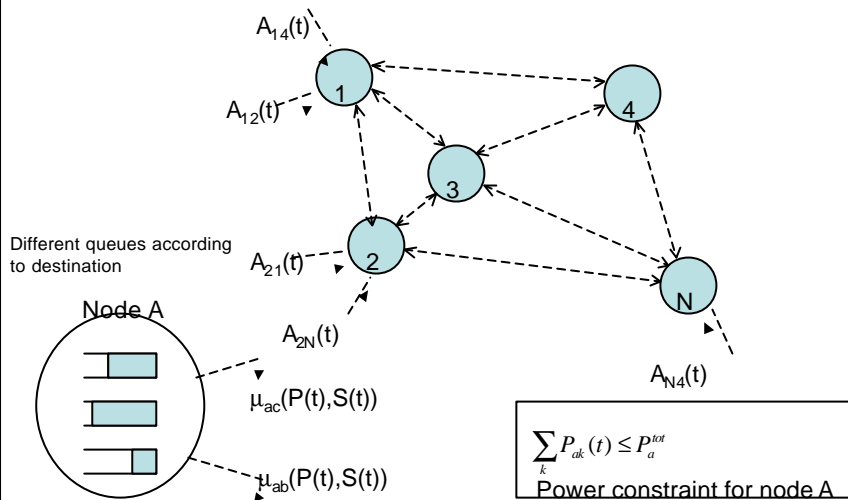
$$L(\underline{U}) = \sum_i q_i U_i^2$$

Sufficient conditions for the system to be stable and have well-defined steady state distribution of unfinished work  $U$

- 1)  $E[L(U(t+T))/U(t)] < \infty \quad \forall U \in R^N$
- 2)  $E[L(U(t+T)) - L(U(t))/U(t)] < -\infty \quad \text{if } U(t) \notin \Lambda$
- 3)  $\text{If } U(t) \in \Lambda \quad p\{U(t+mT) = 0\} \neq 0$

Then a steady state distribution on vector  $U$  exists and hence the system is stable

### Dynamic power allocation and routing





## Assumptions

- Power constrained nodes
- Time is slotted, channel changes from slot to slot
- Variable transmission rates  $\mu(P, S)$
- Cell partitioned network model

$S(t)$ : Channel matrix process  
Ergodic with time average probabilities  $\pi_s$  for each state  $s$

$P(t)$ : power metric  $\in \Pi$ , where  $\Pi$  is a compact set of acceptable power Allocations that includes the power limits for each node

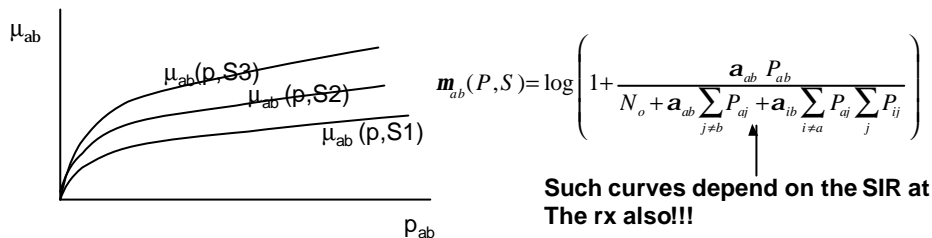
### GOAL:

- Stabilize the system and thereby achieve maximum throughput and maintain acceptably low network delay: by means of joint routing and power allocation policy.
- This work unifies: network capacity, network optimization, network control



**New concept: Network capacity region.** It is a network layer notion. This is distinct from the information theoretic capacity

- Network capacity region:
  - Set of all  $\lambda_{ij}$
  - Considering all possible routing and power allocation strategies
  - Depends on: node power constraints, rate-power function  $\mu(P(t), S(t))$
- Information theoretic:
  - Optimization over all possible modulation and coding schemes
  - Involves many of the unsolved problems of network information theory



The network controller makes the following decisions :

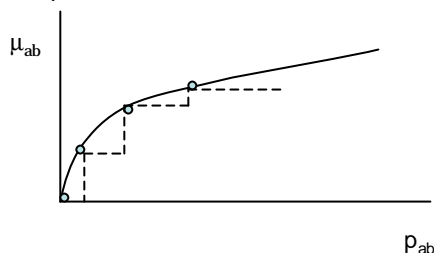
- Power allocation: choose  $P(t)$  such that  $P(t) \in \Pi$
- Routing/Scheduling: choose  $m_{ab}^{(c)}(t)$  such that

$$\sum_c m_{ab}^{(c)}(t) \leq m_{ab}(t) = m_{ab}(P(t), S(t))$$

Note that: if no power allocation,  $m_{ab}(t)$  is purely determined by the dynamic channel states of the network, and any network control algorithm reduces to pure routing and scheduling

The rate of node a will be controlled with the power as the curves indicate if variable length packets can be split and repackaged with new headers for resequencing at the destination (in the analysis extra bits due to such headers will be neglected)

However, splitting and relabeling can be avoided altogether if all packets have fixed lengths and the transmission rates are restricted to integral multiples of the packet length/slot quotient.



## Stability and network capacity region

The goal of the controller is to maintain low backlog and thereby stabilize the System. For that purpose, we take into account the following queuing dynamics

$$U_i^{(c)}(t+1) \leq \max \left[ U_i(t) - \sum_b m_b^{(c)}(t), 0 \right] + \sum_a m_a^{(c)}(t) + A_i^{(c)}(t)$$

### A. Stability of queuing systems

Overflow function: 
$$g(V) = \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \Pr [U(\tau) > V]$$

$$0 \leq g(V) \leq 1$$

A single server queuing system is stable if  $g(V) \rightarrow 0$  as  $V \rightarrow \infty$   
 A network of queues is stable if all individual queues are stable

### B. Network capacity

Let's define the input rate as 
$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} A_i^{(c)}(\tau) = I_{ic}$$

#### Theorem 1:

It is the closure of the set of all rate matrices  $(\lambda_{ic})$  that can be stably supported over the network, considering all possible algorithms. If the flow with destination (c) that enters in node (a) towards (b) is  $f_{ab}^{(c)}$

$$f_{ab}^{(c)} \geq 0 \quad f_{aa}^{(c)} = f_{ab}^{(c)} = 0 \quad \forall a, b, c$$

$$I_{ic} \leq \sum_b f_{ib}^{(c)} - \sum_a f_{ai}^{(c)} \quad \forall i, c \quad i \neq c$$

$$\sum_c f_{ab}^{(c)} \leq R_{ab} \quad \forall a, b \quad R_{ab} \in \Gamma$$

► This constraint ensures that the total flow over any link does not exceed the link capacity

$$\Gamma = \sum_s \mathbf{p}_s \quad \text{convex} \quad \text{Hull} \{ \mathbf{m}(P, S) \mid P \in \Gamma \}$$

Graph family because the rates depend on the power allocation policy



► Compact set of acceptable power allocations that includes the power limits for each node

Lyapunov theory can be used to design stabilizing power allocation and routing algorithms that do not require knowledge of arrival rates or channel statistics in cases where there are only peak power constraints on the wireless devices .

The proof of theorem 1 involves showing that  $\mathbf{I}_{ic} \in \Lambda$  is necessary for stability and that  $\mathbf{I}_{ic}$  interior to  $\Lambda$  is sufficient.

### Centralized systems: dynamic control policy

Generalizes Tassiulas -Ephremides by considering:

- Power allocation
- Networks with general interference
- Time-varying channel

Policy:

1) For all links (a,b) find commodity  $c_{ab}^*(t)$  such that

$$c_{ab}^*(t) = \arg \max_{C \in \{1 \dots N\}} \{U_a^{(c)}(t) - U_b^{(c)}(t)\}$$

and define

$$w_{ab}^*(t) = \arg \max_{C \in \{1 \dots N\}} \{U_a^{(c_{ab}^*)}(t) - U_b^{(c_{ab}^*)}(t), 0\} \quad \text{Maximum differential backlog between a and b}$$

2) Power allocation: choose matrix  $P(t)$  such that

$$P(t) = \operatorname{argmax}_{P \in \Pi} \sum_{a,b} \mathbf{m}_{a,b}(P, S) w_{ab}^*(t)$$

3) Routing: define transmission rates as follows

$$\mathbf{m}_{ab}^{(c)}(t) = \begin{cases} \mathbf{m}_{ab}(P(t), S(t)) & c = c_{ab}^*(t) \quad w_{ab}^*(t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Equalize the differential backlog

It can be proved its stability and that the average bit delay satisfies

$$\bar{D}_{bit} = \left( \frac{1}{NI} \right) \sum_{i,c} \bar{U}_i^{(c)}$$

$$\bar{D}_{bit} \leq \frac{KBN + (K-1)B'N}{eNI} = \frac{KBN + (K-1)B'N}{r(1-r)R^2}$$

### Distributed implementation

Users attempt now to maximize the weighted sum of data rates in

$$P(t) = \operatorname{argmax}_{P \in \Pi} \sum_{a,b} \mathbf{m}_{a,b}(P, S) \quad w_{ab}^*(t) \quad (*)$$

By exchanging information with their neighbors via a low bandwidth control Channel

We define neighbour set  $\Omega_i(t)$  according to best channel conditions

#### **A. Networks with independent channels**

Obtained via orthogonal coding schemes, beamforming or channels are separated such that interference is negligible

The weighted sum in (\*) is maximized by separately maximizing each term  
Nodes making independent power control and routing decisions based only on their local information

$$\max \sum_b w_{nb}^*(t) m_{nb}(P_{nb}, S_{nb})$$

$$s.t. \sum_{b \in \Omega_n} P_{nb} \leq P_n^{Tot}$$

Which can be solved in real-time

## B. Networks with interference

There is an initial interference sensing stage. The idea of randomly choosing users to tx. Is similar to the technique used in the Grossglauer-Tse realy algorithm but now, rather that tx. To the nearest rx., the algorithm chooses the rx with the largest backlog-rate arithmetic.

Dynamic optimization contributes to bridging the gap between theoretical optimization techniques and implementation control algorithms.

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