# ARRAY PROCESSING

# The Course

. Name: Miguel Angel Lagunas

- E-mail: m.a.lagunas@cttc.es
- Course (Notes, Software, Slides) http://www.cttc.es/research-development/training/graduate-and-undergraduate Course: Array Processing I
- Topics: Spatial Diversity. Beamforming. DOA Estimation. Adaptive Arrays. MIMO (?)
- Exam/Control

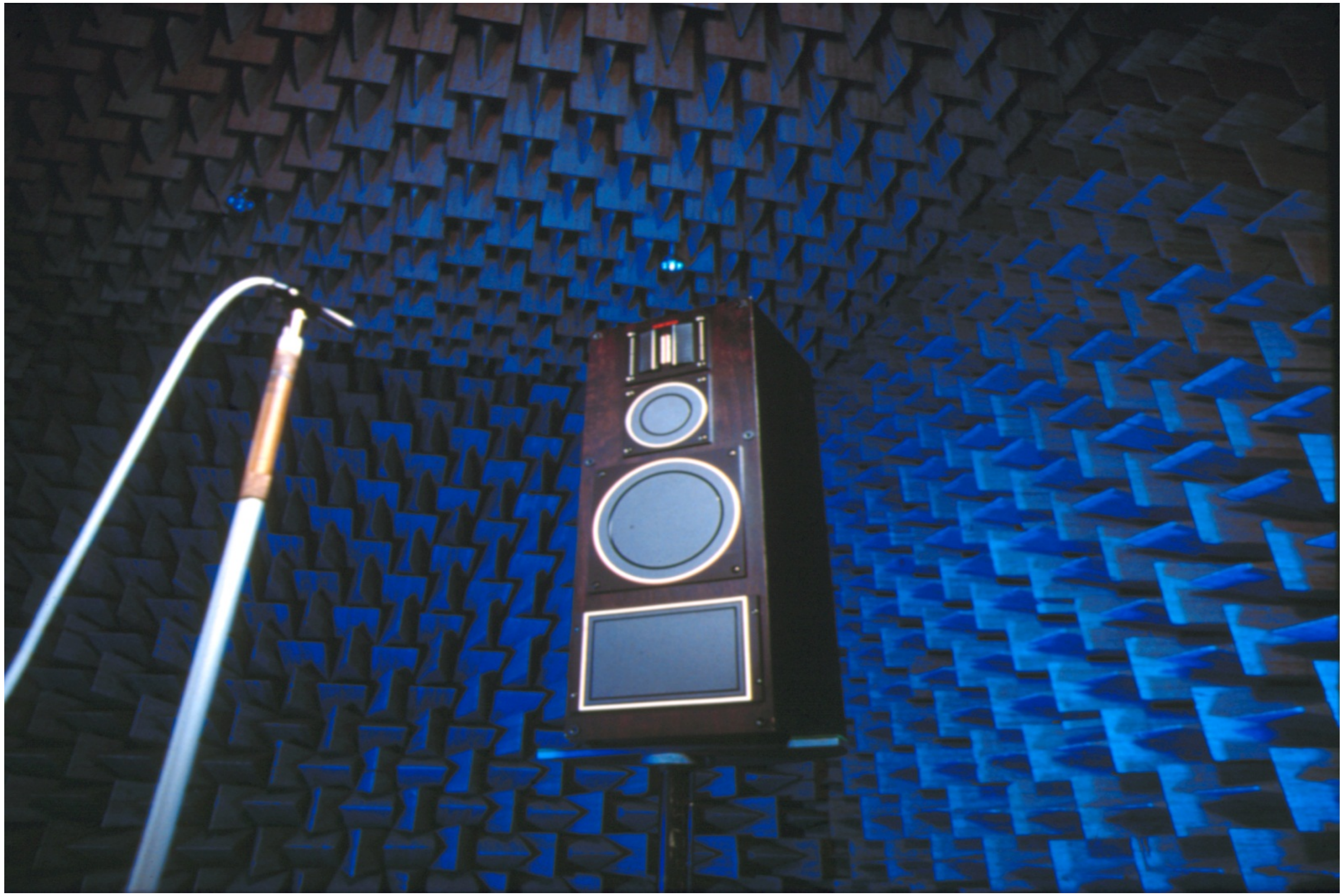
# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

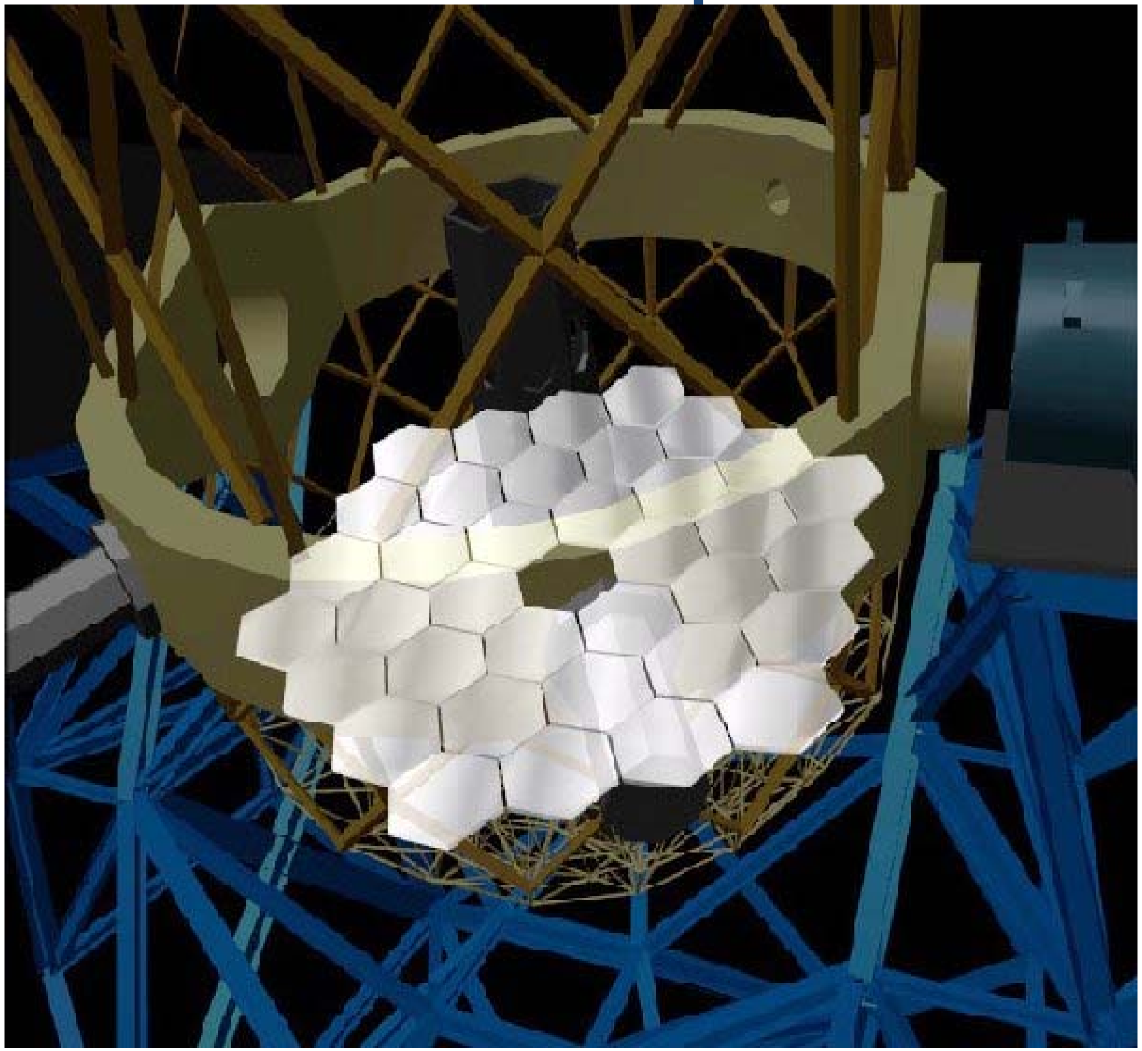
# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

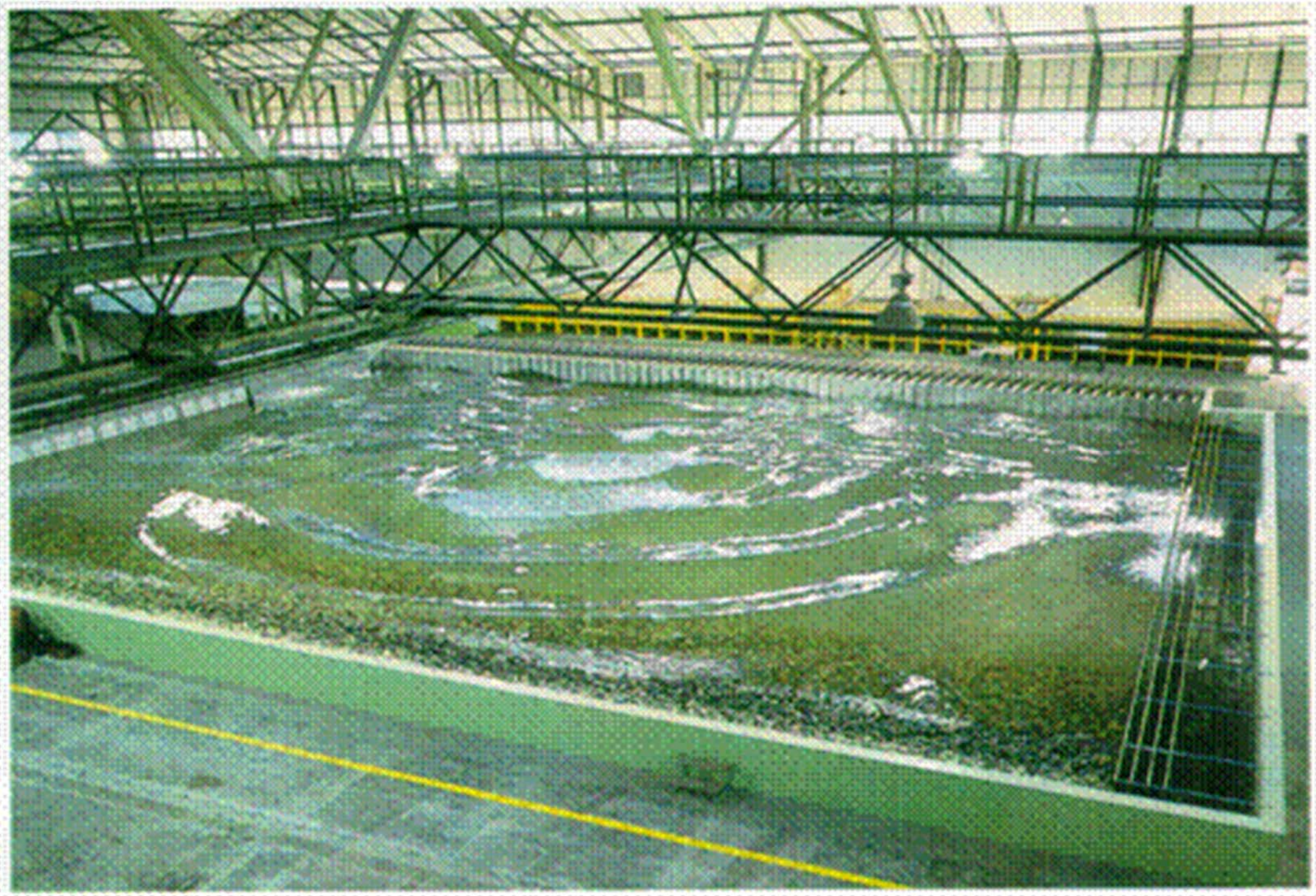
# Electroacoustics



# Telescopes



# Civil Engineering

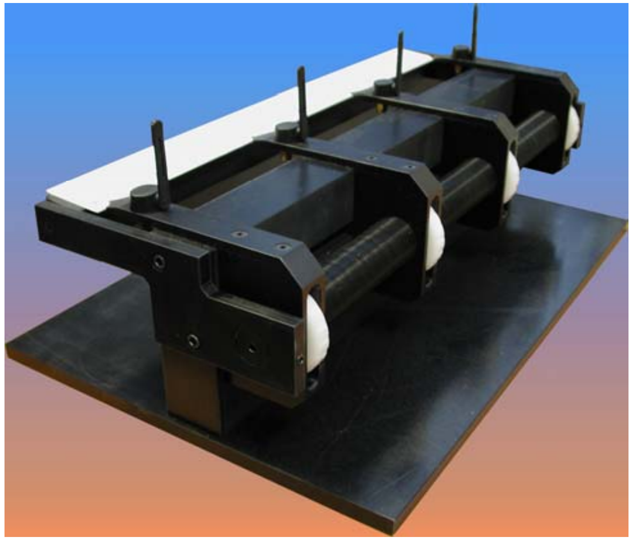


# Home-Audio


PROYECTORES DE SONIDO. El Cine en Casa desde un sólo elemento
Los sistemas YSP de Yamaha incluyen en un sólo componente gran número de altavoces de reducidas dimensiones los cuales y mediante un avanzado método de aplicación de tiempo de retardo entre ellos permite proyectar "haces" sonoros que pueden ser orientados de manera precisa para conseguir un efecto sonoro envolvente óptimo.
Los haces direccionados producen ondas sonoras directas y otras reflejadas creando un verdadero sonido envolvente multicanal así como sonido estereo de alta calidad o en 3 canales para el máximo realismo en conciertos musicales.
La ruptura tecnológica de los sistemas YSP abren una nueva era en el cine en casa facilitando su instalación y adaptándose a cualquier decoración. En cine en Casa sin cables.
¡ SOLICITE UNA DEMOSTRACION !
YSP 4000
PARA 42" Y SUPERIORES
PVP: 1.319€
YSP-3000
PARA 32" Y SUPERIORES
PVP: 849€

# Solar Plants

# Radiocommunications



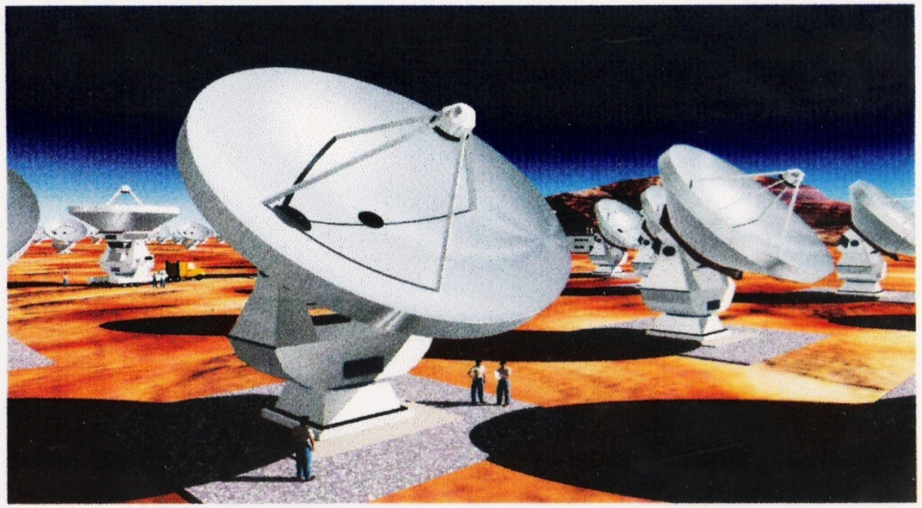
# Radio-Astronomy

## ALMA
## Atacama Large Millimeter Array

Europe-US-Japan collaboration

located in Chile

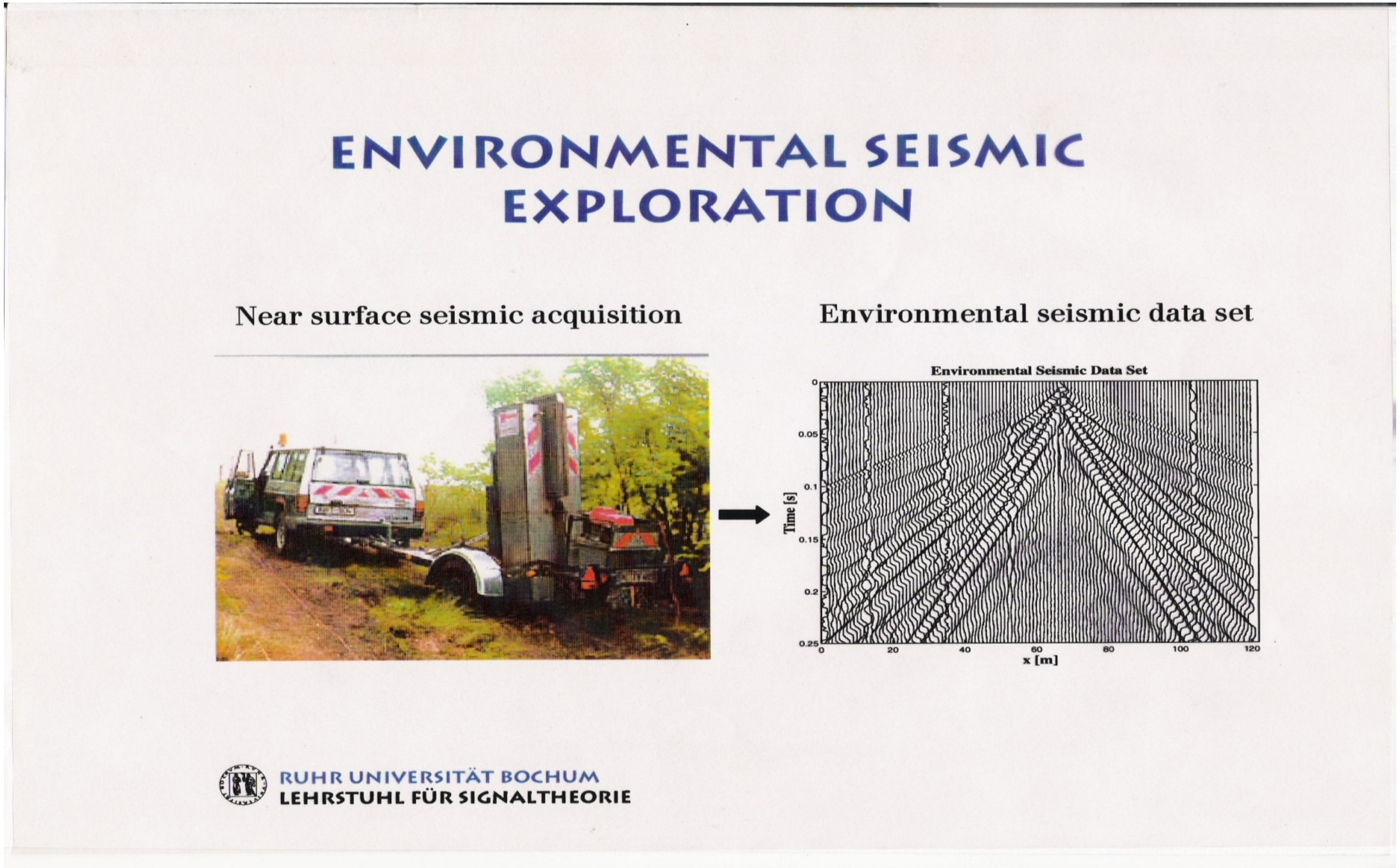
operational in 2012



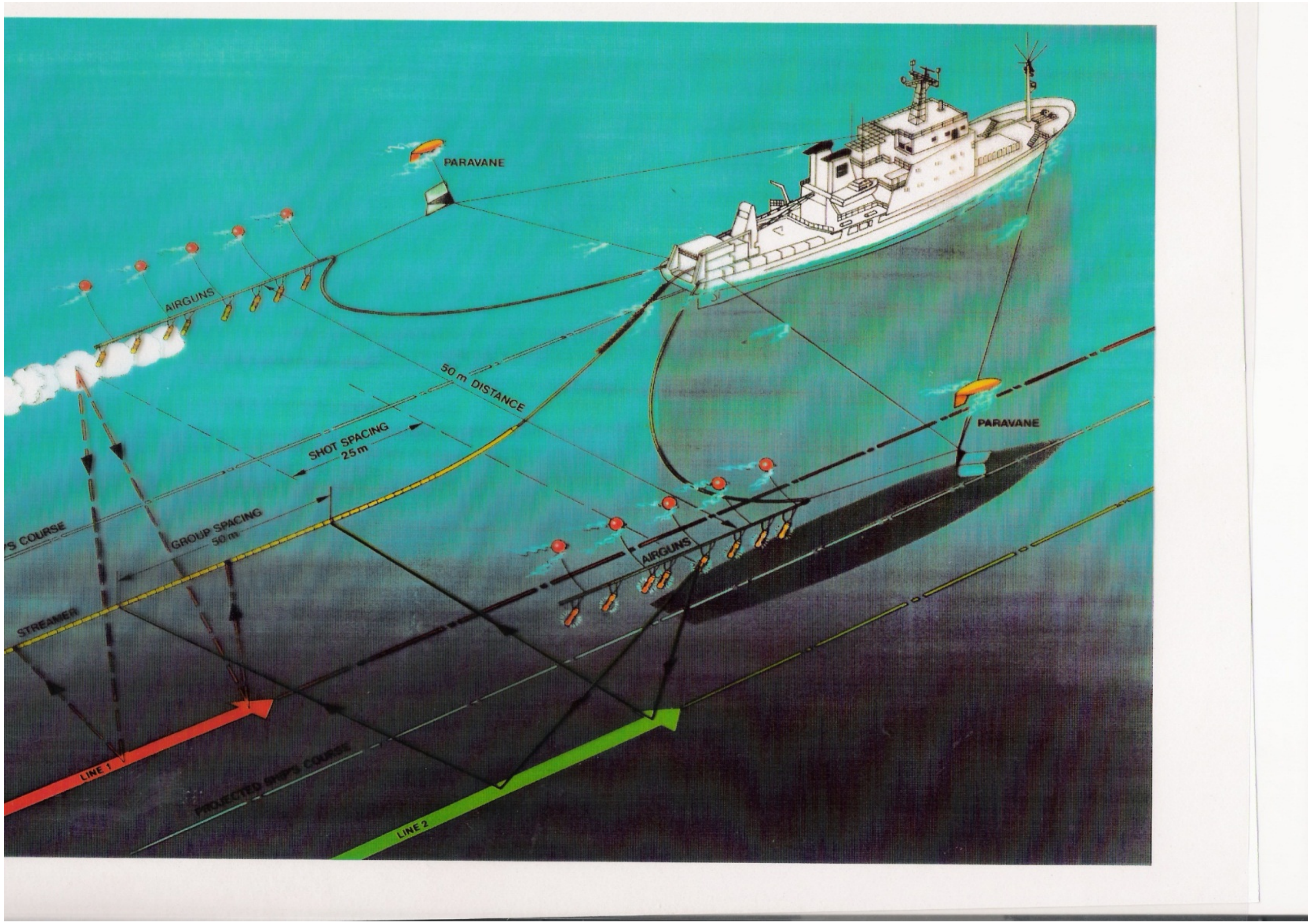
coordinating institute: European Southern Observatory

# Radio_Astronomy

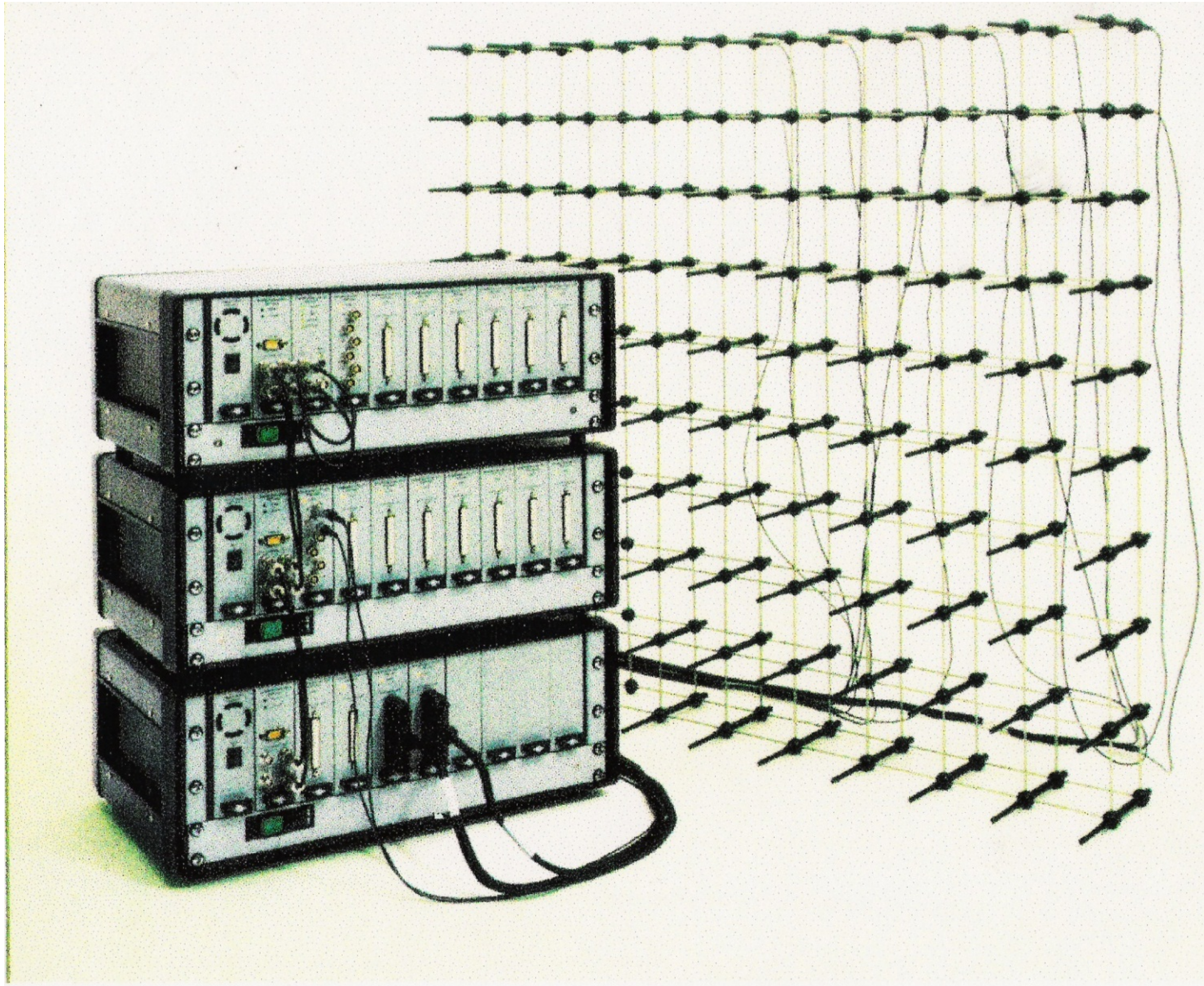
# Geophysics


ENVIRONMENTAL SEISMIC EXPLORATION
Near surface seismic acquisition
Environmental seismic data set
Environmental Seismic Data Set
Time [s]
x [m]
RUHR UNIVERSITÄT BOCHUM
LEHRSTUHL FÜR SIGNALTHEORIE

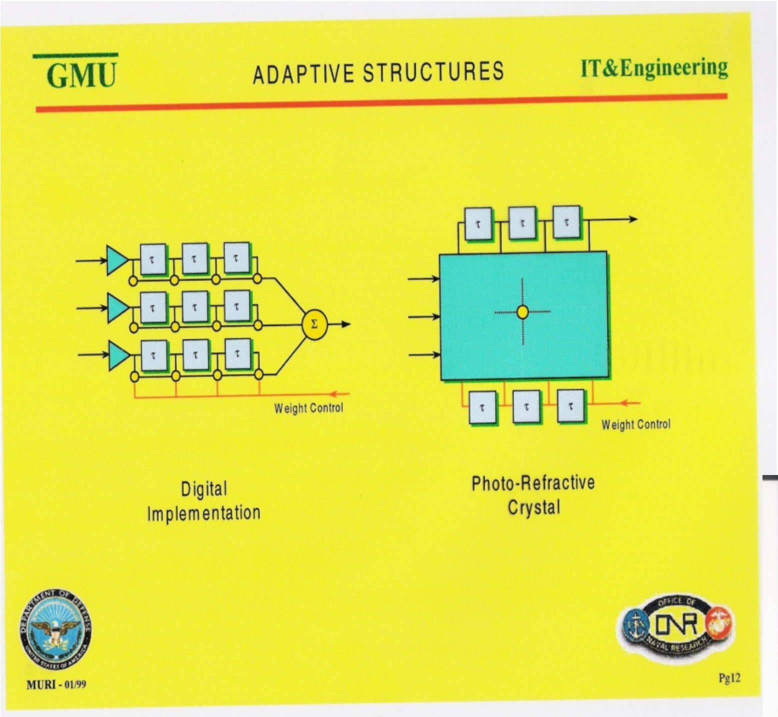
# Sonar

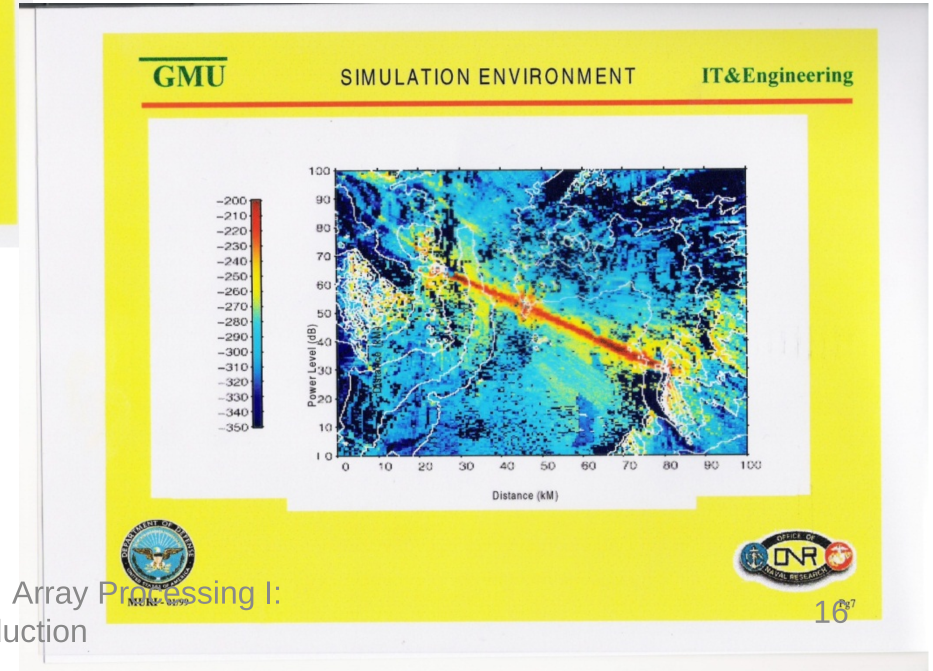

PARAVANE
AIRGUNS
50 m DISTANCE
SHOT SPACING
25 m
PARAVANE
GROUP SPACING
50 m
AIRGUNS
STREAMER
LINE 1
PROJECTED SHIP'S COURSE
LINE 2

# Ambient Acoustics

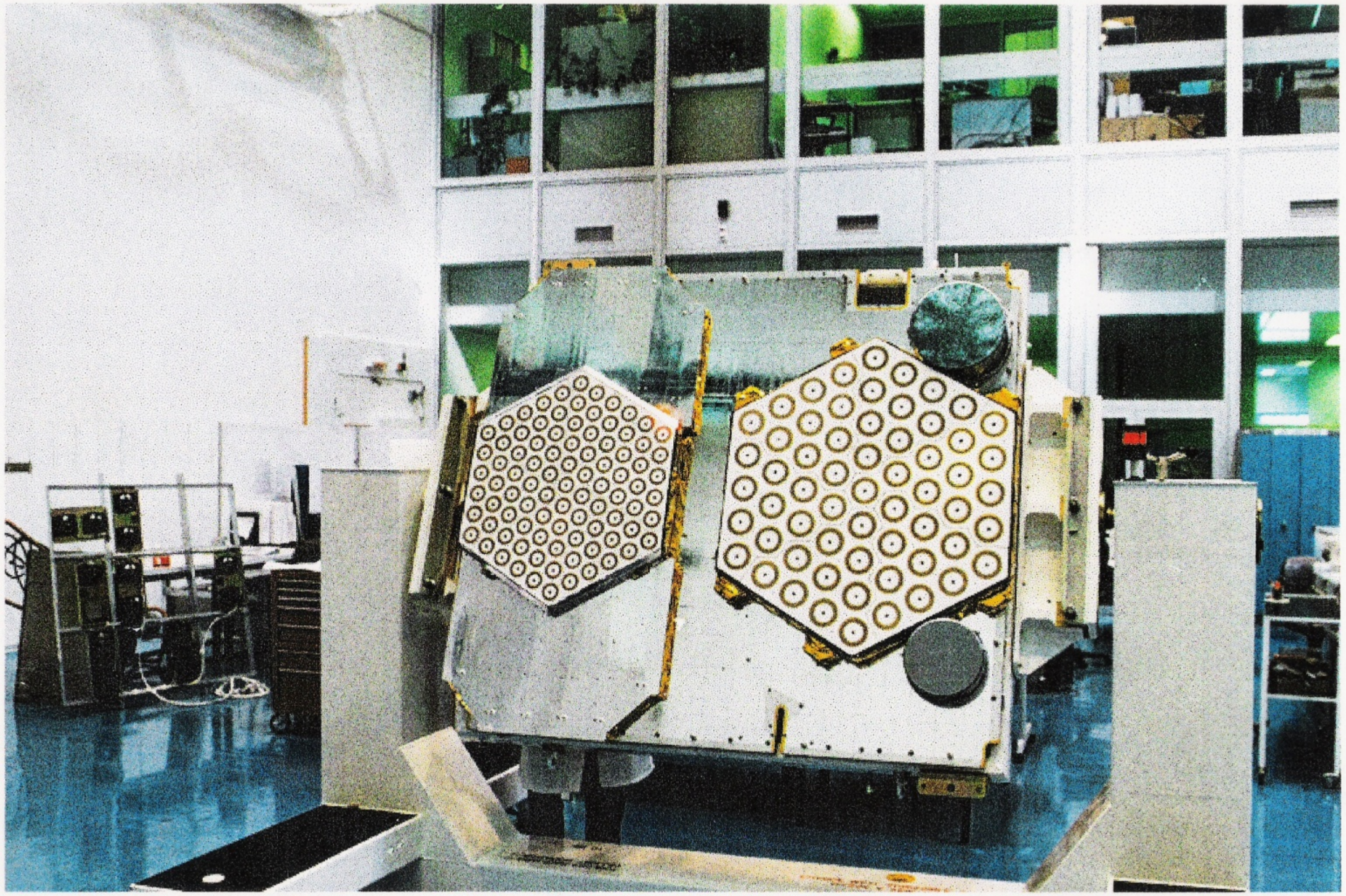


# Optical Guidance Systems



# Satellite

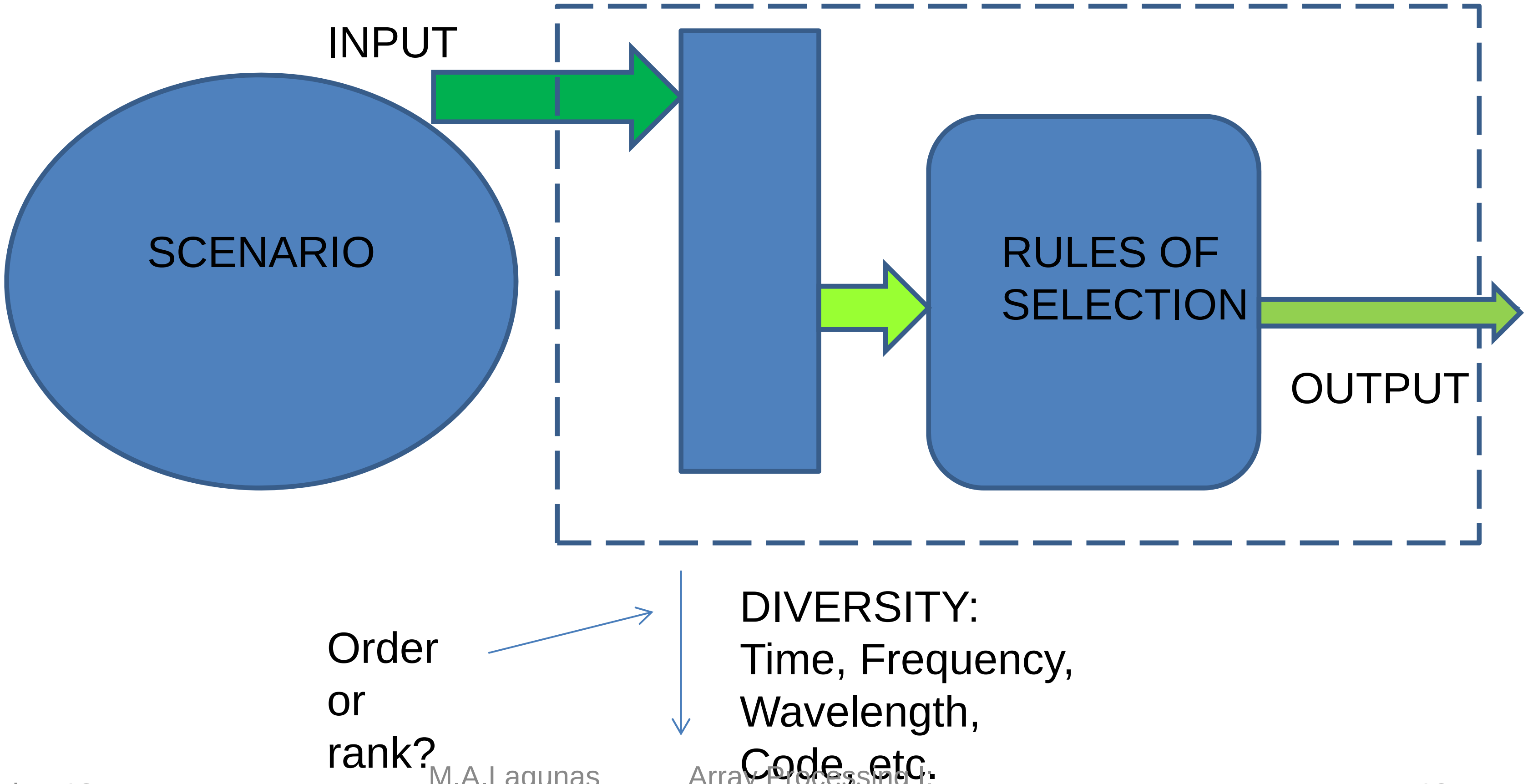


# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# Signal and Diversity Processing

Filtering and, in general signal processing implies always two major steeps: Creating Diversity and Implementing Rules



Traditional signal processing uses time diversity processing as the major outcome of the scenario under analysis

$$\underline{X}^T(t) = \left[ x(t-t_1), x(t-t_2), \ldots\ldots, x(t-t_Q) \right]$$

$$\underline{X}^T(n) = \left[ x(n-n_1), x(n-n_2), \ldots\ldots, x(n-n_Q) \right]$$

(SAW Filters, Digital Filters…)

$$\underline{X}^T(t) = \left[ x(t), \frac{dx}{dt}, \ldots.., \frac{d^{Q-1}x}{dt^{Q-1}} \right]$$

$$\underline{X}^T(n) = \left[ x(n), \Delta x(n), \ldots.., \Delta^{Q-1} x(n) \right]$$

(RLC or passive Filters, Discrete AC Systems)

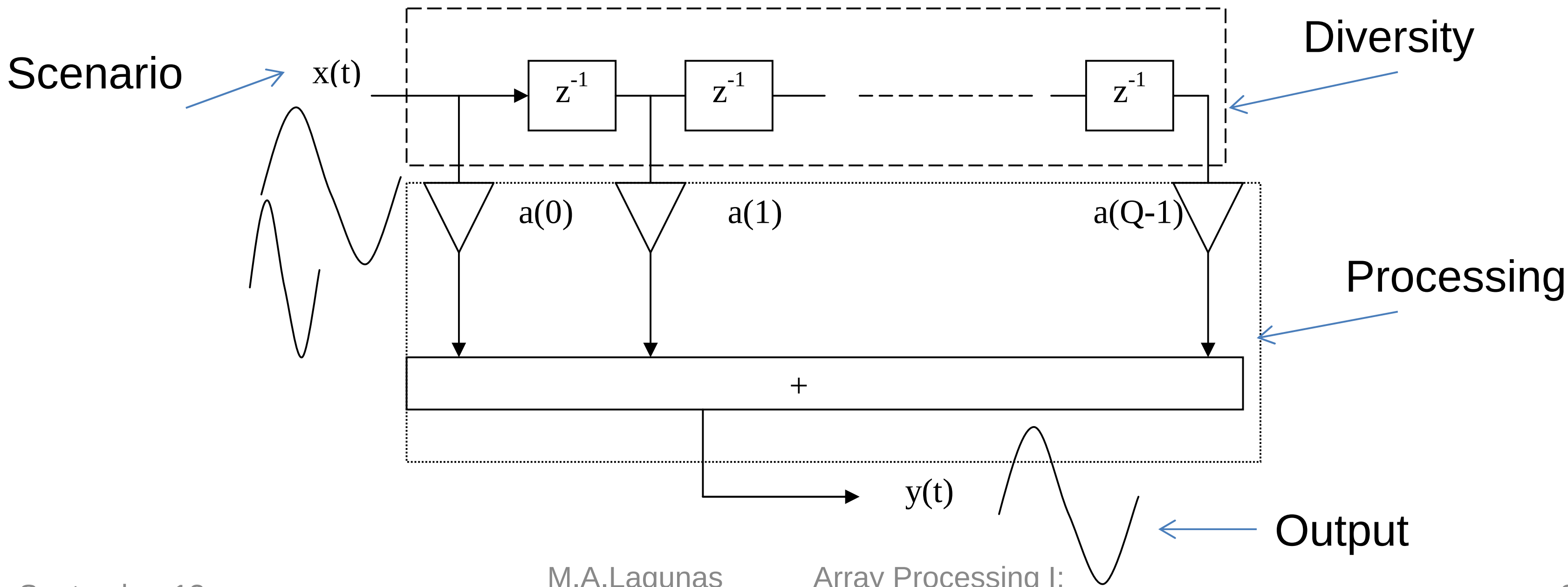
Normally the diversity is stacked with some ordering, being the most familiar choices by time or complexity in getting the FEATURES selected

$$\underline{X}_n^T = \left[ x(n), x(n-1), \ldots, x(n-Q+1) \right]^T$$

Next, over the diversity we apply rules that determine the selection process and imply that some entities present in the scenario are going to show up at the output and other will disappear or are going to be greatly attenuated.

When the rules are, mathematically speaking, linear we refer the overall process as Linear Signal Processing/Filtering

$$y(t) = \sum_{q=0}^{Q-1} a(q).x(t - q.T)$$



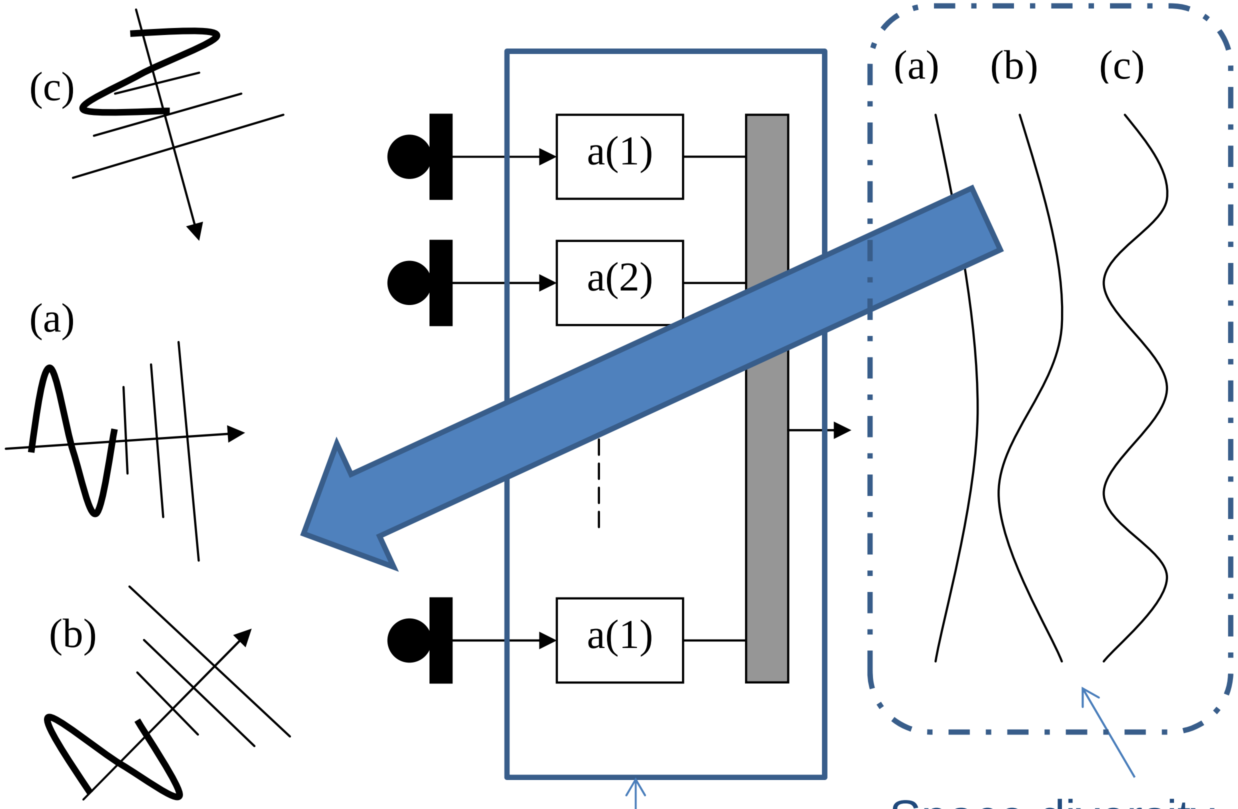
# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# Space Diversity

- Resource Unlimited and not lasting with use.
- No cost, as licensed spectrum for communications.
- Filtering and processing directions of departure and/or arrival (DOAs).
- Expensive in terms of engineering, implementation and deployment cost.
- Performance above expectations.

# A copy of traditional time diversity processing




Sources with the **same frequency** and different directions of arrival

Selection rules of **Beamformer**

Space diversity **signature** of the sources

# Rich Diversity

- Diversity has no order, i.e. sensors or antennas are located everywhere.
- No regular sampling unless linear and uniform array configuration (ULA arrays) is used
- Time-Frequency- Space joint processing (up to 5 dimensions)
- No feedback processing in the beamformer (No sense)
- ……….

# INDEX

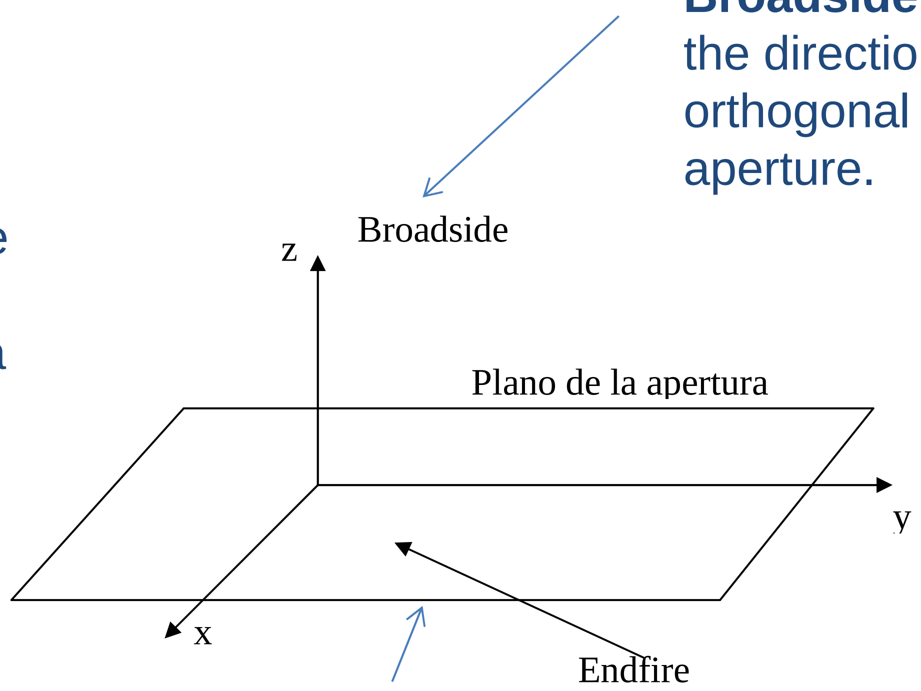
- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# The Space Scenario

Note that the basic feature in the spatial signature is promoted by the delay of a given source in arriving to the sensors in the aperture.

**Broadside** denotes the direction orthogonal to the aperture.

Without loss of generality we will start with planar arrays, i.e. all the antennas stay in a 2D plane.----------------------→

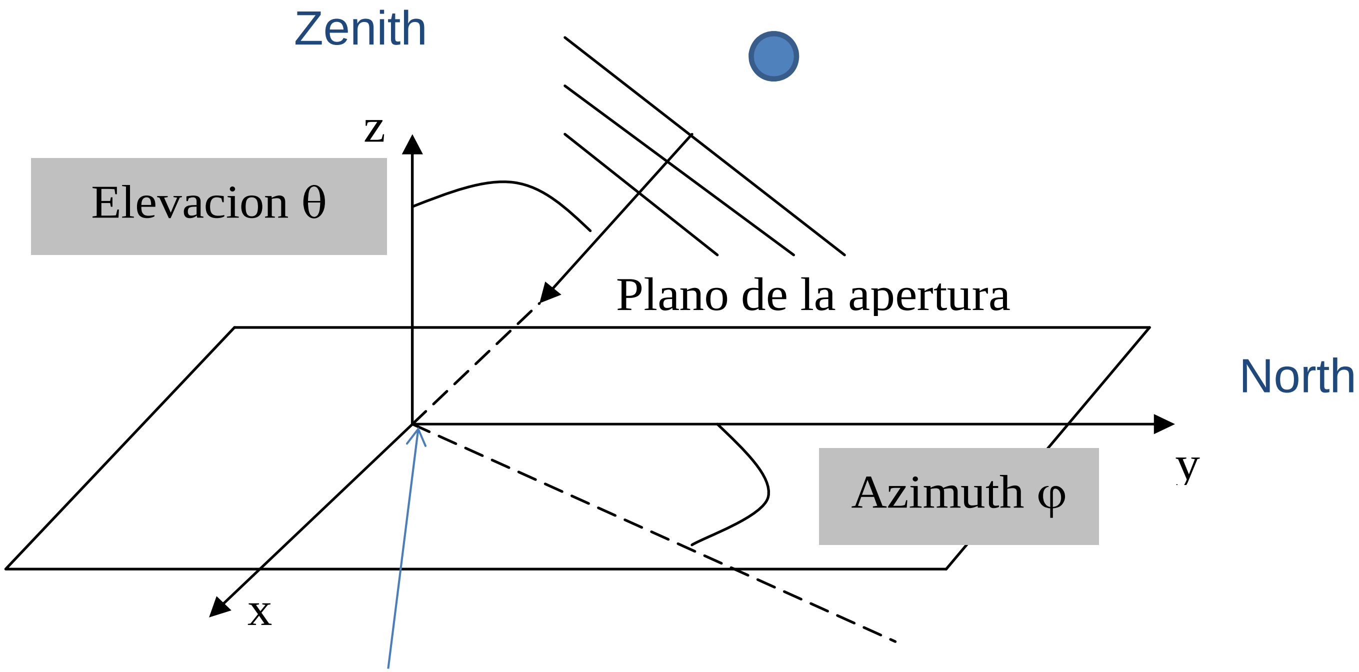



**Endfire** denotes all the directions contained on the array plane

# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# Angles of Arrival




**Phase Center:** A reference point in space not necessary coincident with any antenna position

# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# The Source and Sensor signal

Without propagation loss and flat fading channel, that mostly will affect only the strength of the source arrival, the signal received a sensor #q (remember labeling order is irrelevant), will be a delayed version of the original source

$$sensor \quad \#q -----> x_s\left(t - t_o - \tau_{qs}\right)$$

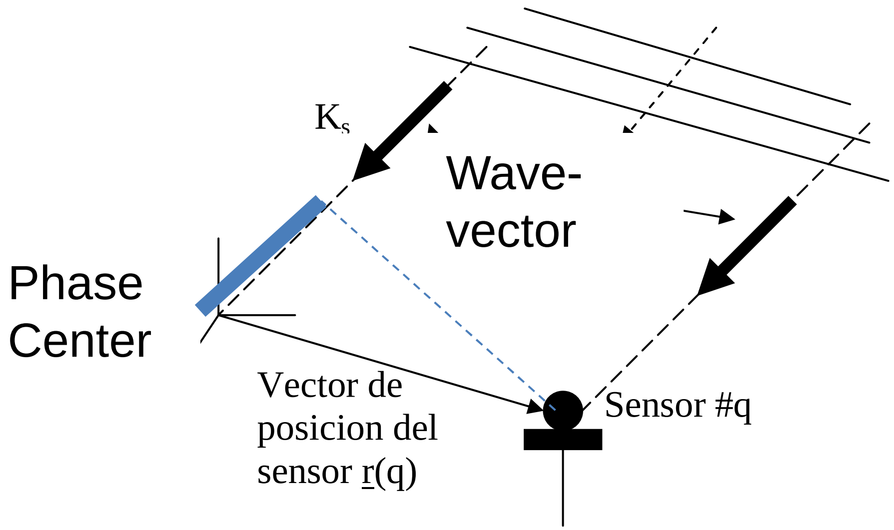
Propagation delay up to the phase-center

Relative (either positive or negative) delay, with respect the phase center, of source #s when arriving to sensor #q

Note that selecting this signal among other impinging the aperture implies that the Beamformer have to process both time and space diversity.

# What parameters influence the relative time delay?

Let us examine, for a given wavelength, which is the relative delay. We will assume a point-source in the far-field so that the wave-front arriving to the aperture is plane




The relative delay is due to the difference distance (blue)

The distance is the projection of the sensor position over the unitary wave-vector

$$\underline{K}_s = \frac{2\pi}{\lambda}\left(\text{sen}(\theta_s).\cos(\varphi_s), \text{sen}(\theta_s).\text{sen}(\varphi_s), \cos(\theta_s)\right)$$

$$\underline{r}_q = d_q\left(\cos(\varphi_q), \text{sen}(\varphi_q), 0\right)$$

Thus, the relative delay depends only in the distance from sensors to the phase center, its location and the angles of arrival of the source. Dividing this distance by the velocity of propagation (in sonar or acoustics it may change also with the frequency), we obtain the relative delay

$$\tau_{qs} = d_q \frac{\text{sen}(\theta_s)}{c}.\cos(\varphi_s - \varphi_q)$$

the source signal will be:

$$\text{Carga de fase} = \phi_{qs} = 2\pi.f.\tau_{qs} = \underline{k}_s \otimes \underline{r}_q = 2.\pi.f.\frac{\text{sen}(\theta_s)}{c}.\cos(\varphi_s - \varphi_q).d_q$$

$$x(t) = a(t).\exp\left(j2\pi.f_c.t\right)$$

Complex Envelope/Carrier formulation of a band-pass signal

And the signal at sensor #q

$$x(t) = a(t-\tau_{qs}).\exp\left(j2\pi.f.(t-\tau_{qs})\right)$$

So we have a **group delay** and a **phase delay.**

$$x(t) = a(t-\tau_{qs}).\exp\left(j2\pi.f_0.(t-\tau_{qs})\right)$$

The phase delay, as denoted bi its name, mostly influences the carrier. Meanwhile the group delay impacts the information carried on by the carrier. Let us examine the phase load due to the relative delay.

$$\tau_{qs} = d_q \frac{\text{sen}(\theta_s)}{c}.\cos(\varphi_s - \varphi_q)$$

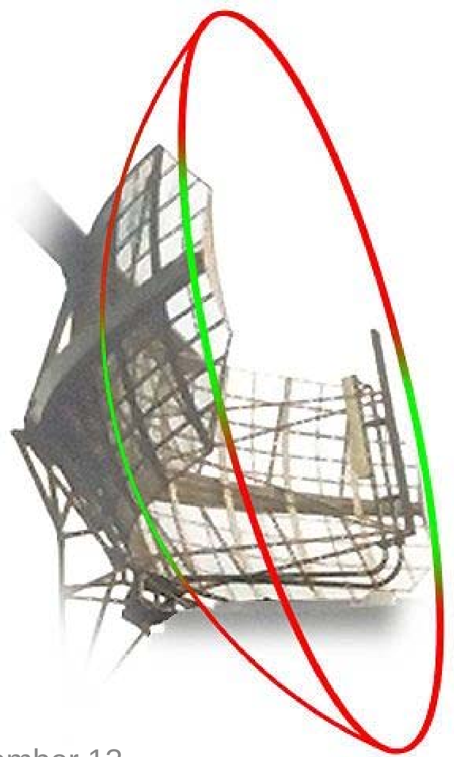
$$\text{Carga de fase} = \phi_{qs} = 2\pi.f.\tau_{qs} = \underline{k}_s \otimes \underline{r}_q = 2.\pi.f.\frac{\text{sen}(\theta_s)}{c}.\cos(\varphi_s - \varphi_q).d_q$$

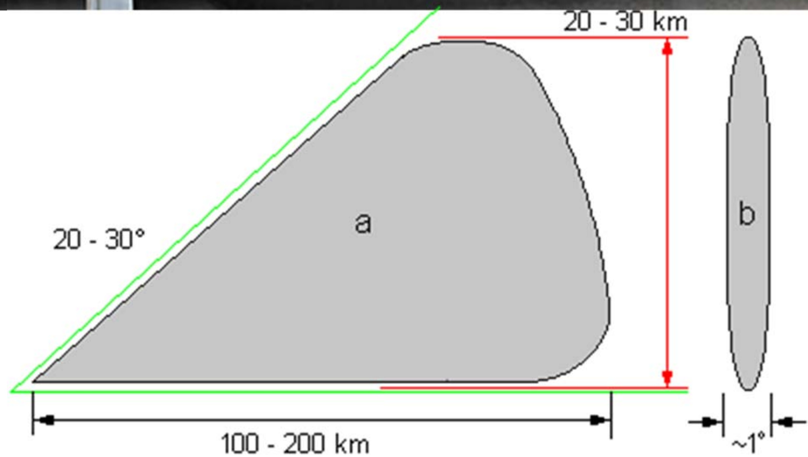
Taking in mind the human auditory system, that basically relays on phase load for locations we will revise the above formula. Note that for location purposes we need large changes on the phase load across our two sensors. Meanwhile for proper listening we need slow changes (to add the incoming signals).

$$\text{Phase load} = 2\pi \frac{f}{c}(\text{sensor separation})\sin(\textit{elevation}).\cos(\text{azimuth difference})$$

- Location: High frequencies are sensed more directive than low frequencies
- Location: At the broadside, we sense much better the location in that at the end-fire
- Location: Large separation of sensors helps for location
- Listening: At the end-fire and/or Azimuth alignment we receive better.
- Location: Distance between sensors in wavelengths is what provides resolution.

Array gain (listening)






Array directivity (Location)

Christian Wolff,
www.itnu.de/radargrundlagen/06.antennas/an11.en.htm

# INDEX

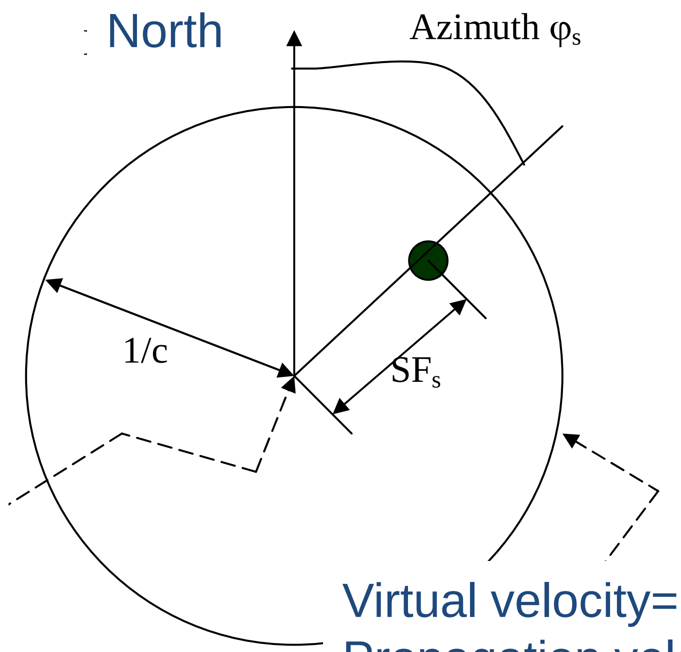
- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# Slowness/Azimuth plane (Geophysics)

Spatial responses and source location are represented, formerly used in geophysics and astronomy, using the co-called slowness/azimuth plane. This plane is the projection of any point in the sphere on the aperture plane.

The projection preserves azimuth, and elevation represented by the so-called slowness factor

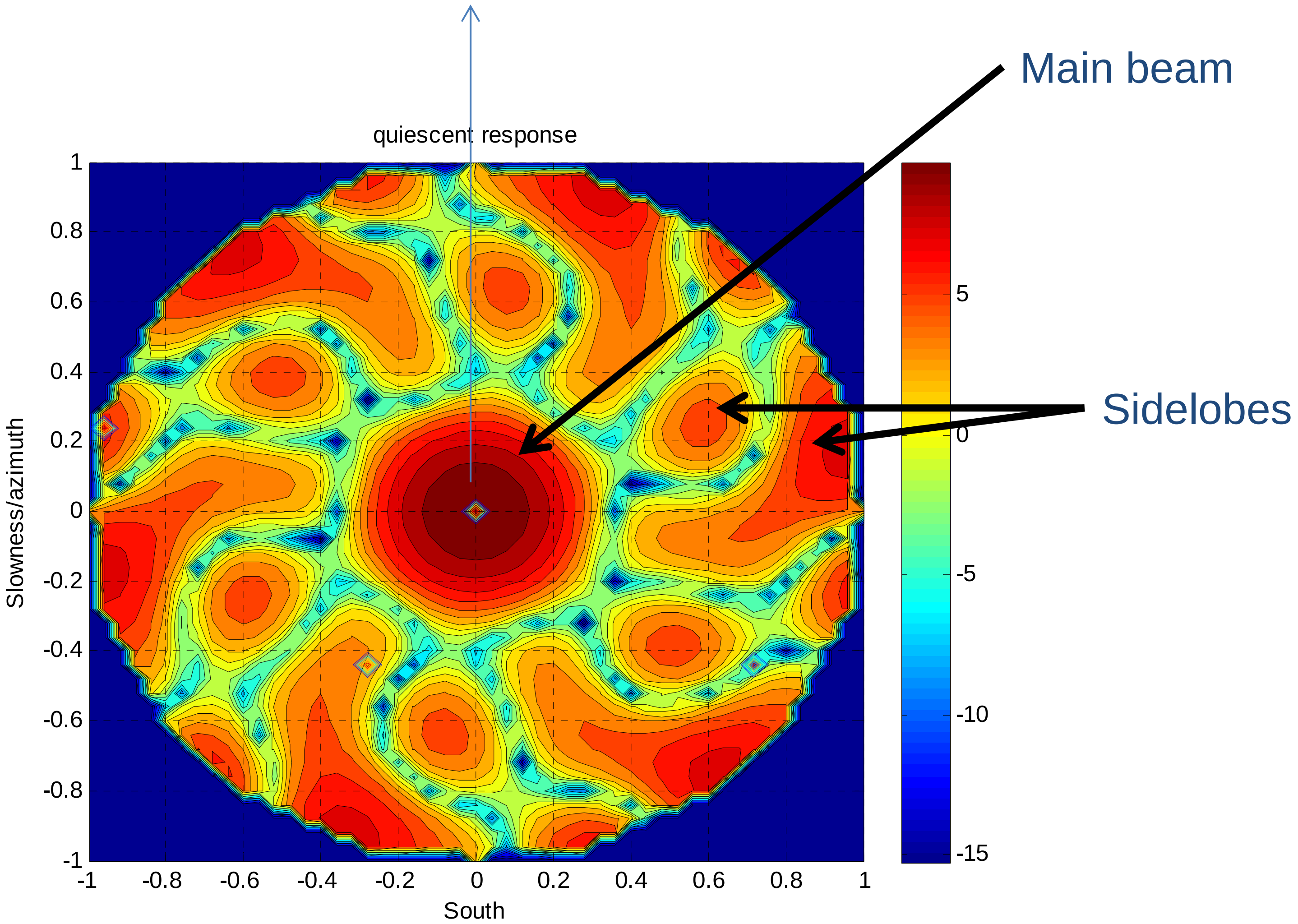
$$SF_s = \frac{\text{sen}(\theta_s)}{c}$$




Virtual velocity=Infinite
“Fast sources”

Virtual velocity=
Propagation velocity
“Slow waves”

Example: Spatial response of a beamformer steered to the broadside in the slowness/azimuth representation



# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

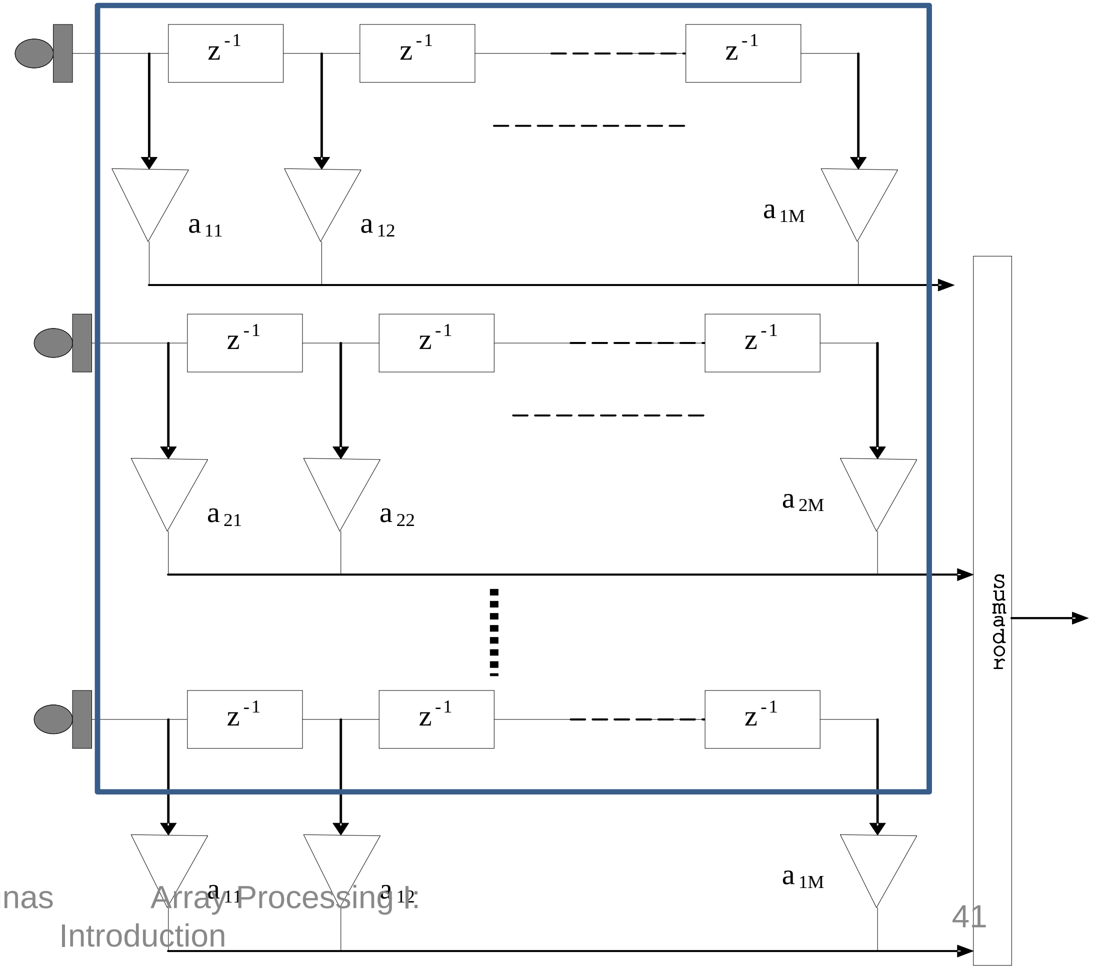
# Wideband/Narrowband processing

The received signal from a single source is:

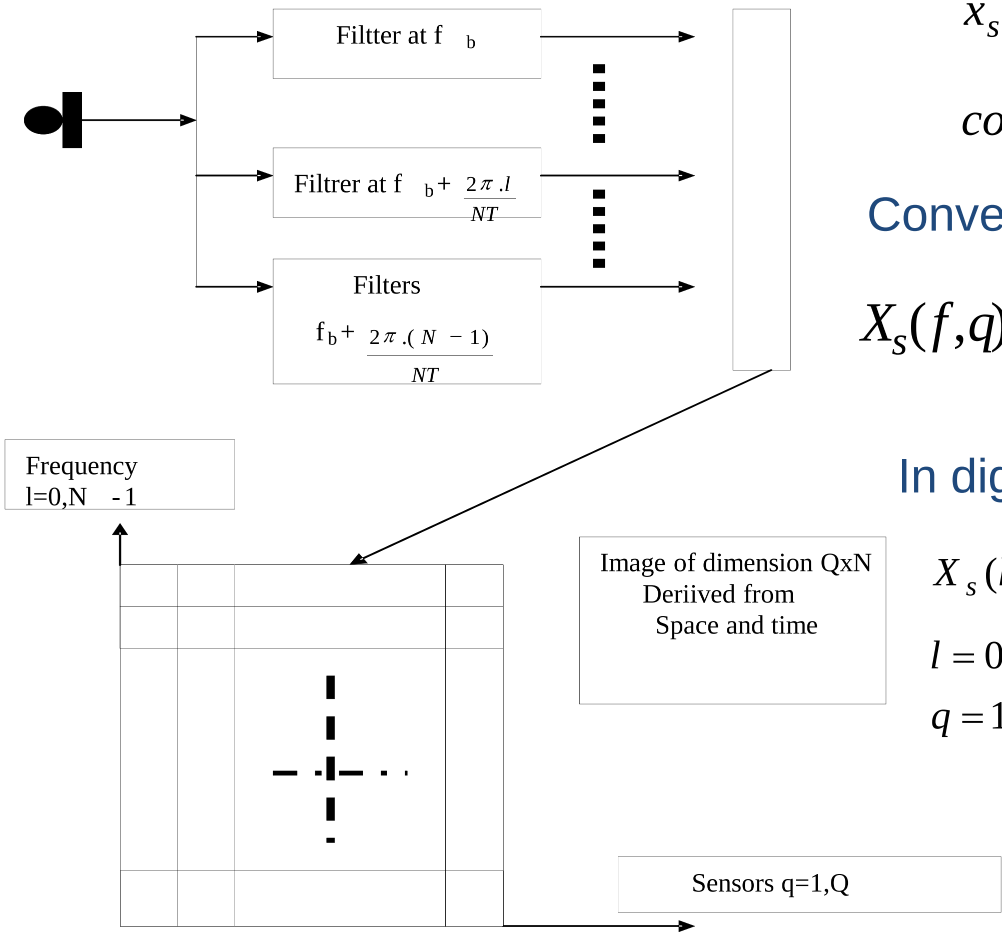
$$x_s(t,q) = x_s(t - t_o - \tau_{qs})$$

$$con \quad \tau_{qs} = d_q \frac{\mathrm{sen}(\theta_s)}{c}.\cos(\varphi_s - \varphi_q)$$

When the source signal is wideband, the proper processing, i.e. detection, location and selection, implies a processing in time and in space. In consequence the (diversity) beamformer exhibits a 2D response (image) where one dimension corresponds to time and the other to space.



# Wideband processing at the frequency domain




$$x_s(t,q) = x_s(t - t_o - \tau_{qs})$$

$$con \quad \tau_{qs} = d_q \frac{sen(\theta_s)}{c}.\cos(\varphi_s - \varphi_q)$$

Converted to narrowband by the filter-bank

$$X_s(f,q) = X_s(f).\exp(-j2\pi f.t_o).\exp(-j2\pi.f.\tau_{qs})$$

In digital filter-bank formulation

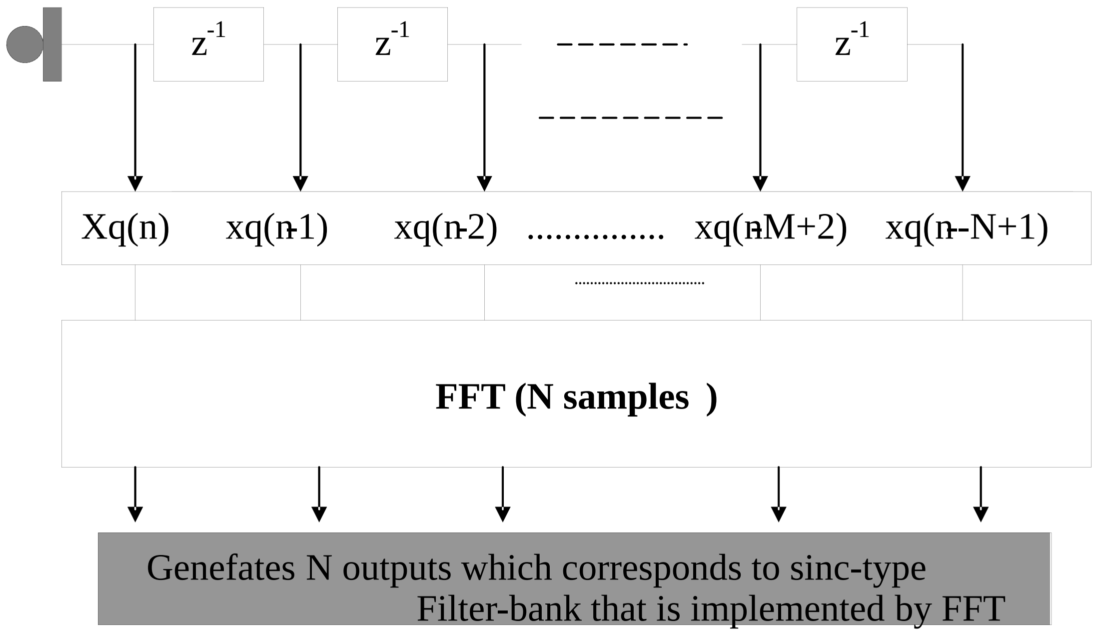
$$X_s(l,q) = X_s(l).\exp(-j2\pi l.\frac{t_o}{NT}).\exp(-j2\pi.l.\frac{\tau_{qs}}{N.T})$$

$$l = 0, N-1 \quad N\ bands$$

$$q = 1, Q \quad Q\ sensors$$

# When the number of bands is high (N large)

In this case, it is easy to use DFT (FFT) to convert the time dimension to the frequency dimension



# Time to frequency: Filter Bank or FFT processing?

- FB produce well isolated bands. Decay faster than 6 dB per octave.
- FB requires fine engineering and does not offers flexibility as its counter part FFT.
- FFT cheaper and easy to implement, mostly for large number of frequency bands. Produce orthogonal filters. The lowest frequency selectivity.
- FB includes FFT and a Poly-phase network
- FB produces delay according with the prototype filter used.
- FB may implement non-uniform frequency bands (Acoustics)

# How we determine the number of bands N, both in time or frequency?

Let us assume that the bandwidth of the source is B, then choosing B0 as the bandwidth for narrowband (???) processing then…

$$N = B / B0\Big|_{\text{round to integer}}$$

This implies that the time diversity is formed by N taps at B=1/T sampling rate. In frequency implies N filters with B0 bandwidth for filter-bank implementation and N samples FFT in the DFT choice.

Note that sometimes (radar, satellite, wireless communications) B0 can be close to B, in such a case, we do not need wideband processing. Staying in narrowband processing the beamformer pass to be a beam vector instead an image. IN SUMMARY, narrowband is much more easy to design, to implement, and cheaper than wideband.

The question is WHEN WE MAY ASSUME THAT WE STAY IN A NARROWBAND PROBLEM. In other words, how given a scenario we may know the B0 bandwidth mentioned before

# Narrowband/Wideband Array Processing

The wideband signal received is:

$$x_q(t) = x_s(t-\tau_{qs}) = a_s(t-\tau_{qs}).\exp(-j2\pi f_o(t-\tau_{qs}))$$

Note that whenever the complex envelope does not depend in the sensor location, the spatial diversity signature only will show up across the aperture, i.e. depending in the sensor location and in the narrowband carrier.

Furthermore, the overall vector of received signal, the so-called SNAPSHOT can be written in closed form as it is shown below:

$$\underline{X}_t = a_s(t).\underline{S}_s$$

$$a_s(t) = a_s(t).\exp(jw_0 t)$$

$$\underline{S}_s = \begin{bmatrix} u1 & .. & uq & uQ \end{bmatrix}$$

STEERING Vector

where

$$with\ uq = 2\pi f_0.d_q.\frac{\sin(\vartheta_s)}{c}\cos(\varphi_q - \varphi_s)$$

Thus the narrowband approach needs that: $$a_s(t-\tau_{qs}) \cong a_s(t)$$

In other words, we stay in the narrowband problem whenever the “GROUP DELAY” is neglectible.

Setting the above condition in frequency we have…..

$$A_s(w).\exp(-j.2\pi.f.\tau_{qs}) \cong A_s(w)$$

Thus this is equivalent to state that the phase load is smaller than one.

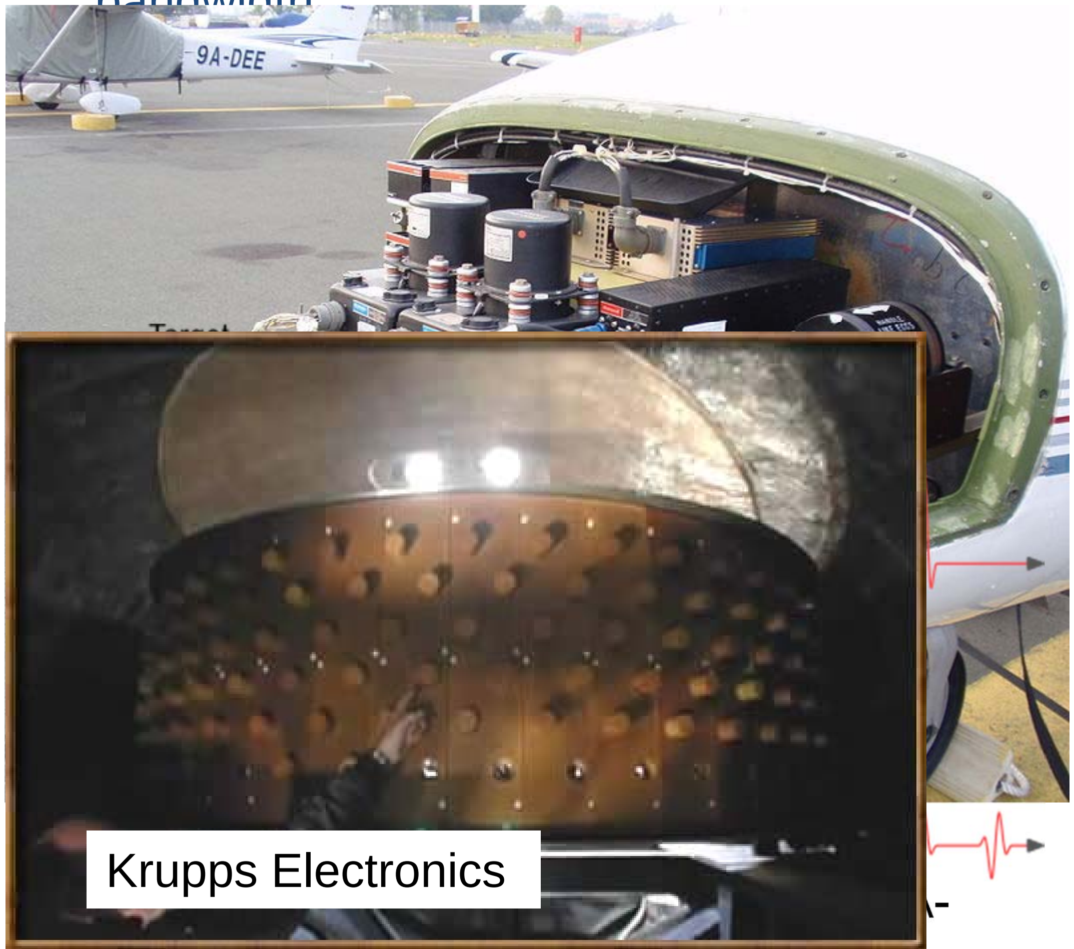
$$f.\tau_{qs} = f.\frac{d_q}{c}.\sin(\theta_s)\cos(\varphi_q-\varphi_s) =$$
$$= f\frac{d_q}{(\lambda_c f_c)}.\sin(\theta_s)\cos(\varphi_q-\varphi_s) << 1$$

Now let us take the worst case as listed below:

- Maximum of f (low pass envelope) which is the bandwidth of the signal B
- Maximum o d is the maximum diameter of the aperture D
- Maximum of sin and cos is one

$$\left(\frac{D}{\lambda_c}\right) << \left(\frac{f_c}{B}\right)$$

In a sentence: The narrowband problem requires that the maximum dimension of the aperture in wavelengths is smaller than the central frequency divide by its bandwidth




Radar: fc=9Ghz, B=1MHz
Lc=3,33 cm
D<<300 mts.
Satelite: fc=21Ghz, B=5MHz
Lc=1.43 cm.
D<<60 mts.
Audio: fc=2KHz. B=4KHz
Lc=0.7 mts
D<<0.35 mts
Narrowband Sonar: fc=200 Hz
B=15 Hz
Lc=7 mts
D<<94 mts.

Date September 2009
Source Own work
Author Dtom

# INDEX

# Narrowband Snapshot

In a narrowband analysis, every source produces a spatial signature formed by the product of the complex envelope by its corresponding steering vector. Thus, including the spatial noise (front-end and down conversion) the received snapshot will be:

$$\underline{X}_t = \sum_{s=1}^{NS} a_s(t).\underline{S}_s + \underline{n}_t$$

Spatial noise highly uncorrelated, unless special cases when distributed sources are present in the scenario

Nevertheless, behind this model is a fine engineering task, covering RF, Baseband, Antenna, etc.

Array calibrado

$$\underline{\underline{G}} = \begin{bmatrix} 1 & 0 & . & 0 \\ 0 & 1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & 1 \end{bmatrix} .g_o$$

Array calibrado (elementos no isotropicos)

$$\underline{\underline{G}} = \begin{bmatrix} 1 & 0 & . & 0 \\ 0 & 1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & 1 \end{bmatrix} .g_o(\theta,\varphi)$$

Array no calibrado (elementos isotropicos)

$$\underline{\underline{G}} = \begin{bmatrix} g_o & 0 & . & 0 \\ 0 & g_1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & g_Q \end{bmatrix}$$

Array no – calibrado (elementos no – isotropicos)

$$\underline{\underline{G}} = \begin{bmatrix} g_o(\theta,\varphi) & 0 & . & 0 \\ 0 & g_1(\theta,\varphi) & . & 0 \\ . & . & . & . \\ 0 & 0 & . & g_Q(\theta,\varphi) \end{bmatrix}$$

Array no – calibrado (elementos no – isotropicos) con acoplamiento mutuo no nulo

$$\underline{\underline{G}} = \begin{bmatrix} g_o(\theta,\varphi) & g_{01} & . & g_{0Q} \\ g_{10} & g_1(\theta,\varphi) & . & g_{1Q} \\ . & . & . & . \\ g_{Q0} & g_{Q1} & . & g_Q(\theta,\varphi) \end{bmatrix}$$

Reliability and fine engineering

Perfect

Equal non-isotropic elements

Not equal gain

Not equal antenna (non isotropic elements)

Mutual Coupling

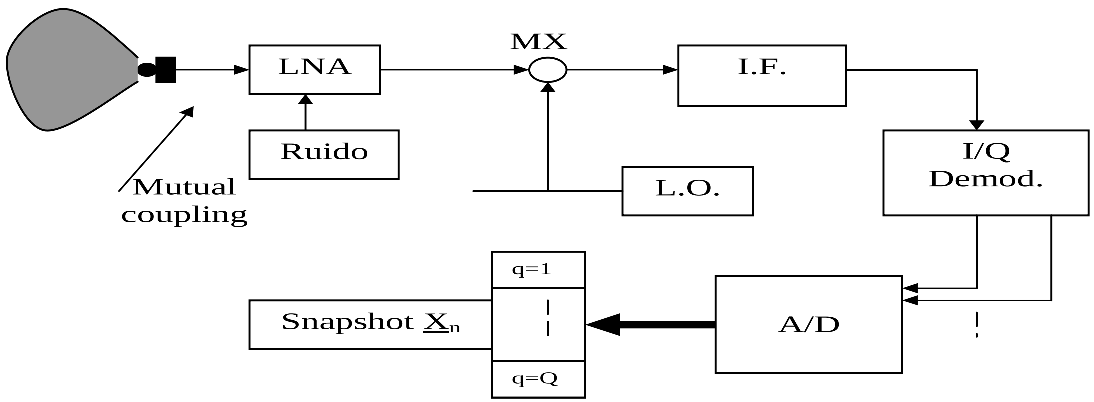
$$\underline{X}_t = \sum_{s=1}^{NS} a_s(t).\underline{\underline{G}}(\theta_s,\varphi_s).\underline{S}_s + \underline{w}_t$$

Non-Calibrated LNA $$E\left[w_q(t).\underline{w}_p^*(t)\right]=\sigma_q^2.\delta(q,p)$$

Perfect LNA $$\sigma_q^2=\sigma^2 \quad \forall q=1,Q$$

Distributed sources (i.e. Radar Clutter) $$\underline{X}_t=\sum_{s=1}^{NS}a_s(t).\int_{-\pi/2}^{\pi/2}f(\theta/s).\underline{S}_s d\theta+\underline{w}_t$$

Almost identical down-conversion chains
Self or unattended calibration
Proper I/Q separation
Few bits per band-pass sampling A/D conversion



# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# The array covariance

The snapshot can be written compactly using the DOA matrix and the source vector

$$\underline{X}_n = \underline{\underline{A}}.\underline{a}_n + \underline{w}_n$$

with

$$\underline{a}_n = \begin{bmatrix} a_1(n) \\ a_2(n) \\ . \\ a_{NS}(n) \end{bmatrix} \qquad \underline{\underline{A}} = \begin{bmatrix} \underline{S}_1 & \underline{S}_2 & . & \underline{S}_{NS} \end{bmatrix}$$

The covariance matrix explores all the auto and cross correlations across the array elements (relative delays)

$$r(q,p) = E\left[x_q(t).x_s^*(t)\right]$$

being $$\underline{X}_n = \underline{\underline{A}}.\underline{a}_n + \underline{w}_n$$

The array covariance matrix is formulated as… $$\underline{\underline{R}} = E\left[\underline{X}_n.\underline{X}_n^H\right]$$

Since

$$E\left[\underline{w}_n.\underline{w}_n^H\right] = \sigma^2.\underline{\underline{I}}$$

$$E\left[\underline{w}_n.\underline{a}_n^H\right] = \underline{\underline{0}}$$ ← always

$$E\left[a_s(n).a_r^*(n)\right] = P_s.\delta(s,r)$$ ← For un-coherent sources

then

$$\underline{\underline{R}} = \sum_{s=1}^{NS} P_s.\underline{S}_s.\underline{S}_s^H + \sigma^2.\underline{\underline{I}} = \underline{\underline{S}}.\underline{\underline{P}}.\underline{\underline{S}}^H + \sigma^2\underline{\underline{I}}$$

$$\underline{\underline{R}} = \underline{\underline{A}}.E\left[\underline{a}_n.\underline{a}_n^H\right].\underline{\underline{A}}^H + \sigma^2.\underline{\underline{I}} = \underline{\underline{A}}.\underline{\underline{P}}.\underline{\underline{A}}^H + \sigma^2.\underline{\underline{I}}$$

s

For un-coherent sources

being

$$= \left[\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_{NS}\right].\begin{bmatrix} P_1 & 0 & . & 0 \\ 0 & P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & P_{NS} \end{bmatrix}.\begin{bmatrix} \underline{S}_1^H \\ \underline{S}_2^H \\ . \\ \underline{S}_{NS}^H \end{bmatrix} + \sigma^2.\underline{\underline{I}}$$

$$\underline{\underline{R}} = \sum_{s=1}^{NS} P_s.\underline{S}_s.\underline{S}_s^H + \sigma^2.\underline{\underline{I}} = \underline{\underline{S}}.\underline{\underline{P}}.\underline{\underline{S}}^H + \sigma^2\underline{\underline{I}}$$

The signal sub-sapce is spanned by NS non-colinear vector. In consequence its rank is the number of sources in the scenario

The noise subspace is full rank equal to Q the number of sensors

# Estimate from the array snapshots the array covariance matrix

Block processing by averaging rank-one matrixes. Usually N>10Q makes no difference between actual and estimate matrix. In any case, convergence to the actual estimate is faster than in time-series processing.

$$\underline{\underline{\hat{R}}}_n = \frac{1}{N}\sum_{q=0}^{N-1} \underline{X}_{n-q} . \underline{X}_{n-q}^H$$

Adaptive estimate

$$\underline{\underline{\hat{R}}}_n = \beta . \underline{\underline{\hat{R}}}_{n-1} + (1-\beta) . \underline{X}_n . \underline{X}_n^H$$

The memory equivalent of this formulation (similar variance) for $N \approx \frac{1}{1-\beta}$

The solution, from a signal processing view, is the so-called SPATIAL SMOOTHING" that will restore the rank of the noise subspace

# INDEX

# The problem of coherent sources

Without loss of generality, let us assume un scenario with two sources which are coherent (Ex. Specular multipath satellite to sea surface

$$E\left[\left|s_1(t)\right|^2\right] = P_1$$
$$E\left[\left|s_2(t)\right|^2\right] = P_2$$
$$E\left[s_1(t).s_2^*(t)\right] = \rho.\sqrt{P_1.P_2}$$

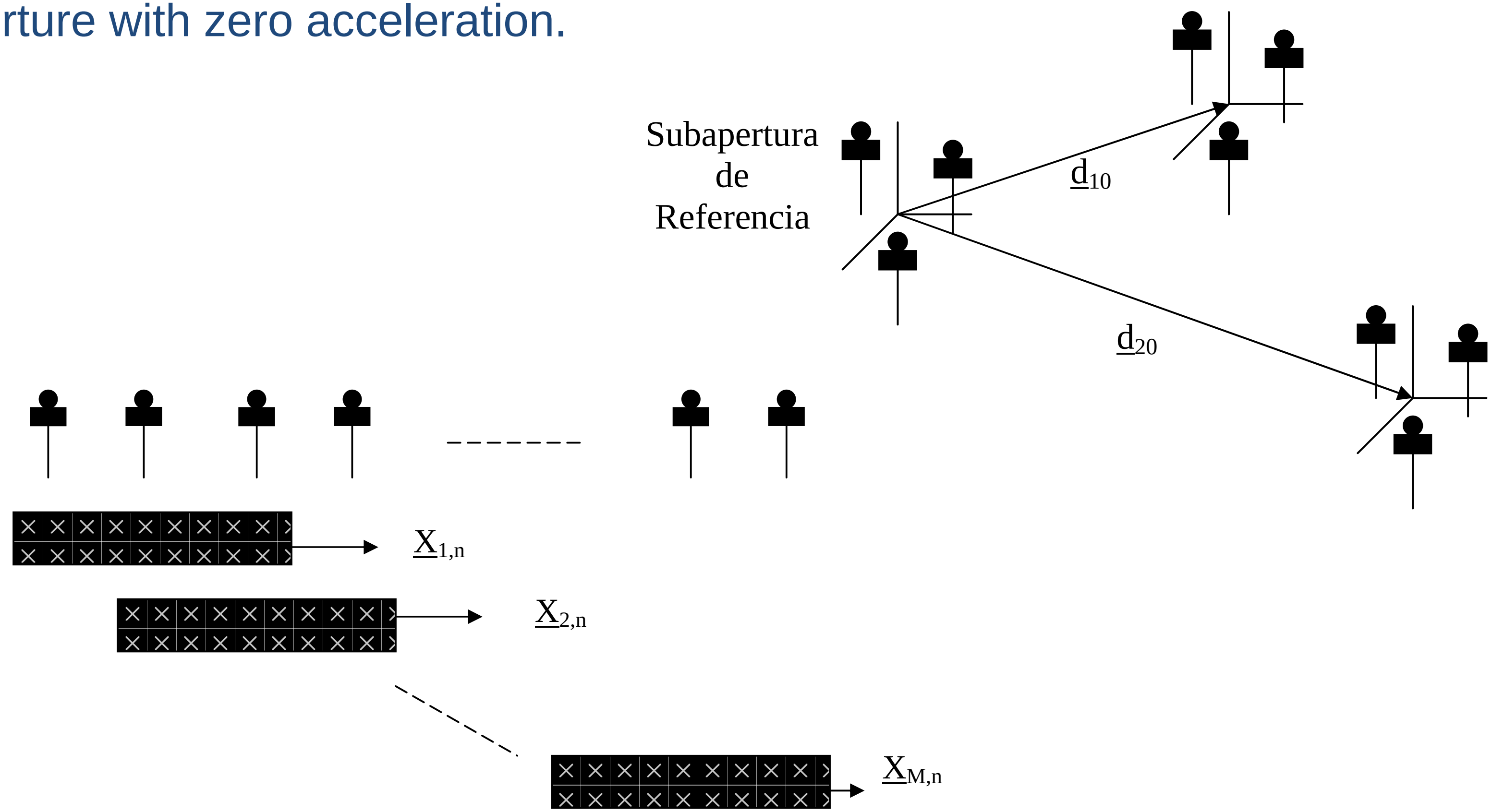
Matrix P is not longer diagonal and reflects off-diagonal elements for those sources that are coherent

$$\begin{bmatrix} P_1 & \rho\sqrt{P_1.P_2} & . & 0 \\ \rho^*\sqrt{P_1.P_2} & P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & P_{NS} \end{bmatrix}$$

The rank of this matrix is NS-1 (i.e. we underestimate the number of sources in the scenario. Basically the array considers that instead two sources, because they provide the same impinging signal is a single source

# Spatial Smoothing

Sub-arraying or with at least two twin apertures. This can be achieved by a mobile aperture with zero acceleration.




$\underline{\underline{R}}_1 = E\left[\underline{X}_{1,n}.\underline{X}_{1,n}^H\right]$ ; $\underline{\underline{R}}_2 = E\left[\underline{X}_{2,n}.\underline{X}_{2,n}^H\right]$ ; .......... ; $\underline{\underline{R}}_M = E\left[\underline{X}_{M,n}.\underline{X}_{M,n}^H\right]$

$$\underline{\underline{R}}_{SPATIAL\ SMOOTHING} = \frac{1}{M}.\sum_{m=1}^{M}\underline{\underline{R}}_m$$

For the case of two twin apertures, the first aperture computes its array covariance matrix, that contains the rank deficient source matrix

$$\underline{\underline{R}}_o = \left[\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_{NS}\right] \begin{bmatrix} P_1 & \rho & . & 0 \\ \rho & P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & P_{NS} \end{bmatrix} \begin{bmatrix} \underline{S}_1^H \\ \underline{S}_2^H \\ . \\ \underline{S}_{NS}^H \end{bmatrix} + \sigma^2 . \underline{\underline{I}}$$

For the second aperture, the difference is that the vector location for ALL the sensors increase on the vector <u>d</u> which is the distance between their phase centers.

$$\underline{r}_{q,1} = \underline{r}_{q,0} + \underline{d}_{10} \quad \Rightarrow \quad \underline{S}_{s,1} = \boxed{\exp(j.\underline{k}_s.\underline{d}_{10})}\, \underline{S}_s$$

In matrix form we have the following where factor gs is given by

$$\underline{\underline{R}}_1 = \left[\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_{NS}\right] \begin{bmatrix} g_1 & 0 & . & 0 \\ 0 & g_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & g_{NS} \end{bmatrix} \begin{bmatrix} P_1 & \rho & . & 0 \\ \rho & P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & P_{NS} \end{bmatrix} . \begin{bmatrix} g_1^* & 0 & . & 0 \\ 0 & g_2^* & . & 0 \\ . & . & . & . \\ 0 & 0 & . & g_{NS}^* \end{bmatrix} . \begin{bmatrix} \underline{S}_1^H \\ \underline{S}_2^H \\ . \\ \underline{S}_{NS}^H \end{bmatrix} + \sigma^2 . \underline{\underline{I}}$$

For the second aperture the covariance matrix is:

$$\underline{\underline{R}}_1 = \left[\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_{NS}\right] . \begin{bmatrix} P_1 & \rho.g_1.g_2^* & . & 0 \\ \rho.g_1^*.g_2 & P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & P_{NS} \end{bmatrix} . \begin{bmatrix} \underline{S}_1^H \\ \underline{S}_2^H \\ . \\ \underline{S}_{NS}^H \end{bmatrix} + \sigma^2 . \underline{\underline{I}}$$

Being

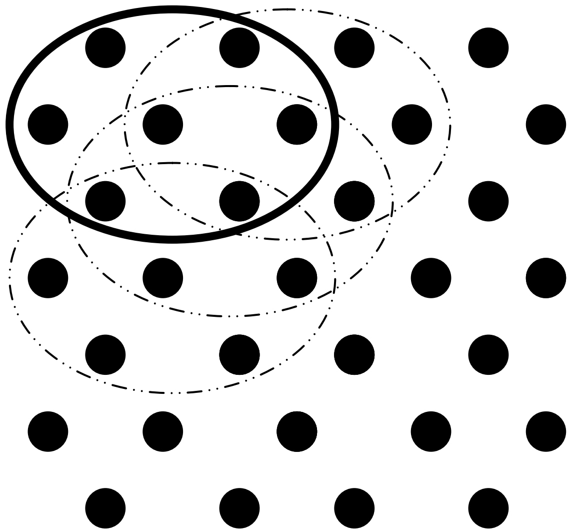
$$g_1.g_2^* = \left\{ \sum_{na=0}^{2-1} \exp\left[ j.\left(\underline{k}_1 - \underline{k}_2\right).\underline{d}_{na,0} \right] \right\} = \cos\left(\left(\underline{k}_1 - \underline{k}_2\right).\underline{d}_{na,0}\right) < 1$$

When adding the two matrixes

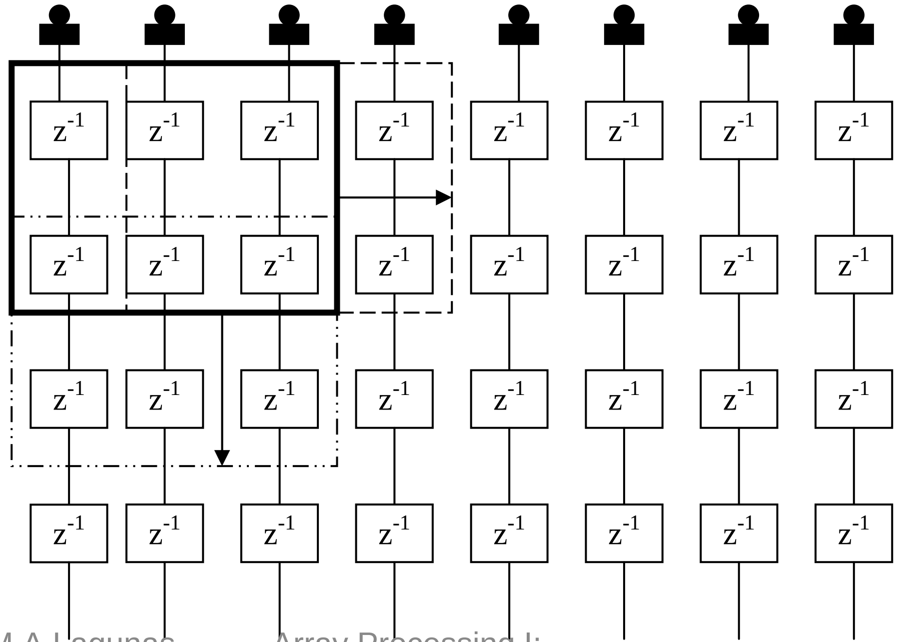
¡¡¡The range is restored!!!

$$\underline{\underline{R}}_1 = \left[\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_{NS}\right] . \begin{bmatrix} 2P_1 & \rho.(1 + g_1.g_2^*) & . & 0 \\ \rho.(1 + g_1^*.g_2) & 2P_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & 2P_{NS} \end{bmatrix} . \begin{bmatrix} \underline{S}_1^H \\ \underline{S}_2^H \\ . \\ \underline{S}_{NS}^H \end{bmatrix} + 2\sigma^2 . \underline{\underline{I}}$$

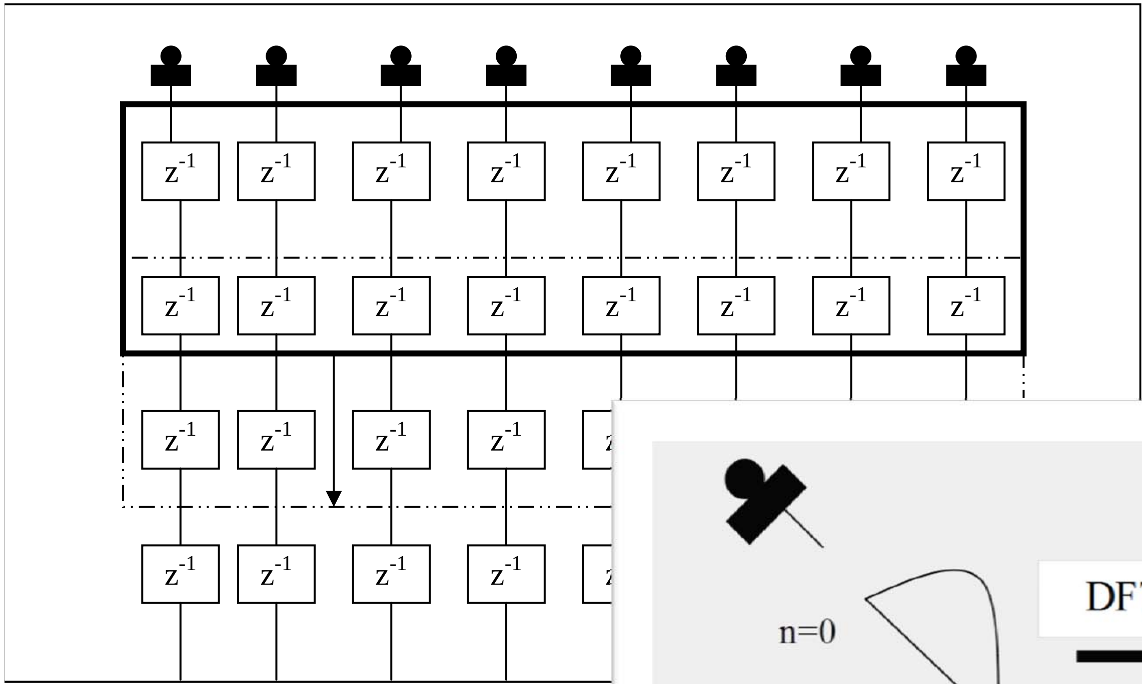
## Other example of self arraying

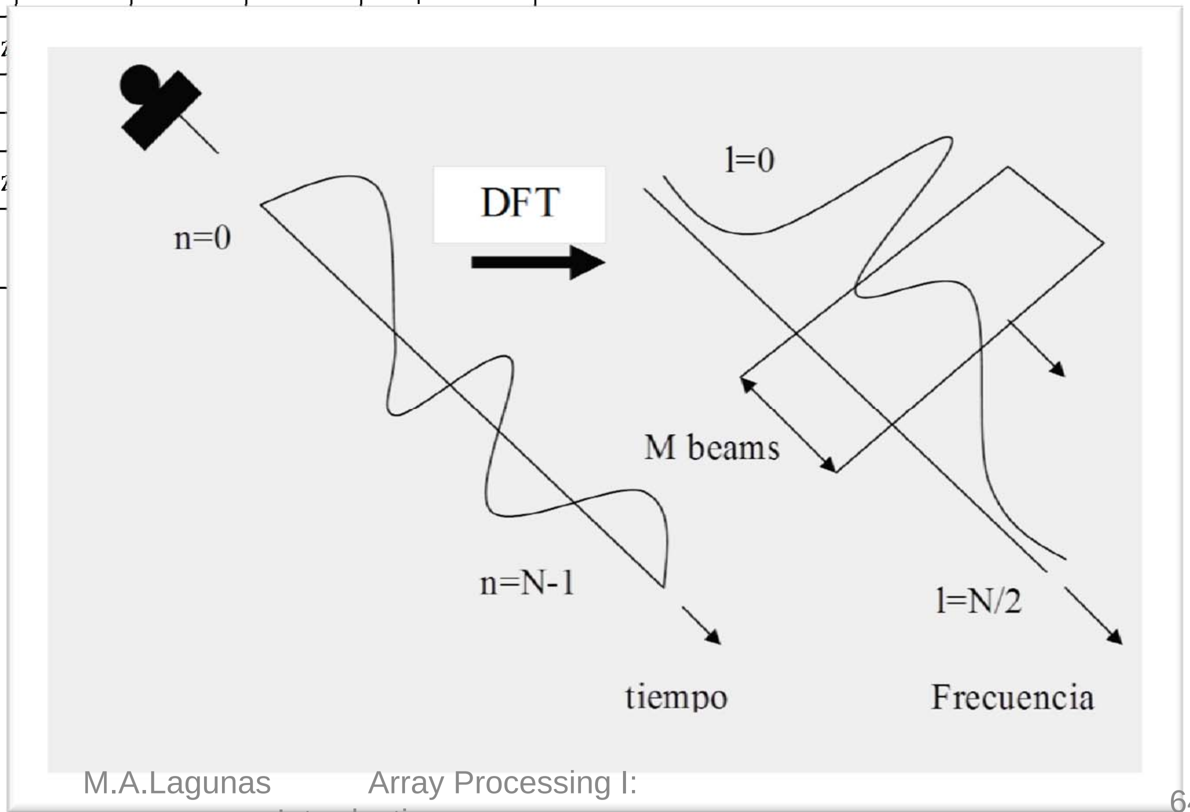


## Wideband sub-arraying



# The smoothing is easy to implement in wideband arrays

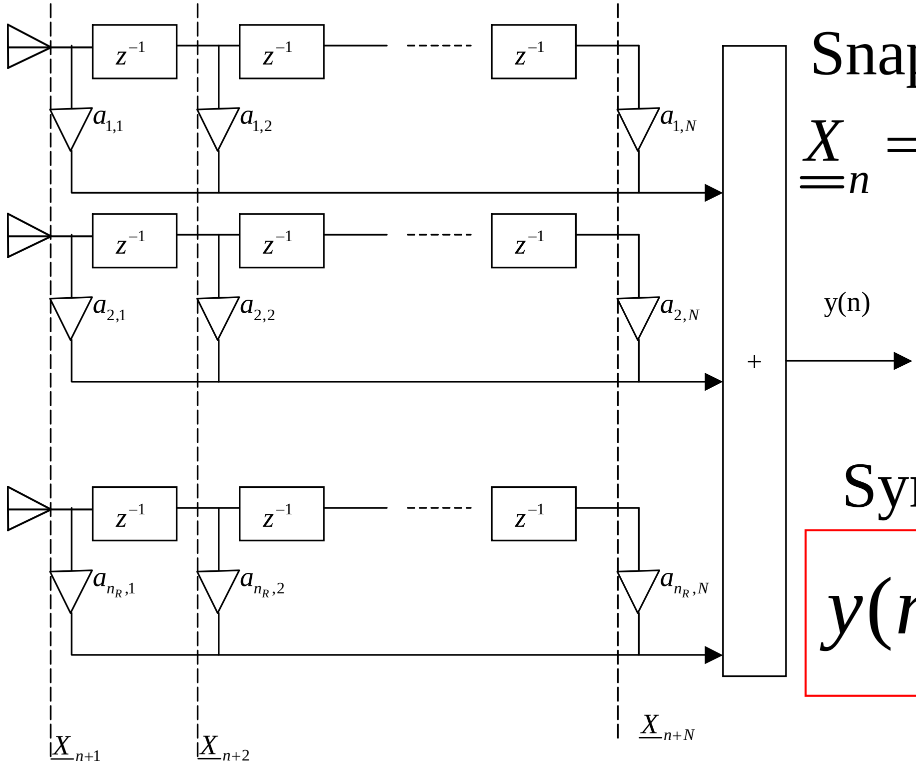


# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot

# Wideband beamforming formulation




Snapshot

$$\underline{\underline{X}}_n = \left[ \underline{X}_n \quad \underline{X}_{n-1} \quad .. \quad \underline{X}_{n-N+2} \quad \underline{X}_{n-N+1} \right]$$

Symbol output

$$y(n) = traza\left( \underline{\underline{A}}^H . \underline{\underline{X}}_n \right)$$

$$\underline{\underline{A}} = \left[ \underline{a}_1 \quad \underline{a}_2 \quad \dots \quad \underline{a}_N \right] = \begin{bmatrix} a_{1,1} & a_{1,2} & .. & a_{1,N} \\ a_{2,1} & a_{2,2} & .. & a_{2,N} \\ .. & .. & .. & .. \\ a_{n_R,1} & a_{n_R,2} & .. & a_{n_R,N} \end{bmatrix}$$

Receiver Matrix

Also another linear formulation can be used by defining the extended snapshot and beamformer as:

$$\underline{X}_{ne} = \begin{bmatrix} \underline{X}_n \\ .. \\ \underline{X}_{n-q} \\ .. \\ \underline{X}_{n-N+1} \end{bmatrix} \qquad \underline{A}_e = \begin{bmatrix} \underline{A}_1 \\ .. \\ \underline{A}_q \\ .. \\ \underline{A}_N \end{bmatrix}$$

In summary, the two formulations for wideband beamforming are:

$$y(n) = trace\left(\underline{\underline{A}}^H \underline{\underline{X}}_n\right) = \underline{A}_e^H . \underline{X}_{ne}$$

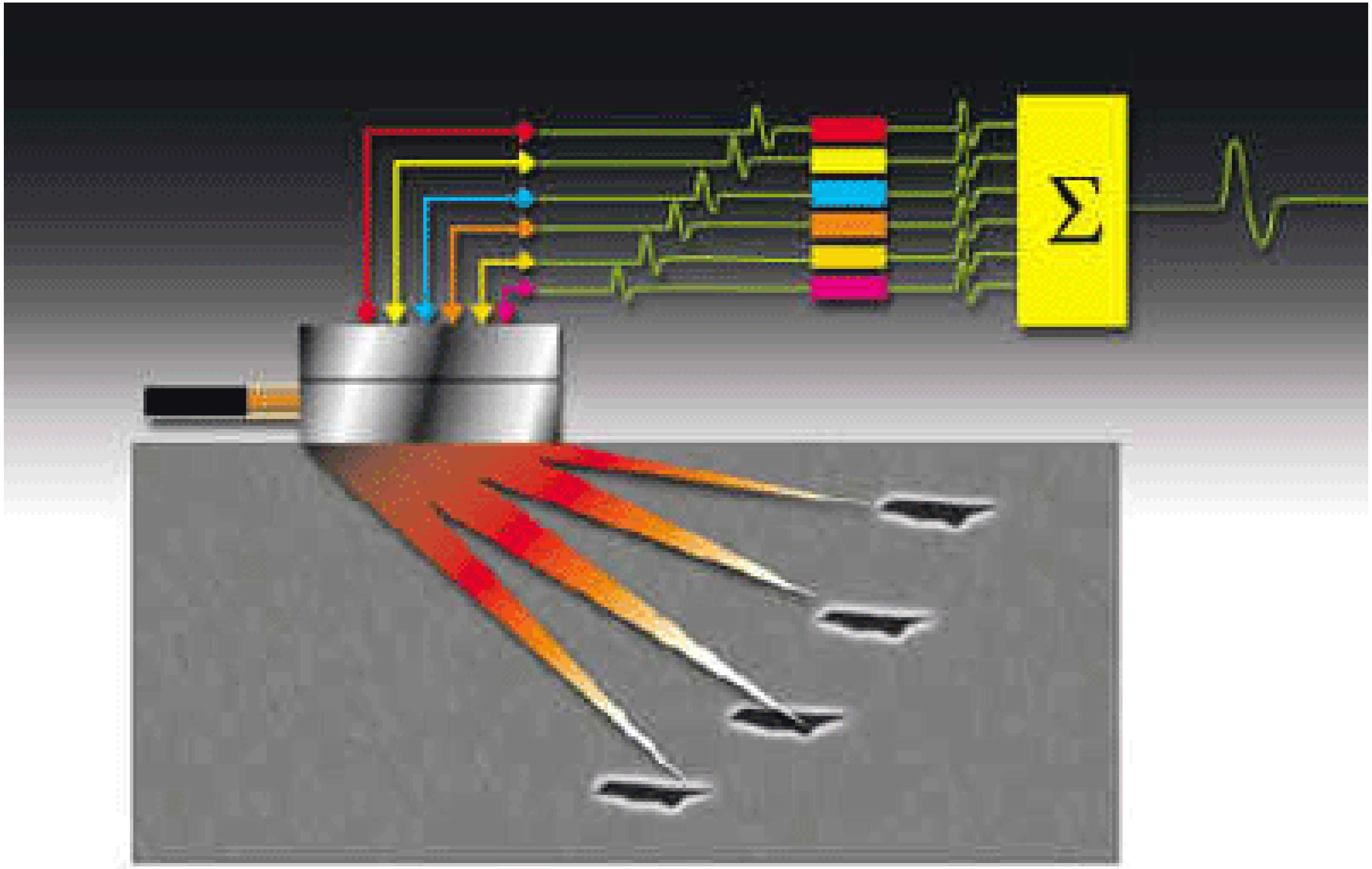
The steering vector, is defined for every frequency as the signature in the diversity plane of a single frequency source on the actual location of the wideband source

$$\phi_{sq} = \frac{w_m}{c} . d_q . \mathrm{sen}(\theta_s)$$

$$\begin{aligned} \underline{S}_{m,s} = [&1.\exp(j\phi_{s1}),.\exp(j\phi_{s1})\exp(-jw_m),..,.\exp(j\phi_{s1})\exp(-jw_m(M-1)), \\ &1.\exp(j\phi_{s2}),\exp(-jw_m).\exp(j\phi_{s2}),..,\exp(-jw_m(M-1)).\exp(j\phi_{s2}),....... \\ &......,1.\exp(j\phi_{sQ}),\exp(-jw_m).\exp(j\phi_{sQ}),..,\exp(-jw_m(M-1)).\exp(j\phi_{sQ}),]^T \end{aligned}$$

# INDEX

- Array examples
- Signal/Diversity processing
- Space Diversity versus time-diversity
- The space scenario
- Angles of arrival (DOA)
- Source and sensor signal: Significant parameters
- Slowness/Azimuth plane
- Wideband and Narrowband arrays
- The narrowband snapshot
- The array covariance
- Coherent sources: Spatial smoothing
- The wideband snapshot


Σ

## ARRAYS

EJERCICIO

Considere una apertura lineal  de 8 elementos y de separación uniforme de 2 metros. La frecuencia central de trabajo es de 4Kz y la velocidad de propagación de 320 metros por segundo.

1.- Determine el mayor ancho de banda de trabajo para que pueda considerarse un “array” trabajando en banda estrecha.

Si se desea trabajar en el margen de 10Hz a 8KHz. usando la DFT de la señal en cada sensor.

2.-  ¿Cual ha de ser la duración temporal del “snapshot” de banda ancha necesario para que cada “beam” de la FFT pueda considerarse un “snapshot” de banda estrecha?

Suponga que en el escenario aparece una fuente que incide a 30º sobre el “broadside” y una replica coherente con la misma amplitud a 60º del “broadside”. Al mismo tiempo, tan solo se esta interesado en el contenido que la fuente a 30º presenta a la frecuencia de 60Hz.

3.- Cual es la duración de señal a grabar con sensor para estimar la matriz de covarianza a del “snapshot” de banda estrecha a 60Hz.

4.- Indique cual es el “steering” o vector de enfoque de la fuente a 30º a 5 KHz en el elemento 4 tomando como centro de fase de la apertura el elemento 1.

Considerando que los vectores de enfoque no cambian sustancialmente del la frecuencia de 60 Hz a la frecuencia de 80 Hz. (es decir del “beam” 6 de la FFT al “beam”  8) y que las matrices de covarianza estimadas a ambos son las que siguen:

$$\underline{\underline{R}}_{60Hz} = \underline{\underline{S}}\begin{bmatrix} 9 & 6 \\ 6 & 9 \end{bmatrix}\underline{\underline{S}}^H + 0.15\underline{\underline{I}} \qquad \underline{\underline{R}}_{80Hz} = \underline{\underline{S}}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\underline{\underline{S}}^H + 0.2\underline{\underline{I}}$$

5.- Demuestre que al promediar ambas matrices (“spatial smooting” en banda ancha) la matriz de covarianza resultante tiene el rango igual al numero de angulos de llegada, es decir, dos uno para la deseada y otro para la coherente. Es decir, se restituye el rango.

1.- $\lambda = \frac{c}{f} = \frac{320}{4000} = 80.10^{-3}$ $\frac{16}{80}10^3 << \frac{4000}{Bw} \Rightarrow Bw << 20Hz.$

2.-

La frecuencia de muestreo ha de ser mayor que 16 KHz. Tomaremos 20 KHz. de muestreo. Para que cada columna del snapshot transformado por la DFT ha de tener un ancho de banda menor que 20 Hz, tomaremos 10 Hz. de resolución para la FFT. Esto daría un numero de muestras N según sigue:

$$N = \frac{f_{maz}}{Bw} = \frac{20000}{10} = 2000 \qquad Duracion = 2000\frac{1}{f_{muestreo}} = 100\ mseg.$$

3.-

Cada snapshot de banda ancha requiere 100 mseg. La estimación de la matriz de covarianza en banda esctrecha requiere al menos 10 Q snapshots, es decir 80 snapshots. Tomaremos 100 en total.

$$100 snpahots\ de\ banda\ estrecha \Rightarrow 100 snpahots\ de\ banda\ ancha = 100.100.10^{-3} = 10 seg.$$

4.- El elemento cuatro esta separado una distancia de 3.d=6mts. del centro de fase.

$$s_{30^\circ,5KHz}(4) = \exp\left(-j\frac{2\pi.60}{320}6.\sin(30^\circ)\right) = \exp(-j.3{,}53)$$

5.- Al sumar, la matriz resultante es:

$$\underline{\underline{R}}_{60,80Hz} = \underline{\underline{S}}\begin{bmatrix} 11 & 8 \\ 8 & 6 \end{bmatrix}\underline{\underline{S}}^H + 0.35\underline{\underline{I}}$$

como el determinante de la matriz central es 66-64=2 diferente de cero el rango de la matriz del subespacio de señal es 2. Notese que la SNR de cada fuente pasa de 17.78 dB y 16 dB. Para deseada y coherente a 14.97 dB y 13,59 dB en la matriz promedio, es decir el precio a restituir rango conlleva un deterioro leve en la SNR de las fuentes.

# Ejercicio
# Banda ancha/ Banda Estrecha

- La estructura de señal recibida
- Definición de banda estrecha
- Impacto de dimensión máxima de la apertura en la condición de banda estrecha
- El "snapshot" de banda estrecha
- "Beamformer" para banda estrecha
- "Beamformer" para banda ancha (space-time, space frequency)
- Validación experimental de que nos encontramos en banda estrecha